Amplitude Modulation and Demodulation in Strain Dependent Diffusive Semiconductors

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In communication processes, amplitude modulation is very helpful to save power by using a single band transmission. Thus in this paper authors have explored the possibility of amplitude modulation as well as demodulation of an electromagnetic wave in a transversely magnetized electrostrictive semiconductor. The inclusion of carrier diffusion and phenomenological damping coefficient in the nonlinear laser–semiconductor plasma interaction adds a new dimension to the analysis present in this paper. This problem is analyzed in different wave number regimes over a wide range of cyclotron frequencies. It is found that the complete absorption of the waves takes place in all the possible wavelength regimes when the cyclotron frequency ($\omega_c$) becomes exactly equal to $(\nu^2 + \omega_0^2)^{1/2}$ in absence of damping parameter. It has also been seen that diffusion of charge carriers modifies amplitude modulation and demodulation processes significantly. The damping parameter plays a very important role in deciding the parameter range and selecting the side band mode that will be modulated by the above-mentioned interaction.

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1. Introduction

Modulational instability of propagating beams has been a field of interest since the origin of physical optics with its concentration of diffraction and wave guiding processes. This is due to the fact that the scattering of light from sound or low frequency electromagnetic wave affords a convenient and widely used means of controlling the frequency, intensity, and direction of an optical beam [1]. This type of modulation makes possible a large number of applications involving the pulse shaping, optical beam deflection, display, and processing of information [2, 3]. The fabrication of some optical devices such as acousto-optic modulators is based on...
the interaction of an acoustic wave or a low frequency electromagnetic wave with the incident laser beam.

The problem of propagation of optical radiation in crystal in the presence of an electric field or acoustic strain field has been an active area of research especially when modulation of light beam is produced by sound waves. It is a well-known fact that, when an unmodulated electromagnetic wave propagates through a plasma with periodically varying parameters, it gets modulated in amplitude. This periodic variation in propagation parameters may be induced by the propagation of an acoustic wave in plasmas [4]. The propagation of an acoustic wave in plasmas leads to the periodic variation in its electron density, which further leads to modulation at the acoustic wave frequency.

Amplitude modulation is commonly encountered as a preliminary step in many complex modulation schemes. One of the important problems in communication system is that of developing an effective method for modulation as well as demodulation of waves. This often makes for maximum simplicity and economy, particularly at low power outputs. The problem of modulation in semiconductor plasmas has been studied by a number of workers [5, 6]. In nonlinear acoustics an important field of study is amplification/attenuation and frequency mixing of waves in semiconductors [7, 8] because of its immediate relevance to problems of optical communication systems. The modulation of a laser beam produced due to certain plasma effects in semiconductors was reported by Sen and Kaw [9].

Semiconductors also provide a compact and less expensive medium to model nonlinear interactions which can be classified as modulational interaction. The resulting amplification of decay channels produced by modulational interactions is generally referred to as an instability of wave propagating in nonlinear dispersive medium such that the steady state becomes unstable and involves into a temporally modulated state [10]. In most of the investigations of modulational interactions with nonlocal effects such as diffusion of excitation density that is responsible for nonlinear refractive index change has normally been ignored. They assumed that the nonlinearity arises from the nonlinear polarization locally. But in the case of electrostrictive effect, the nonlinearity is solely due to the motion of free carriers and therefore the effect is nonlocal. The high mobility of excited charge carriers makes diffusion effects particularly relevant in semiconductor technology as they travel significant distances before recombining. Hence the diffusion of carrier is however expected to have a strong influence on the nonlinearity of the medium particularly in high mobility semiconductors, viz. III–V compound semiconductors, in which carriers can easily be moved. Therefore, the inclusion of carrier diffusion in theoretical studies of modulational phenomenon seems to be important from both the fundamental and application points of view and thus attracted attention of many workers in the last couple of decades [11–14]. Motivated by the above discussion, in this paper, we have presented the electrostrictive amplitude modulation of an intense electromagnetic beam in a strain dependent diffusive
semiconductor crystal. The effect of diffusion of the charge carriers on this nonlinear interaction of the laser beam adds a new dimension to the analysis presented in \( n \)-type semiconductors [12]. The intense pump beam electrostrictively generates an acoustic wave within the semiconductor medium that induces an interaction between the free charge carriers and the acoustic phonons. This interaction induces strong space charge field that modulates the pump beam. Thus, the applied optical and generated acoustic wave in an electrostrictive modulator can produce amplitude modulation and demodulation effect at acoustic wave frequency. It is also found from the literature that the application of magnetic field is favorable for the phenomenon under study.

2. Theoretical formulation

The hydrodynamical model of a homogeneous semiconductor plasma applicable for the medium of infinite extent (i.e. \( k_a l < 1 \), where \( k_a \) is the wave number of acoustic mode and \( l \) is the mean free path of electron) has been considered for the theoretical formulation of modulation index of amplitude modulated laser beam in \( n \)-type diffusive semiconductor plasma. The medium is immersed in a static magnetic field \((B_0)\) pointing along \( z \)-axis that is normal to the propagation vectors of parametrically generated acoustic \((k_a)\) and the pump \((k_0)\) waves. Here the low frequency perturbations are assumed to be due to acoustic wave \((\omega_a, k_a)\) generated through acoustic polarization in the crystal. The physics of the problem can be explained as follows: the electron concentration oscillates at the acoustic wave frequency due to electrostrictive potential fields accompanying the acoustic wave. The pump wave then gives rise to a transverse current density at frequencies \( \omega_0 \) and \((\omega_0 \pm \omega_a)\), where \( \omega_0 \) is the frequency of the pump wave. The transverse current densities produced at frequency \((\omega_0 \pm \omega_a)\) are known as side band current densities. These side band current densities produce side band electric field vectors and consequently the pump wave gets modulated. In the following analysis the side band will be represented by the suffixes \( \pm \), where + stands for the mode propagating with frequency \((\omega_0 + \omega_a)\) and – stands for \((\omega_0 - \omega_a)\) mode.

The pump field \( E_0 \) gives rise to a time-varying electrostrictive strain and is thus capable of deriving the acoustic waves in the medium, with the strain given by \( \partial u/\partial x \), where the deviation of a point \( x \) from its equilibrium position is \( u(x, t) \). If the derived space charge field is \( E_a \) then the net electrostrictive force in the positive \( x \)-direction acting on a unit volume is given by [15]:

\[
F = \frac{\gamma}{2} \frac{\partial}{\partial x} (E_0 E_a^*),
\]

and consequently, the equation of motion for \( u(x, t) \) in the electrostrictive semiconductor is given by

\[
\rho \frac{\partial^2 u}{\partial t^2} - c \frac{\partial^2 u}{\partial x^2} + 2 \Gamma_a \rho \frac{\partial u}{\partial t} = -\frac{\gamma}{2} \frac{\partial}{\partial x} (E_0 E_a^*),
\]

where \( \gamma \) is a phenomenological constant describing the change in the optical dielectric constant and is typically of the order of \( 10^{-11} \) mks units [15, 16], \( \rho \) is the mass
This acoustic wave generated due to electrostrictive strain modulates the dielectric constant and gives rise to a nonlinear induced polarization as

\[ P_{\text{es}} = -\gamma E_0 \frac{\partial u^*}{\partial x}. \] (3)

At very high frequencies of the field which is quite large as compared to the frequencies of the motion of electrons in the medium, the polarization is considered on neglecting the interactions of the electrons with one another and with nuclei of the atoms. Thus, the electric displacement in the presence of electrostrictive polarization \( P_{\text{es}} \) is simply given by \( D = \varepsilon E_a + P_{\text{es}} \) [17]. Then the space charge field \( E_a \) is determined by the Poisson equation as

\[ \frac{\partial E_a}{\partial x} = \frac{n_{1e}}{\varepsilon} + \frac{\gamma}{\varepsilon} E_0 \frac{\partial^2 u^*}{\partial x^2}, \] (4)

where \( \varepsilon \) is the dielectric permittivity of the crystal.

To compute the perturbed current densities in \( n \)-type semiconductor in the presence of electrostrictive coupling, using Eqs. (2) and (4) the authors obtained the perturbed carrier density as

\[ n_1 = -2\rho e u \left[ -k_a^2 \frac{\partial^2}{\partial x^2} (1 - A^2) + \omega_a^2 + 2i\omega_a \Gamma_a \right] \frac{e\gamma E_0}{c}, \] (5)

where \( A^2 = (\gamma|E_0|)^2/(2\varepsilon c) \) is the dimensionless electrostrictive coupling coefficient due to the electrostrictive interaction in the medium under study.

The electron momentum transfer equation including diffusion effect is

\[ \frac{\partial \vartheta_j}{\partial t} + \left( \frac{\partial_0}{\partial x} \right) \vartheta_j + \nu \vartheta_j = -\frac{e}{m} [E_j + (\vartheta_j \times B_0)] - D\nu \frac{\Delta n_{1j}}{n_{1j}}. \] (6)

The subscript \( j \) stands for 0, + and – modes. The above equation describes the electron motion under the influence of the electric fields associated with the pump and side band modes, in which \( m \) and \( \nu \) are, respectively, the effective mass of electrons and the phenomenological electron collision frequency. \( n_{1j} \) stands for the perturbed and unperturbed electron densities. \( D \) is the carrier diffusion coefficient which may be expressed using Einstein’s relation \( D = k_B T \mu_1 / e \) in which \( k_B, T, \mu_1, \) and \( e \) are the Boltzmann constant, electron temperature, electron mobility, and electronic charge, respectively. In the above relation, the pump magnetic field is neglected by assuming that the electron plasma frequency of the medium is of the order of the pump frequency.

The above momentum transfer equation shall be used to obtain the oscillatory electron fluid velocity in the presence of pump electric field \( (E_0) \) and field of the side band modes \( (E_{\pm}) \). By linearizing Eq. (6) these velocity components may be obtained as
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\[ \vartheta_{jx} = -\frac{e}{m} E_j \left( 1 + \frac{D v k^2}{\omega_p^2} \right) \frac{\nu - i \omega_j}{(\nu - i \omega_j)^2 + \omega_c^2} \]  
(7)

and

\[ \vartheta_{jy} = -\frac{e}{m} E_j \left( 1 + \frac{D v k^2}{\omega_p^2} \right) \frac{\omega_c}{(\nu - i \omega_j)^2 + \omega_c^2}, \]  
(8)
in which \( \omega_c = (eB_0)/m \) is the electron cyclotron frequency.

In deriving above equations, we have assumed an \( \exp(i(\omega_j t - k_j x)) \) dependence on the field quantities.

The total transverse current density in this medium is given by

\[ J_{\text{total}} = e \left[ \sum_j n_0 \vartheta_{jx} + \sum_j n_0 \vartheta_{jy} \exp(i(\omega_j t - k_j x)) \right], \]  
(9)

where \( n_0 \exp(i(\omega_j t - k_j x)) \) represents the current generated due to the interaction of the pump with acoustic wave. In order to obtain the modulation indices of the modulated side band modes, the authors use the general wave equation which under the chosen configuration reduces to

\[ \frac{\partial^2 E_{\text{total}}}{\partial x^2} - \mu \varepsilon \frac{\partial^2 E_{\text{total}}}{\partial t^2} - \mu \frac{\partial J_{\text{total}}}{\partial t} = 0, \]  
(10)

where \( \mu \) is the permeability of the medium. \( E_{\text{total}} \) will be given by

\[ E_{\text{total}} = E_\pm (-ikx) + E_\pm \exp(-i(k \pm k_a)x). \]  
(11)

Now neglecting \( \exp(-i k x) \) in comparison to 1, we obtain the following expressions for modulation indices:

\[ \frac{E_{\pm}}{E_0} = -\frac{2 i \omega_0 \varepsilon \mu A c (\omega^2_c - \nu^2 - \omega_0^2)}{m \beta (\omega^2_c + \nu^2 - \omega_0^2)} \frac{1 + D v k^2}{\omega_p^2}, \]  
(12)
in which \( \beta = \gamma E_0 \). In deriving above relation, we have assumed \( \exp(-i k x) \ll 1 \). This approximation is justifiable as \( k \approx 10^7 \) to \( 10^8 \) m\(^{-1} \) and \( \exp(-i k x) \) will remain negligible for very high values of \( x \).

By rationalization of Eq. (12) one obtains the real parts of modulation indices as

\[ \frac{E_{\pm}}{E_0} = -\frac{2 i \omega_0 \varepsilon \mu A c (\omega^2_c - \nu^2 - \omega_0^2)}{m \beta (\omega^2_c + \nu^2 - \omega_0^2)} \frac{1 + D v k^2}{\omega_p^2} \times \frac{(\omega^2_c + \nu^2 + \omega_0^2)}{(\omega^2_c + \nu^2 - \omega_0^2)^2 + 4 \omega_0^2 \nu^2 (k_a \pm 2k)}. \]  
(13)

From Eq. (13) it is clear that carrier diffusion plays a significant role in deciding on the magnitudes of the modulation indices in semiconductor plasma.

3. Results and discussion

In this section the amplitude modulation and demodulation due to electrostrictive interaction in the presence and absence of phenomenological damping constant \( \Gamma_a \) shall be analyzed through Eq. (13). Equation (13) may be analyzed under two different wave number regimes, i.e. 1) \( k_a > 2k \) and 2) \( k_a < 2k \).
Case 1: $k_a > 2k$

Case 1.1. In the presence of phenomenological damping constant ($\Gamma_a \neq 0$)

In this wave number regime, the amplitude of the side band modes ($E_\pm$) always remains in phase with the pump wave in the permissible cyclotron range. These in-phase side bands interact with the pump wave under this wave number regime to produce modulated acoustic wave. Hence in $k_a > 2k$ regime one can always get modulation. The modulation index is found to be a maximum at a particular value of magnetic field when $\omega_c \approx \omega_0$.

The numerical estimation has been made for an $n$-type III–V semiconductor assumed to be duly irradiated by 10.6 $\mu$m pulsed CO$_2$ laser at 77 K. The physical parameters used are: $m = 0.014m_0$ ($m_0$ being the free electron mass), $\nu = 4 \times 10^{11}$ s$^{-1}$, $\omega_0 = 1.6 \times 10^{13}$ s$^{-1}$, $\rho = 5.8 \times 10^3$ kg m$^{-3}$, $\Gamma_a = 2 \times 10^{10}$ s$^{-1}$, $\omega_a = 10^{12}$ s$^{-1}$.

The variations of ($E_+/E_0$) and ($E_-/E_0$) with the applied magnetostatic field ($\omega_c$) are depicted in Figs. 1 and 2. In both the figures two curves are drawn; the one — in the presence of diffusion ($D \neq 0$, $-$ $-$ $-$) and the second — in the absence of diffusion ($D = 0$, $-$ $-$ $-$) of the carriers. It may be seen from Figs. 1 and 2 that the modulation indices of both the side bands increase with cyclotron frequency up to $\omega_c < \omega_0$. These indices yield a maximum value at $\omega_c \approx \omega_0$. On further increasing the magnetic field ($\omega_c > \omega_0$) they abruptly decrease. It may also be inferred from these curves that the modulation indices of both the modes get enhanced due to the presence of carrier diffusion.

![Fig. 1: Variation of modulation index of plus mode (for $k_a > 2k$) with magnetic field, when $\Gamma_a \neq 0$.](image1)

![Fig. 2: Variation of modulation index of minus mode (for $k_a > 2k$) with magnetic field, when $\Gamma_a \neq 0$.](image2)
Case 1.2. In the absence of phenomenological damping constant ($\Gamma_a = 0$)

In the absence of phenomenological damping constant $\Gamma_a$, Eq. (13) becomes

$$
\frac{E_\pm}{E_0} = -\frac{2\omega_0^2 e\mu c k_a c A^2 (\omega_c^2 - \nu^2 - \omega_0^2)}{m\beta \left[(\omega_c^2 + \nu^2 - \omega_0^2)^2 + 4\omega_0^2 \nu^2\right]} \frac{(k_a \pm 2k)}{\omega_0^2} \left(1 + \frac{D\nu k^2}{\omega_0^2}\right).
$$  (14)

For this case the variations of $(E_+/E_0)$ and $(E_-/E_0)$ with the applied magnetostatic field ($\omega_c$) are depicted in Figs. 3 and 4. It can be seen from Figs. 3 and 4 that at $\omega_c \approx \omega_0$ the modulation indices of both the modes become zero. When one applies weak magnetic field so that the cyclotron frequency becomes smaller than the carrier frequency, both the modes are found in phase with pump wave. Hence one gets modulation of both the side bands in the absence of phenomenological damping when $\omega_c < \omega_0$. If one increases the magnetic field and achieves a range where $\omega_c > \omega_0$ both side bands go out of phase with pump wave and hence get demodulated. This nature can be explained as follows: let us express the amplitude ratio in the form $(E_\pm/E_0) = -R$. One can incorporate the negative sign in phase factor as $(E_\pm/E_0) = -R \exp(i\pi)$. It indicates that the modulated wave and the pump wave has a phase difference of $\pi$ or the two waves are out of phase. As a result, the modulated wave again interacts with the pump wave to produce the demodulated acoustic wave. Therefore, the phenomenon of demodulation takes place. Thus, the authors can conclude that in the regime $k_a > 2k$ with $\omega_c > \omega_0$ and $\Gamma_a = 0$, the phenomenon of demodulation can be observed. It may also be inferred from these graphs that the carrier diffusion increases the modulation indices but decreases the demodulation indices. It can also be seen that the modulation index of the minus mode is always greater than that of the plus mode.
Fig. 5. Variation of modulation index of plus mode (for $k_a > 2k$) with magnetic field.
Fig. 6. Variation of modulation index of minus mode (for $k_a > 2k$) with magnetic field.

To appreciate the role of phenomenological damping constant ($\Gamma_a$) on modulation/demodulation process, the indices of both the modes in the presence and absence of $\Gamma_a$ are illustrated in Figs. 5 and 6. One may directly infer from these figures that both the side bands are always in phase with pump wave (modulated) in the presence of $\Gamma_a$. But when $\Gamma_a = 0$, these side bands are in modulated state only when $\omega_c < \omega_0$, otherwise ($\omega_c > \omega_0$) both are in demodulated state. Hence phenomenological damping constant has a positive influence over this phenomenon when $k_a > 2k$.

Case 2: $k_a < 2k$

Case 2.1. In the presence of phenomenological damping constant ($\Gamma_a \neq 0$)

The variations of $(E_+/E_0)$ and $(E_-/E_0)$ with the applied magnetostatic field ($\omega_c$) are depicted in Figs. 7 and 8. It may be seen that the amplitude of the plus mode is in phase with pump wave which confirms the modulation process. The maximum value of the modulation index of the plus mode is achieved at $\omega_c \approx \omega_0 \approx 2 \times 10^{13}$ s$^{-1}$. Figure 8 shows the variation of $(E_-/E_0)$ with applied magnetostatic field ($\omega_c$). The behavior of the modulation index of the minus mode is opposite the plus mode in nature. Hence the minus mode is always demodulated in this region. The carrier diffusion increases both modulation and demodulation indices.

Case 2.2. In the absence of phenomenological damping constant ($\Gamma_a = 0$)

Figures 9 and 10 show the variations of the plus $(E_+/E_0)$ and minus $(E_-/E_0)$ modes with magnetostatic field ($\omega_c$). Here the amplitudes of the plus and minus side band modes are found in phase with the pump wave under the condition $\omega_c < (\omega_0^2 + \nu^2)^{1/2}$ and $\omega_c > (\omega_0^2 + \nu^2)^{1/2}$, respectively. For a particular magnetic field ($\omega_c$) if one gets modulation of the minus mode, the plus mode gets demodulated and vice versa. Hence both the modes have opposite behavior in the $k_a < 2k$ wave number regime. This is so because $E_+$ mode gets modulated at low magnetic field when $\omega_c < (\omega_0^2 + \nu^2)^{1/2}$ whereas $E_-$ mode gets modulated in the opposite
Fig. 7. Variation of modulation index of plus mode (for $k_a < 2k$) with magnetic field, when $\Gamma_a \neq 0$.

Fig. 8. Variation of modulation index of minus mode (for $k_a < 2k$) with magnetic field, when $\Gamma_a \neq 0$.

Fig. 9. Variation of modulation index of plus mode (for $k_a < 2k$) with magnetic field, when $\Gamma_a = 0$.

Fig. 10. Variation of modulation index of minus mode (for $k_a < 2k$) with magnetic field, when $\Gamma_a = 0$.

condition. It can also be seen from both Figs. 9 and 10 that the carrier diffusion coefficient always increases the magnitude of modulation indices for both the plus and minus modes. At very high magnetic fields when $\omega_c \gg \omega_0, \nu$ the amplitude modulation starts decreasing whereas demodulation starts increasing. This can directly be inferred from Eq. (14) where the magnitude of amplitude-ratio becomes inversely proportional to the square of the electron cyclotron frequency when $\omega_c$ becomes quite larger than pump and collision frequencies.

The effect of phenomenological damping constant on the modulation process has been depicted in Figs. 11 (plus mode) and 12 (minus mode) when $k_a < 2k$. In the presence of damping, the plus mode remains in phase whereas the minus mode
is always out of phase with pump wave in the magnetic field range under study.

The absence of damping shows both types of characteristics, i.e. modulation and
demodulation for the plus and minus modes depending upon the relative magni-
tudes of cyclotron and carrier frequencies. For smaller values of magnetic field, the
plus mode is in the modulated state whereas the minus mode gets demodulated.

Fig. 11. Variation of modulation index of plus mode (for \( k_a < 2k \)) with magnetic field.
Fig. 12. Variation of modulation index of minus mode (for \( k_a < 2k \)) with magnetic field.

A direct dependence of the ratio of amplitudes on pump frequency \( \omega_0 \) and the acoustic intensity \( u \) is observable from Eq. (14). However, in the vicinity
of the wave number range \( k_a = 2k \), authors find a singularity in the field ratio.

Lashmor-Davies [18] has explained the behaviour in this range and accordingly,
the decay instability takes place showing the absence of modulation. The authors
can compare the amplitude ratio for magnetized and unmagnetized plasma by
using Eq. (14) as

\[
\frac{(E_x/E_0)_{B_0 \neq 0}}{(E_x/E_0)_{B_0 = 0}} = \frac{1 - \omega_c^2/\omega_0^2}{1 + \omega_c^2/\omega_0^2}.
\]

The numerator is always less than one but denominator can become either greater
or less than one depending upon weather \( \omega_c^2 > 2(\omega_0^2 + \nu^2) \) or \( \omega_c^2 < 2(\omega_0^2 + \nu^2) \),
 respectively. The authors further divide their discussion in two parts: (i) when
\( \omega_c^2 > 2\omega_0^2 \), and (ii) when \( \omega_c^2 < 2\omega_0^2 \).

Neglecting the collision term in Eq. (15), the amplitude ratio at \( \omega_c^2 > 2\omega_0^2 \)
becomes

\[
\frac{(E_x/E_0)_{B_0 \neq 0}}{(E_x/E_0)_{B_0 = 0}} = \frac{1 - (\omega_c/\omega_0)^2}{1 + (\omega_c/\omega_0)^2}.
\]

A decrease in the amplitude ratio due to higher magnetic fields (at \( \omega_c^2 > 2\omega_0^2 \))
(with negative sign) can be obtained from Eq. (16). As \( \omega_c^2/\omega_0^2 > 2 \), the numerator
becomes negative. Hence, one may conclude that the demodulation can take place
in this range of magnetic field.
In the range of magnetic field when $\omega^2 < 2\omega_0^2$ the expression reduces to
\[
\frac{(E_x/E_0)_{B_0\neq0}}{(E_x/E_0)_{B_0=0}} = \frac{1 - (\omega_c/\omega_0)^2}{1 - 2\omega^2/\omega_0^2}.
\]
This ratio will be always greater than one and it can be inferred that the modulation of the wave can be increased by increasing the magnitude of externally applied magnetic field.

The above discussion reveals that the amplitude modulation/demodulation of an electromagnetic wave by an acoustic wave can be easily achieved in electrostrictive III–V semiconductor plasma medium. It is also found that the damping plays a very important role in deciding the parameter range and selecting the side band mode, which will be modulated by the above-mentioned interaction. The presence of carrier diffusion alters the result favorably. It always increases the value of modulation/demodulation indices for both the modes around $\omega_c \approx \omega_0$.

The consideration of a diffusive crystal with electrostrictive polarization thus possibly offers an interesting area for the purpose of investigations of different modulational interactions and one hopes to open a potential experimental tool for energy transmission and solid state diagnostics in electrostrictive diffusive crystals.

References
