Representations, Not Revolutions: Czachor's Calculus and Bell's Theorem

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We examine recent claims that "non-Newtonian" arithmetic and calculus topple Bell's theorem. Our basic point regarding such a claim is straightforward: the expectation functional used in those papers is linear only with respect to the deformed sum $\oplus$, not the ordinary $+$. Consequently, the familiar Clauser–Horne and Clauser–Horne–Shimony–Holt derivations — which lean on linearity under ordinary addition — do not apply. Within a single arithmetic level, a Bell-type *analogue* can be formulated *if* the outcomes and expectation values are defined in that level and satisfy linearity with respect to the level's addition $\oplus$; however, the standard Clauser–Horne/Clauser–Horne–Shimony–Holt proof for "+" is inapplicable. The eye-catching "beyond-Tsirelson" effects show up only when levels are mixed — thus, one computes with standard rules on quantities defined in a deformed calculus, producing out-of-range *aggregates* (e.g., totals exceeding one) rather than single-event probabilities. The touted "relativity of observed probabilities" also splices together two different moves, conditioning on a restricted sample space versus pushing everything through a scalar remapping. A simple horizon toy model already shows that there is no single-valued remapping that accomplishes this globally. The analogy with Einstein velocity addition helps a little in one dimension; in three dimensions, it collapses. There, the composition is non-commutative and non-associative, and the right language is a gyrogroup structure, not a pullback of ordinary addition. Bringing in Lambare's measurement-independence critique, we further argue that Czachor's reply (built from a hand-tuned bijection and a non-additive integral) addresses neither that objection nor Bell's own premises. In short, the program amounts to a representational re-encoding, not a counterexample of local hidden-variable.

topics: Bell and Clauser–Horne–Shimony–Holt (Bell-CHSH), non-Newtonian arithmetic, deformed expectation, level mixing

1. Introduction

For some time, Czachor has promoted a new paradigm, which he calls *non-Newtonian calculus* and *generalized arithmetics*, arguing that standard probability and analysis should be replaced by a hierarchy of level-indexed calculi and operations [1, 2]. In this picture, ordinary addition, multiplication, differentiation, and integration are not fundamental at all but artefacts of a particular "language" (conventionally, $\ell = 0$). As a mnemonic, think of $\ell = 0$ as the textbook or lab-notebook dialect. Quantum "paradoxes" — including the violation of Bell inequalities and the appearance of Tsirelson bounds — are said to stem from a confusion of languages, i.e., from modeling and aggregating empirical data at the incorrect arithmetic level [3]. If, instead, one consistently adheres to an appropriate level and its matching calculus, it is claimed, these paradoxes more or less disappear. Czachor casts this as an "arithmetic relativity" move, a rhetoric deliberately echoing the shift from Euclidean to non-Euclidean geometry [2] — an ambitious comparison, perhaps intentionally so.

In the current paper, we take a careful look at that program, following the outline presented in [3]. We concentrate on two pillars that carry the strongest conclusions. For orientation, we outline these pillars below before proceeding to the detailed analysis:

(i) the use of a *non-additive* integral (non-additive with respect to ordinary "+", yet additive relative to a level-dependent operation $\oplus_\ell$) [1]; and

(ii) the construction of a hierarchy of mutually isomorphic arithmetics R_ℓ, each level tied to the standard one via a bijection [2].

We argue that these design choices relocate the analysis *outside* the probabilistic framework assumed by Bell, and that the headline effects — including the touted "beyond-Tsirelson" values — appear only when one *mixes* levels (i.e., reads level-ℓ objects using level-0 rules) [4–6]. Within any fixed level, an appropriate Clauser–Horne/Clauser–Horne–Shimony–Holt-type *analogue* can be formulated if linearity holds with respect to $\oplus_\ell$; the standard inequalities under "+" do not carry over automatically. In that sense, the program is a representational reparameterization of probabilities, not a Bell-local hidden-variable model in the usual (Kolmogorov) sense [7]. For related beyond-Tsirelson claims in the same vein, see also [8].

Our critique is not about pathological bijections. In the works under review, the required maps are explicitly bijective (often monotone), and the internal algebra is tuned so that familiar laws survive within each level [3]. For ease of reference, the main sticking points are as follows:

(i) a departure from Kolmogorov additivity and linearity of expectation under ordinary "+" [7];

(ii) an equivocation between 'locality' in the redefined calculus and Bell-locality (factorizability) in standard probability [4–6];

(iii) a reliance on reparameterized angular differences to secure rotational invariance;

(iv) the lack of an operational story about how laboratory devices would implement $\oplus_\ell$ aggregation rather than ordinary addition. (That, the authors think, matters.)

2. Main themes in Czachor's paper [2], with brief critical notes

To guide the reader, the following list synthesizes the paper's central theses and our concise critical remarks on each of them.

(i) Thesis: Bell's theorem as a "confusion of languages."
Critique: This reframes the premises rather than refuting Bell. Since the expectation in the paper is linear only with respect to a deformed sum $\oplus$, not ordinary "+", the usual CH/CHSH bounds (which rely on linearity under "+") are inapplicable; any "violations" lie *outside* Bell's framework [2–6].

(ii) Analogy with Einstein velocity addition.
Critique: This is valid only in 1D (rapidity). In 3D the Einstein composition is noncommutative and non-associative (Thomas–Wigner rotation), so it cannot arise from any global pullback $f^{-1}(f(u) + f(v))$. It is not a universal template for "how nature adds" [9, 10].

(iii) Relativity of observed probabilities (moving observers/black holes).
Critique: The mapping $p \mapsto g_r(p)$ is a change of *representation*, not of labolatory arithmetic. Treating both p and $g_r(p)$ as operational probabilities requires monotonicity, endpoint conditions and a device-level aggregation rule — however, none of these are provided in [3]. A finite-N horizon bound is given in Remark 2 (see Sect. 5.4.1).

(iv) Section 3 — Lemma on binary probabilities.
Critique: The identity $g(p) + g(1 - p) = 1$ is correct but minimal and underdetermines g. The specific $g(p) = \sin^2\left(\frac{\pi}{2}p\right)$ is an *ansatz* imported from the Born/Fubini-study side, not a prediction of the hierarchy [3, 11].

(v) Projective (pulled-back) arithmetic for events.
Critique: Requires *dual normalization* of the same data (once with ordinary "+" and once with projective $\oplus_X$), creating a two-additions operational gap. Experiments sum counts with "+"; no mechanism is given for $\oplus_X$ at the device level [3].

(vi) Non-Newtonian (non-additive) integral.
Critique: By construction, the integral is linear only with respect to $\oplus$, not "+". This already places the model outside Kolmogorov additivity, so standard Bell/CH/CHSH derivations do not apply [1, 7].

(vii) Hierarchy of levels and "Copernican" lemmas.
Critique: Within a fixed level one can *formulate* level-consistent Bell-type inequalities provided linearity under the level's addition holds; but no *selection principle* is given for which level (or which g) nature uses [2].

(viii) Bell/CH at a fixed level vs mixing levels.
Critique: The paper's own results show that violations arise only from *level-mixing* (reading level-ℓ objects with level-0 rules). This is a representation mismatch, not new physics [3].

(ix) Beyond–Tsirelson scenarios.
Critique: Out-of-range *aggregates* (e.g., totals exceeding 1 or CH/CHSH algebraic combinations that behave as if outside $[0, 1]$) appear only when level-ℓ quantities are aggregated with ordinary "+". They do not arise within a fixed level; they are artifacts of mixing calculi [8].

(x) Applications/implications (cryptography, "arithmetic relativity").
Critique: Speculative without a realizable $\oplus$-aggregating device or testable departures from quantum mechanics (QM). "Locality" is redefined (commutativity) rather than shown as Bell-factorization $P(A, B|a, b, \lambda) = P(A|a, \lambda)\, P(B|b, \lambda)$ on a Kolmogorov space [3, 6].

3. On equations (1)–(4) and the "Einstein addition" analogy

Either something is authentic or it is unauthentic, it is either false or true, make-believe or spontaneous life; yet here we are faced with a prevaricated truth and an authentic fake, hence a thing that is at once the truth and a lie.[†1]

3.1. The "Einstein addition" analogy

Czachor's starting point (i.e., (1)–(4) in [1]) defines a pulled-back arithmetic by means of a bijection $f\colon \mathbb{R} \to \mathbb{R}$:

$$\begin{aligned} x \oplus_f y &= f^{-1}\big(f(x) + f(y)\big), \\ x \ominus_f y &= f^{-1}\big(f(x) - f(y)\big), \\ x \odot_f y &= f^{-1}\big(f(x)f(y)\big), \\ x \oslash_f y &= f^{-1}\big(f(x)/f(y)\big) \qquad \text{(with } f(y) \neq 0). \end{aligned} \tag{1}$$

(and analogously on other sets X, Y bijective with $\mathbb{R}$).[†2] This yields an arithmetic $R_f = \{\mathbb{R}, \oplus_f, \ominus_f, \odot_f, \oslash_f\}$ *isomorphic* to the standard one via f. In particular, $\oplus_f$ and $\odot_f$ are *commutative and associative*, and multiplication distributes over addition, since f is an isomorphism onto $\{\mathbb{R}, +, -, \cdot, /\}$.[†3]

Czachor then suggests that the "laws of nature (addition)" might follow a general formula patterned on relativistic composition of velocities. Even if one chooses $f(\beta) = \operatorname{arctanh}(\beta)$ — so that

$$\beta_1 \oplus_f \beta_2 = \tanh\big(\operatorname{arctanh}(\beta_1) + \operatorname{arctanh}(\beta_2)\big) = \frac{\beta_1 + \beta_2}{1 + \beta_1\beta_2}, \tag{2}$$

— the identification is *not* physically sound for the following reasons.

- The 1D case constitutes a special reparameterization rather than a general law. For collinear velocities, the above is merely *rapidity* addition, i.e., a benign change of variables tailored to the 1D setting. It says nothing about higher dimensions or about non-velocity observables [9].
- In 3D the Einstein velocity composition is neither commutative nor associative. Let $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w} \in \mathbb{R}^3$ be (non-collinear) velocities with $c = 1$. Working with metric $\operatorname{diag}(+1, -1, -1, -1)$, the pure boost by $\boldsymbol{u}$ has the block form

$$\Lambda(\boldsymbol{u}) = \begin{pmatrix} \gamma_u & \gamma_u \boldsymbol{u}^{\mathrm{T}} \\ \gamma_u \boldsymbol{u} & I + (\gamma_u - 1)\, \hat{\boldsymbol{u}} \cdot \hat{\boldsymbol{u}}^{\mathrm{T}} \end{pmatrix}, \tag{3}$$

where $\hat{\boldsymbol{u}} = \boldsymbol{u}/\parallel \boldsymbol{u} \parallel$ and $\gamma_u = 1/\sqrt{1 - \parallel \boldsymbol{u} \parallel^2}$. Applying $\Lambda(\boldsymbol{u})$ to $U' = (\gamma_v, \gamma_v \boldsymbol{v})$ yields

$$\begin{aligned} U^0 &= \gamma_u \gamma_v \Big(1 + \boldsymbol{u} \cdot \boldsymbol{v}\Big), \\ \boldsymbol{U} &= \gamma_u \gamma_v \Big(\boldsymbol{u} + \boldsymbol{v}_\parallel + \frac{\boldsymbol{v}_\perp}{\gamma_u}\Big), \end{aligned} \tag{4}$$

so the observed 3-velocity is

$$\boldsymbol{u} \oplus_E \boldsymbol{v} = \frac{\boldsymbol{u} + \boldsymbol{v}_\parallel + \boldsymbol{v}_\perp/\gamma_u}{1 + \boldsymbol{u} \cdot \boldsymbol{v}}. \tag{5}$$

Here, $\gamma_u = (1 - \parallel \boldsymbol{u} \parallel^2)^{-1/2}$ and we split $\boldsymbol{v}$ into parallel and perpendicular parts relative to $\boldsymbol{u}$, respectively, by

$$\boldsymbol{v}_\parallel = \frac{\boldsymbol{u} \cdot \boldsymbol{v}}{\parallel \boldsymbol{u} \parallel^2}\, \boldsymbol{u} \quad (\boldsymbol{u} \neq 0), \quad \text{and} \quad \boldsymbol{v}_\perp = \boldsymbol{v} - \boldsymbol{v}_\parallel, \tag{6}$$

with the convention $\boldsymbol{v}_\parallel = 0$ when $\boldsymbol{u} = 0$. Following this convention, (5) reduces to $\boldsymbol{u} \oplus_E \boldsymbol{v} = \boldsymbol{v}$ for $\boldsymbol{u} = 0$. Using the identity $1 - 1/\gamma_u = \parallel \boldsymbol{u} \parallel^2 \gamma_u/(1 + \gamma_u)$, one can rewrite the numerator as $\boldsymbol{u} + \boldsymbol{v}/\gamma_u + \gamma_u(\boldsymbol{u} \cdot \boldsymbol{v})\, \boldsymbol{u}/(1 + \gamma_u)$, showing that (5) is equivalent to a standard textbook form.

In general, $u \oplus_E v \neq v \oplus_E u$ and $(u \oplus_E v) \oplus_E w \neq u \oplus_E (v \oplus_E w)$ (Thomas–Wigner rotation). Because we pull back *scalar* addition via scalar bijection f, any $x \oplus_f y = f^{-1}(f(x) + f(y))$ is commutative and associative by construction; thus, no such scalar f can reproduce the genuine 3D Einstein law [9, 10].

- The correct algebraic setting is a gyrogroup, not a pulled-back sum. Relativistic velocity space in 3D forms a *gyrogroup*. It is gyrocommutative/gyroassociative with a gyration $\operatorname{gyr}[u, v]$ (the Wigner rotation) such that

$$u \oplus_E (v \oplus_E w) = (u \oplus_E v) \oplus_E \operatorname{gyr}[u, v](w). \tag{7}$$

A global pullback of $+$ necessarily erases this rotational content [10].
- There is a category error across observables. Velocity composition is a kinematic group law tied to Lorentz symmetry and *rapidity*. It is not a template for adding arbitrary physical quantities: (i) energy–momentum adds *linearly* as a four-vector (no Einstein-like nonlinearity in conservation laws); (ii) counts and probabilities in laboratories are aggregated by ordinary $+$, which underwrites the law of large numbers and the affine steps in CH/CHSH derivations [5, 7, 9].
- The framework provides no selection principle for f. Equation (1) holds for *any* bijection f, generating infinitely many non-Newtonian additions, all mutually isomorphic. The framework offers no empirical or structural criterion that singles out the "Einstein" choice beyond the collinear velocity niche. Indeed, the same paper [3] invokes an "arithmetic relativity" in which no level is privileged, thus undercutting the idea that nature does privilege a specific f.
- There is an operational mismatch involving two distinct addition rules. In actual experiments, data aggregation (frequencies, counts, normalization) is done with ordinary $+$. The construction

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[†1] The quote comes from the book [12] (originally published in Polish in 1971).

[†2] See (1)–(4) and context in [3].

[†3] In the paper [3] itself, it is noted that the isomorphism carries over commutativity/associativity, e.g., $f(x \oplus_f y) = f(x) + f(y)$ (equivalently, $x \oplus_f y = f^{-1}(f(x) + f(y))$).

itself from [3] later requires the same probabilities to satisfy *both* non-Newtonian normalization (with $\oplus_f$) and standard normalization (with $+$) to match the observed frequentist addition. In the absence of a device-level mechanism that makes apparatuses compute via $\oplus_f$ rather than $+$, the identification remains representational, not physical [1–3].

From the above we conclude as follows. Equation (1) formalize a *reparameterization* of arithmetic, not a derivation of physical addition laws. Identifying the pulled-back $\oplus_f$ with relativistic velocity composition is viable only in the collinear case (rapidity addition) and fails in the physically generic 3D setting, where the Einstein composition is non-abelian and non-associative. As a universal account of "how nature adds," the proposal is therefore *incorrect* and, operationally, conflicts with how laboratory data are actually aggregated.

3.2. The 1D case: What Damski's (1+1)D analysis implies for pulled–back arithmetic

Damski [13] parametrizes the (1+1)D Lorentz transformations by a two-component parameter $U = (U_0, U_1)$ (not the physical four-velocity), with branches corresponding to subluminal and superluminal sectors; in standard conventions the orbits sit on unit-hyperbola branches whose sign depends on the chosen metric signature. For subluminal u, v (with $u = U_1/U_0$ and $c = 1$), the composition reduces to the standard Einstein formula $u' = (u - v)/(1 - uv)$, equivalently, the additivity of rapidity [9]. *Note:* this $u' = (u - v)/(1 - uv)$ is the velocity *transformation* into a frame moving at v; composing two co-directed velocities uses $(u + v)/(1 + uv)$. Damski further shows that some proposed superluminal extensions fail the group property, whereas particular sign-branched forms do satisfy it once a "clockwork" postulate fixes time orientation [13].

Implication for the f-pullback $x \oplus_f y = f^{-1}(f(x)+f(y))$ are as follows. With $f(u) = \mathrm{artanh}(u)$ on $(-1, 1)$, the pulled–back sum reproduces exactly the 1D Einstein addition; this is an ordinary change of value (rapidity) and is physically correct in the subluminal, collinear case. However, Damski's (1+1)D analysis shows that extending to superluminal frames requires *branch choices and sign conventions* to preserve group structure; a single global bijection $f : \mathbb{R} \to \mathbb{R}$ cannot encode these branches by the pullback rule alone. In practice, one needs at least a piecewise f (or multiple charts) plus explicit orientation rules (to be exact, the additional structure that Damski explicitly presents). Therefore, the "Einstein analogy" for $f^{-1}(f(x)+f(y))$ is valid, but strictly limited to the 1D subluminal sector; it is not a universal template even in 1D once superluminal kinematics is admitted [13]. (For additional analysis see also [14].)

4. Pathological bijections and why an admissibility condition is necessary

The pullback scheme $x \oplus_f y = f^{-1}\left(f(x) + f(y)\right)$ and its matching calculus only behave like a viable "change of language" if f has enough regularity. Without it, the derivative, integral, and even basic normalization either degenerate or decouple from laboratory procedures [15]. This section isolates the minimal assumptions that prevent pathologies and exhibits two failures when they are dropped.

Definition 1. (Admissible bijection) A bijection $f : I \to f(I) \subseteq \mathbb{R}$ (with $I \subseteq \mathbb{R}$ an interval) is admissible if:

- (i) f is strictly monotone and onto $f(I)$;
- (ii) f is absolutely continuous on I;
- (iii) (local regularity) for every compact $J \Subset I$ there exist $0 < m_J \leq M_J < \infty$ with $m_J \leq f'(x) \leq M_J$ for almost every $x \in J$;
- (iv) (closure (sufficient for totality on the chosen domain)) $f(I)$ is closed under ordinary $+$ (i.e., $f(I) + f(I) \subseteq f(I)$), which guarantees that $x \oplus_f y$ is defined for all $x, y \in I$. Alternatively, one may restrict to a maximal $I' \subseteq I$ for which $f(I')$ is closed under $+$ and hence $\oplus_f$ is everywhere-defined on I'.

Under (i)–(iii), the change of variables is nonsingular on compacta, the chain rule holds almost everywhere (a.e.), and the pullback derivative/integral agree with the usual ones up to the f reparameterization.

Remark 1. (Why Definition 1 is not cosmetic) Conditions (ii)–(iii) exclude *singular* monotone maps (continuous, increasing, but with $f'(x) = 0$ a.e.) and wild homeomorphisms. Such maps push Lebesgue measure to a singular measure, so "uniform density" in the f calculus becomes concentrated on a null set in x. The fundamental theorem of calculus in the deformed language then fails in the most literal way. The (sufficient) closure item ensures $\oplus_f$ is globally defined on the chosen domain.

4.1. Pathology 1: Singular (Cantor-type) maps and measure-mismatch in differentiation

Let $C : [0, 1] \to [0, 1]$ be the (Devil's staircase) Cantor distribution function: continuous, nondecreasing, with $C'(x) = 0$ for $\mathrm{d}x$-a.e. x, and let $\mathrm{d}C$ denote the (singular) Borel measure it induces. Interpreting the "deformed derivative" via the Radon–Nikodym (RN) derivative under the change of variable $r = C(x)$, we define

$$\frac{\mathrm{D}A}{\mathrm{D}x} := \frac{\mathrm{d}\,(A \circ C)}{\mathrm{d}C} \qquad \text{(defined } \mathrm{d}C\text{-a.e. } x). \tag{8}$$

Since $\mathrm{d}C \perp \mathrm{d}x$ and $C'(x) = 0$ for $\mathrm{d}x$-a.e. x, we simultaneously have, for $A \in C^1$,

$$\frac{\mathrm{d}\,(A \circ C)}{\mathrm{d}x} = 0 \quad \text{for } \mathrm{d}x\text{-a.e. } x,$$
$$\frac{\mathrm{d}\,(A \circ C)}{\mathrm{d}C} = A'(C(x)) \quad \text{for } \mathrm{d}C\text{-a.e. } x. \tag{9}$$

Thus, the fundamental theorem/chain rule with respect to $\mathrm{d}x$ fails for $A \circ C$ (because $\mathrm{d}C$ is singular to $\mathrm{d}x$), even though the RN chain rule holds *with respect to* $\mathrm{d}C$. The pathology is therefore due to the *singular change of variable* $x \mapsto C(x)$; mixing differentiation rules tied to different base measures ($\mathrm{d}x$ vs $\mathrm{d}C$) creates a premise-shift that yields spurious "effects."[†4]

Example (identity). Taking $A = \mathrm{id}$ gives

$$\frac{\mathrm{D}\,\mathrm{id}}{\mathrm{D}x} = \frac{\mathrm{d}C}{\mathrm{d}C} = 1 \quad (\mathrm{d}C\text{-a.e. } x),$$
$$\frac{\mathrm{d}\,(\mathrm{id} \circ C)}{\mathrm{d}x} = \frac{\mathrm{d}C}{\mathrm{d}x} = 0 \quad (\mathrm{d}x\text{-a.e. } x). \tag{10}$$

Hence the statement "$\frac{\mathrm{D}\,\mathrm{id}}{\mathrm{D}x} = 1$" is true in the RN/$\mathrm{d}C$ sense, but not as a $\mathrm{d}x$-a.e. statement; conflating the two notions of "derivative" is precisely the error.

4.2. Pathology 2: Smooth but unphysical f yields incorrect kinematics (or fails closure)

Even when f is C^1, the pullback can be physically meaningless without a *selection principle* and closure. Take $I = [-1, 1]$ and $f(\beta) = \beta^3$. Then $f(I) = [-1, 1]$, but for $\beta_1 = \beta_2 = 0.9$ we have $f(\beta_1) + f(\beta_2) = 2 \cdot 0.9^3 = 1.458 \notin f(I)$, so $\beta_1 \oplus_f \beta_2$ is undefined, i.e., the operation is not closed on I. Extending I to $\mathbb{R}$ to regain closure, then

$$\beta_1 \oplus_f \beta_2 = f^{-1}\left(f(\beta_1) + f(\beta_2)\right) = \left(\beta_1^3 + \beta_2^3\right)^{1/3}, \tag{11}$$

and the same numbers give $(0.9^3 + 0.9^3)^{1/3} = (0.729 + 0.729)^{1/3} = 1.458^{1/3} \approx 1.1336 > 1$, since $0.9^3 = 0.729$ and $1.458^{1/3} \approx 1.1336$. Thus, a subluminal pair ($|\beta_1| = |\beta_2| = 0.9$) composes to a superluminal value under this f, which is physically incorrect. By contrast, the rapidity map $f(\beta) = \mathrm{artanh}(\beta)$ (with $I = (-1, 1)$) reproduces Einstein's addition exactly in 1D. Hence, without regularity and a symmetry/operational selection principle (plus closure), the calculus can "predict" arbitrary outcomes by tuning f, undermining physical content.

[†4]See Cantor's worked-out example and the discussion of the measure mismatch underlying the $\mathrm{D}A/\mathrm{D}x$ claims in our comment preprint [15].

4.3. Diagnostic: An f-admissibility checklist for physics claims

Before reading any Bell/relativity/thermo claim in a deformed calculus, use the following quick checklist:

a. Regularity (local): Is f absolutely continuous with $0 < m_J \leq f' \leq M_J$ a.e. on each compact $J \Subset I$? If not, expect singular derivatives/integrals and loss of normalization.
b. Closure (sufficient): Is $f(I)$ closed under ordinary "+" (or has the domain been restricted so that $\oplus_f$ is everywhere-defined)?
c. Symmetry: Does f implement the correct group parameter (e.g., rapidity for boosts)? If not, expect incorrect composition laws (e.g., superluminal sums).
d. Operational bridge: Does any device-level aggregation rule realize $\oplus_f$ in the laboratory, or are the data still added with ordinary "+"?

The summary of the section is as follows. The "generalized arithmetic" program must, at minimum, restrict to admissible f in the sense of Definition 1. Singular (Cantor-type) maps make the deformed calculus ill-defined; smooth but unconstrained maps yield empirically false laws or fail closure. In the absence of both regularity and a symmetry/operational selection principle, the framework is not a physical theory but a representational relabeling scheme.

5. Why Czachor's main conclusion about Bell's theorem is incorrect

What actually fails in the premise audit? We distinguish two failure modes of Czachor's construction. *Physically decisive* is the violation of Bell factorizability in the standard Kolmogorov calculus (see (27) in Sect. 5.5). By contrast, the *nonlinearity of expectations* arises only inside the deformed hidden-layer calculus and serves as a diagnostic for why the usual CH/CHSH algebraic derivations do not go through (see Sect. 5.4.1). In what follows, we treat factorizability as the primary locality premise and nonlinearity as the secondary, algebraic reason CH/CHSH proofs fail here (see also Sect. 5.4).

5.1. Sketch of Czachor's local-realist model of singlet correlations

5.1.1. Ingredients

Choose a strictly monotone bijection $f : \mathbb{R} \to \mathbb{R}$ and define a *non-Diophantine* (deformed) arithmetic on the same underlying set

$$x \oplus y := f^{-1}\big(f(x) + f(y)\big),$$
$$x \odot y := f^{-1}\big(f(x)\, f(y)\big). \tag{12}$$

Define the associated (non-Newtonian) integral so that the fundamental theorem of calculus holds in the deformed system

$$\int_{x_1}^{x_2} \mathrm{D}x\ A(x) = f^{-1}\bigg(\int_{f(x_1)}^{f(x_2)} \mathrm{d}r\ (f \circ A \circ f^{-1})(r)\bigg). \tag{13}$$

This identity holds provided that f satisfies the regularity needed for a well-defined pullback of Lebesgue integration and the fundamental theorem (strict monotonicity and absolute continuity on its domain with locally bounded derivative, together with a domain in which $f(I)$ is closed under $+$ so that $\oplus$ is defined); see Definition 1.

We now recall how the non-Newtonian ("deformed") integral is defined in Czachor's formalism. Let $f : \mathbb{R} \to \mathbb{R}$ be a strictly monotone bijection generating the deformed arithmetic. Following Czachor, we define the deformed integral by

$$\int^f \mathrm{d}^f\lambda\ \varphi(\lambda) := f^{-1}\left(\int \mathrm{d}\lambda\ f\big(\varphi(\lambda)\big)\right). \tag{14}$$

This equation is the definition of the deformed (non-Newtonian) integral.

It is important to stress that, except when f is affine, the map $\varphi \mapsto \int^f \mathrm{d}^f\lambda\ \varphi(\lambda)$ is *not linear* in φ. In particular, for generic non-affine f,

$$\int^f \mathrm{d}^f\lambda\ (\varphi_1+\varphi_2) \neq \int^f \mathrm{d}^f\lambda\ \varphi_1(\lambda) + \int^f \mathrm{d}^f\lambda\ \varphi_2(\lambda). \tag{15}$$

Thus in general there is no countably additive measure μ_f on the λ-space such that $\int^f \mathrm{d}^f\lambda\ \varphi(\lambda) = \int \mathrm{d}\mu_f(\lambda)\ \varphi(\lambda)$ for all bounded measurable φ; any such representation would necessarily be linear in φ. The deformed integral is therefore best understood as a *non-linear expectation functional*

$$E_f[\varphi] := f^{-1}\big(E[f(\varphi)]\big), \tag{16}$$

where $E[\cdot]$ is the ordinary expectation with respect to the underlying measure on λ (for instance, the uniform measure on S^1).

In Czachor's hidden-variable model on $\Lambda = S^1$, the joint quantities, which are called "probabilities", are then obtained by applying this non-linear functional to the products of the indicator functions and the density. Writing $\chi_i(a,\lambda)$ and $\chi_j(b,\lambda)$ for the local response functions and $\rho(\lambda)$ for the hidden-variable density, one has

$$P_{ij}(a,b) := \int^f \mathrm{d}^f\lambda\ \rho(\lambda)\,\chi_i(a,\lambda)\,\chi_j(b,\lambda) =$$
$$f^{-1}\left(\int \mathrm{d}\lambda\ f\big(\rho(\lambda)\,\chi_i(a,\lambda)\,\chi_j(b,\lambda)\big)\right). \tag{17}$$

The inner Lebesgue integral here is completely standard; all the deformation is carried by the outer application of f^{-1}. This observation will be important later when we emphasize that the hidden-variable part of the model lives on an ordinary Kolmogorov probability space, while non-linearity only enters through the final "reading" of that model back into quantities interpreted as probabilities.

5.1.2. Hidden variable space and locality

The hidden variable λ is an angle on the unit circle, $\lambda \in [0, 2\pi)$, with a distribution that is uniform in the $\mathrm{D}\lambda$ sense. Precisely, let $r = f(\lambda)$ and define $\mathrm{D}\lambda$ as the pullback of Lebesgue measure $\mathrm{d}r$ under f, i.e., for any integrable A,

$$\int \mathrm{D}\lambda\ A(\lambda) := \int \mathrm{d}r\ (A \circ f^{-1})(r). \tag{18}$$

"Uniform in $\mathrm{D}\lambda$" means that the density ρ is constant with respect to $\mathrm{D}\lambda$ (equivalently, the pushforward $f_\#(\rho\,\mathrm{D}\lambda)$ is the normalized Lebesgue measure in the r-variable). For analyzer settings α (party 1) and β (party 2), deterministic local response functions split the circle into outcome arcs via characteristic functions $\chi^{(1)}_{\alpha,\pm}(\lambda)$ and $\chi^{(2)}_{\beta,\pm}(\lambda)$, depending only on (α,λ) and (β,λ), respectively.

5.1.3. Joint probabilities in the deformed calculus

Joint probabilities are computed within a Clauser–Horne (CH) formulation, but using the deformed product/integral. For clarity, we write the deformed CH expression explicitly before giving its consequences. Thus, for $j,k \in \{+,-\}$, one has

$$p'_{jk}(\beta-\alpha) = \int D\lambda\ \ \chi^{(1)}_{\alpha,j}(\lambda) \odot \chi^{(2)}_{\beta,k}(\lambda) \odot \rho(\lambda), \tag{19}$$

with ρ being the uniform density with respect to $\mathrm{D}\lambda$.

5.1.4. Choice of f (the matching condition)

To recover the singlet statistics for a relative angle $\Delta = \beta-\alpha$, f is fixed by the mapping from (ordinary) arc-length ratios on the circle to the *target* quantum probabilities,

$$f^{-1}\Big(\frac{\theta}{2\pi}\Big) = \frac{1}{2}\sin^2\Big(\frac{\theta}{2}\Big), \qquad \theta \in [0,\pi]. \tag{20}$$

(Concrete realizations use a piecewise-smooth inverse expressed via arcsin; the exact form is not needed for the sketch.)

5.1.5. Result

With the uniform $\mathrm{D}\lambda$-measure and f satisfying (20), one obtains the singlet joint probabilities

$$p'_{++} = p'_{--} = \tfrac{1}{2}\sin^2\left(\tfrac{\Delta}{2}\right),$$
$$p'_{+-} = p'_{-+} = \tfrac{1}{2}\cos^2\left(\tfrac{\Delta}{2}\right), \tag{21}$$

hence the correlation

$$E(\Delta) = p'_{++} + p'_{--} - p'_{+-} - p'_{-+} = -\cos(\Delta). \tag{22}$$

The marginals are $p'_{+\bullet} = p'_{-\bullet} = \frac{1}{2}$ and $p'_{\bullet+} = p'_{\bullet-} = \frac{1}{2}$ (no-signalling at the laboratory level). Indeed, for each local setting the $+$ and $-$ arcs functions occupy equal Dλ-measure, and since the push-forward via f^{-1} acts uniformly, these 1/2 marginals remain setting-independent after mapping back.

5.1.6. *Why Bell derivations don't go through (pointer)*

Standard CH/CHSH proofs use ordinary linearity and factorization in *Kolmogorov* calculus. Here, linearity holds with respect to $\oplus$, and the products are $\odot$ under Dλ. When translating back to standard probabilities using f^{-1}, the joint distribution reproduces (21) and thus *cannot* satisfy Bell factorizability (see Sect. 5.5).

5.2. What Bell's theorem actually assumes (standard form)

Throughout Sect. 5, we treat non-factorization in standard calculus as a core physical failure. Nonlinearity in the deformed calculus is an algebraic obstruction explaining the breakdown of CH/CHSH derivations.

For clarity, we collect the standard premises used in deriving the CH/CHSH inequalities:

(i) Measurement independence, $\rho(\lambda|x,y) = \rho(\lambda)$.
(ii) Locality (factorizability), $P(a,b|x,y,\lambda) = P(a|x,\lambda)\,P(b|y,\lambda)$.
(iii) Kolmogorov probability and linear expectation, i.e., ρ is finitely additive (countable additivity in the full Kolmogorov framework) and $\mathbb{E}[X+Y] = \mathbb{E}[X] + \mathbb{E}[Y]$.

Under (i)–(iii) one derives the CHSH bound [4–6]

$$S = \mathbb{E}[A_0B_0] + \mathbb{E}[A_0B_1] + \mathbb{E}[A_1B_0] - \mathbb{E}[A_1B_1] \tag{23}$$

which satisfies the condition $|S| \le 2$.

5.3. What Czachor changes

Czachor replaces ordinary addition/expectation by a level-dependent calculus (via a bijection f), and/or redefines "locality" so that the standard *factorizability* need not hold on a single σ-algebra. He also introduces nonlinear maps $g(\cdot)$ on marginal probabilities (binary lemma), and horizon/observer maps g_r that conflate conditioning with scalar transformation [1–3].

Immediate consequence of Czachor's change is as follows. If you alter (2) *or* (3), the CHSH derivation does not go through. Exceeding 2 with deformed addition or with a redefined "locality" does *not* refute Bell's theorem; it shows you are no longer in its assumptions [5, 6].

5.4. Two precise failure points (each is enough to void the CHSH proof)

Lemma 1. Factorizability $\Rightarrow$ CH/CHSH; hence singlet $\Rightarrow$ non-factorization). Assume standard (Kolmogorov) probability with the ordinary $+$ and $\times$, measurement independence, and Bell factorizability (see (27)). Then, the Clauser–Horne/Clauser–Horne–Shimony–Holt inequalities follow. Therefore any model reproducing the singlet correlations (e.g., $E(\Delta) = -\cos(\Delta)$) must violate (27) in standard calculus.

Proof sketch. By Bell's inequality (27), one constructs a single joint distribution for all counterfactual outcomes, or applies Fine's theorem to obtain CH/CHSH inequalities. Since the singlet statistics violate these inequalities, at least one premise fails; here the premises of standard probability and measurement independence are maintained at the level of observed frequencies, so the failing assumption must be factorizability. The nonlinearity in the deformed hidden-layer calculus explains why the usual algebra cannot be carried out within that formalizm, yet nevertheless the physical violation remains precisely the lack of Bell's inequality once probabilities are read in the ordinary sense.

Lemma 2. Nonlinearity of the deformed expectation breaks CHSH inequalities. The step that fails without linearity under the ordinary "+" is the identification of

$$S = \sum_{x,y} s_{xy}\,\mathbb{E}[A_xB_y] = \mathbb{E}\Big[\sum_{x,y} s_{xy}\,A_xB_y\Big], \tag{24}$$

where $s_{00} = s_{01} = s_{10} = +1$ and $s_{11} = -1$. The pointwise bound $|A_0(B_0{+}B_1) + A_1(B_0{-}B_1)| \le 2$ does not require linearity, and $\mathbb{E}[2] = 2$ follows from the preservation of constants; but relating the pointwise bound to S via the above identification *does* require linearity under the ordinary "+" (and, in some presentations, the triangle inequality/Jensen to handle absolute values). Hence, any $|S| > 2$ obtained with a non-additive expectation indicates the failure of this identification, not a refutation of Bell. Neither monotonicity nor preservation of constants, nor convexity/concavity alone, suffices; one needs additivity (at least affine linearity) in the ordinary sense for this step.

5.4.1. Concrete diagnostics (drop-in checks)

We draw attention to the following two issues:s

- Additivity check (probability/expectation). If the paper uses a nonlinear g on $[0,1]$ (e.g., $g(p) = \sin^2(\frac{\pi}{2}p)$) and treats both p and $g(p)$ as probabilities on the same event space, then for a 3-outcome partition $\{A, B, C\}$, one has
$$g\left(P(A \cup B)\right) \stackrel{?}{=} g\left(P(A)\right) + g\left(P(B)\right). \quad (25)$$
This generally fails (e.g., when $P(A) = 0.2$, $P(B) = 0.3$, $g(0.5) = 0.5 \neq g(0.2) + g(0.3) \approx 0.302$), so Kolmogorov additivity is broken. The CHSH proof — which *requires* linearity — cannot be invoked [5, 7].
- Locality check (factorizability vs horizon/representation). If Bob's probabilities are produced by horizon truncation/conditioning, they live on a different σ-algebra (or under a different measure). Writing $\tilde{p} = g_r(p)$ treats a change of measure as a scalar map on $[0,1]$. Then the joint probabilities $P(a, b|x, y, \lambda)$ are not defined/combined on a single common space with factorizability — again, it is outside Bell's assumptions [3].

Remark 2. Horizon bounds, finite N. Let N trials contain $S = pN$ successes and let the observer see a fraction $r = m/N$. Over all orderings with the same (S, N), the visible count S_{vis} ranges from $\max\{0, S - (N-m)\}$ to $\min\{S, m\}$. Hence
$$\tilde{p} = \frac{S_{\text{vis}}}{m} \in \Big[\frac{\max\{0, S - (N-m)\}}{m}, \frac{\min\{S, m\}}{m}\Big], \quad (26)$$
with endpoints realized by packing successes fully after/before the horizon. For integer constraints, interpret the endpoints with $\lceil\cdot\rceil/\lfloor\cdot\rfloor$ as appropriate.

5.4.2. Assumption audit (Bell–CHSH vs Czachor): Point list

To keep a narrative linear, we first list the audit points. The nested bullets under each item clarify the role in CH/CHSH framework and indicate how the reviewed papers [1–3] modify it. Thus:

(i) Kolmogorov probability and linear expectation
- Standard: a single probability space (Λ, Σ, ρ); finite additivity suffices for CHSH (countable additivity in the full Kolmogorov framework); linearity $\mathbb{E}[X+Y] = \mathbb{E}[X] + \mathbb{E}[Y]$ [7].
- Used in CHSH: to pass from $|S|$ to $\mathbb{E}[\,|A_0(B_0+B_1)+A_1(B_0-B_1)|\,] \leq \mathbb{E}[2] = 2$ [5].
- Czachor changes: deformed arithmetic via a bijection f and a nonlinear map $g(\cdot)$ on $[0,1]$; the induced expectation is non-additive in $\mathbb{R}_0$ [1, 3].
- Consequence/diagnostic: the CHSH derivation is no longer invalid; test additivity on a 3-outcome partition, where typically $g(P(A \cup B)) \neq g(P(A)) + g(P(B))$.

(ii) Locality as factorizability
- Standard: $P(a, b|x, y, \lambda) = P(a|x, \lambda)\, P(b|y, \lambda)$ on the *same* (Λ, Σ, ρ) [6].
- Used in CHSH: to define joint terms and bound S under local hidden variables [5].
- Czachor changes: "locality" weakened to complement-preserving/no-signalling relations on marginals; horizons/observers live on different σ-algebras [3].
- Consequence/diagnostic: outside Bell's assumptions; ask: can all four setting pairs be put on a common space with factorizability? If not, CHSH is inapplicable.

(iii) Conflation of sample spaces (horizon/conditioning)
- Standard: observer change modeled as a *kernel* $P \mapsto \tilde{P}$ (conditioning on a sub-σ-algebra).
- Used in CHSH: requires one common event field for all probabilities [4].
- Czachor changes: asserts a scalar map $\tilde{p} = g_r(p)$ and treats both as "physical probabilities" at the same level [3].
- Consequence/diagnostic: category error; verify with the $r = \frac{1}{2}$ two-block counterexample: same $p = \frac{1}{2}$, but $\tilde{p} \in \{0, 1\}$.

(iv) Non-identifiability of g_r from p alone
- Standard: if $\tilde{p} = g_r(p)$, then g_r must be single-valued given (p, r).
- Used in CHSH: implicit in treating $\tilde{p}$ as a function of the scalar parameter p.
- Czachor changes: $\tilde{p}$ depends on time-profile/censoring, not just p.
- Consequence/diagnostic: no unique g_r; bounds in Remark 2 are sharp.

(v) Measurement independence
- Standard: $\rho(\lambda|x, y) = \rho(\lambda)$ [4].
- Used in CHSH: at the outset to define a setting-independent hidden-variable distribution [5].
- Czachor changes: typically not addressed/changed.
- Consequence/diagnostic: even if maintained, the previous deviations already void CHSH applicability.

5.5. Locality, factorizability, and why the deformation matters

Bell's "local causality" is usually captured by factorizability

$$P(A,B\,|\,a,b,\lambda) = P(A\,|\,a,\lambda)\,P(B\,|\,b,\lambda), \tag{27}$$

equivalently Jarrett's split [16–20] into parameter independence (PI) and outcome independence (OI). Czachor's construction is *deterministic and setting-local at the level of response functions* ($A = A(a,\lambda)$, $B = B(b,\lambda)$), so PI holds at the level of mappings $\lambda \mapsto A, B$. However, the probability calculus composing the local responses is not the Kolmogorov product measure. Instead, it is a deformed product $\odot$ accompanied by a deformed integral $\mathrm{D}\lambda$. In that internal calculus one has a *formal* factorization

$$P_f(A,B\,|\,a,b,\lambda) = P_f(A\,|\,a,\lambda) \odot P_f(B\,|\,b,\lambda), \tag{28}$$

but after translating results to laboratory probabilities via f^{-1}, the joint distribution $P(A,B|a,b)$ fails to factorize as in (27) with the *ordinary* product.

Because experimental frequencies and statistical tests are compiled with standard (Diophantine) arithmetic, the relevant notion of locality is encoded in (27). Hence Czachor's model is *Bell-nonlocal*, namely it preserves no-signalling (marginals depend only on local settings), yet it violates factorizability when probabilities are read in the usual sense. The "non-linearity of expectations" identified in our review explains why the CH/CHSH algebraic steps fail *within the deformed calculus*; but the physically decisive point is that, once brought back to standard probabilities, the model reproduces the quantum singlet statistics and therefore cannot satisfy (27).

At the level of deterministic response functions one typically has OI in a classical model. Still, here the deformation moves the violation to the *probability composition rule*, namely PI (insensitivity to the remote *setting*) is retained in the response maps. In contrast, factorizability — equivalent to PI + OI in the standard Kolmogorov calculus — fails after translation to ordinary probabilities. Operationally, that is sufficient to call the model nonlocal in Bell's sense, despite its non-signalling character.

We therefore emphasize in this review that the core physical shortfall is *failure of factorizability in standard probability theory*. The non-linearity of expectations is a useful diagnostic for why CH/CHSH proofs do not go through in the deformed algebra, but it is the non-factorization (and thus Bell-nonlocality) that carries the primary physical significance.

6. Lambare's critique and why Czachor's response is non-responsive

> The whole plan hinged upon
> the natural curiosity of potatoes.[†5]

For quick reference, we list two central points raised by Lambare [22]:

(i) Measurement independence (MI). Bell/CHSH framework assumes a setting-independent hidden-variable distribution

$$\rho(\lambda|x,y) = \rho(\lambda), \tag{29}$$

equivalently $P(\lambda,x,y) = \rho(\lambda)P(x,y)$. Lambare shows that in Czachor-style constructions the domain or density of λ *changes with the settings* (x,y), so MI is violated. Any "violation" obtained under such models does not contradict Bell; it departs from a premise of the theorem [22].

(ii) Counterfactuals not required. The standard bounds of 2 (or $2\sqrt{2}$) arise directly at the level of expectations, without adding outcomes of unperformed measurements. Thus, attacking a counterfactual gloss of Bell does not touch the actual derivation [22].

6.1. What Czachor's response does

Czachor explicitly replaces ordinary arithmetic with a *deformed* calculus. Expectations are computed via a bijection f,

$$E_f[X] = f^{-1}\left(E[f(X)]\right), \tag{30}$$

which is linear with respect to a deformed sum $\oplus_f$ but, in general,

$$E_f[X+Y] \neq E_f[X] + E_f[Y]. \tag{31}$$

In particular, constants are fixed points, i.e., for any constant c, $E_f[c] = c$. Czachor then *chooses* f (piecewise trigonometric) so that E_f reproduces $\sin^2$-type quantum probabilities, and claims a Bell-type "violation" within this calculus [23].

6.2. Why this does not answer Lambare

The manoeuvre described in the previous section does not engage Lambare's [22] points. Instead, it shifts the framework in which Bell's assumptions are formulated, leaving the original issues untouched.

1. Premise switch instead of rebuttal. The Bell/CHSH approach uses linearity under ordinary "+" to pass from the pointwise inequalities to their expectation form. By abandoning additivity under "+", the response puts itself *outside* Bell's premises. That does not rebut Lambare's MI point; it avoids it [5, 6, 22].

[†5] The quote comes from [21] (various editions since 1957).

2. MI remains unaddressed. Regardless of whether arithmetic is deformed, Bell's MI is a statement about *setting-independence of* $\rho(\lambda)$ *on a common probability space*. Czachor's framework still lets the measure/domain depend on (x, y) (e.g., by re-labeling ranges or densities in the f space), so Lambare's objection persists [22].
3. Engineered ansatz, no selection principle. The specific f is tuned to yield trigonometric correlations. Infinitely many bijections are available; no physical or structural criterion uniquely picks this f. Hence the result lacks predictive content and cannot serve as a general reply to Bell or to Lambare [23].
4. Operational gap. Laboratory probabilities are relative frequencies aggregated by ordinary "+". The response supplies no device-level rule showing that real counting implements $\oplus_f$ or that indicator products obey the deformed multiplication implied by f. Without such a bridge, claims about "probabilities" in the deformed calculus do not correspond to measured frequencies [7].
5. Counterfactuals still irrelevant. Because the response never re-establishes the Bell premises (MI + common σ-algebra + ordinary additivity), disputing counterfactual narratives remains beside the point of Lambare's critique [22].

Lambare identifies a concrete premise failure (MI) and clarifies that Bell's derivation does not rely on counterfactual bookkeeping. Czachor's response neither restores MI nor engages the expectation-level proof. Instead, it *changes the probability calculus* and hand-tunes f to fit quantum trigonometry. As a reply to Lambare (and to Bell), it is therefore non-responsive and incorrect; it changes the rules rather than meeting them [22, 23].

7. Conclusions

In this paper, we critically assess claims that non-Newtonian arithmetic and calculus overturn Bell-type results that do not pass. The central observation is that the expectation functional used in this line of work is linear only with respect to a deformed sum (not ordinary addition), which removes the standard CH/CHSH derivations from applicability. Within a single level, a suitable Bell-type analogue can be formulated if linearity holds with respect to that level's addition; "beyond-Tsirelson" values arise only from level mixing [1–7].

The following items synthesize the main results of our analysis:

(i) Inapplicability of CH/CHSH inequalities in the case of deformed addition. The proof steps that bound the CHSH expression rely on the linearity of the expectation under ordinary addition; replacing the expectation with a deformed, non-additive functional indicates these steps. Within a single level, an appropriate inequality may be formulated if linearity holds relative to $\oplus$ [4–6].

(ii) Einstein-addition analogy fails beyond 1D. The pullback law $x \oplus y = f^{-1}(f(x) + f(y))$ is commutative/associative, whereas the 3D relativistic composition exhibits Thomas–Wigner rotation (non-commutative, non-associative) and is properly modeled by gyrogroups, not by global pullbacks [9, 10].

(iii) Admissibility condition for f. We introduce a minimal regularity class (strict monotonicity, absolute continuity, local bounds on f') together with a convenient closure condition ensuring $\oplus_f$ is well-defined across the chosen domain. Singular choices (e.g., Cantor-type) invalidate the deformed derivative/integral, while smooth but unphysical choices (e.g., $f(\beta) = \beta^3$) yield incorrect kinematics or fail closure. Hence both regularity and a symmetry-based selection principle are necessary [15n].

(iv) Diagnostics and a qualified normal form. Non-additivity of expectation and loss of factorizability each are each sufficient to invalidate CHSH inequalities. Practical tests include an additivity check for remapped probabilities and a horizon/conditioning audit. A *qualified* bijection normal form shows that when the level-ℓ correlator set (with zero marginals) lies within the local correlator polytope, a standard local vidden-variable (LHV) thoery representation exists; otherwise, only context-dependent (pairwise) matchings are possible [5, 7, 24].

(v) Lambare and the non-response. Lambare's critique identifies measurement independence as the key premise and notes that counterfactual bookkeeping is unnecessary for Bell' theorem. Czachor's response shifts premises by adopting a deformed calculus and adjusting a bijection to reproduce trigonometric correlations, without addressing measurement independence or the expectation-level proof [22, 23].

The non-Newtonian framework is best uderstood as a representational re-encoding rather than a physical counterexample to Bell's theorem. Without an operational device-level rule that aggregates data via the deformed sum, and without a principled selection of the bijection compatible with relevant symmetries, the approach lacks empirical traction [7].

Our analysis does not reject deformed calculi as mere changes of variables within a fixed level; rather, it targets the stronger claim that such calculi, without additional structure and operational justification, refute Bell's theorem or predict new physics.

A recent proposal by Czachor [25] (see also our critical comments [26] and [27]) introduce a so-called "τ-time" framework, anchoring its dimensionless evolution parameter by positing that the timelike thickness of the hyperlayer is Planck-scale — but only when viewed over cosmological (i.e., Hubble-time) intervals. Czachor's anchoring [25] leads him to staggeringly large values (a cosmically and physically *baroque* figure), i.e., $\tau \sim 10^{243}$ in today's terms — a figure as extravagant as it is speculative — and about $\tau \sim 10^{203}$ per human year (*sic*).

While internally consistent under Czachor's chosen normalization, these numbers don't stem from any fitted dynamics or empirical calibration. In other words, the model picks a scale, but it doesn't justify this choice by operational linkage. We noted that there is no proposed mapping from τ to measurable quantities like clock rates, redshifts, or spectral shifts with defined magnitudes or experimental constraints. Any derived phenomenology — say, τ-dependent oscillator widths that hint at drifting masses — rests on that same ungrounded normalization. Until there is a way to calibrate or connect τ to observables, such features should be treated as formal outcomes of the mathematical structure rather than as testable, empirical forecasts.

For the reader's interest, an even more detailed analysis of the presented topic, encompassing a broader scope of Czachor's research (fifteen papers), will appear in an upcoming publication [27].

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