Vol. 98 (2000) ACTA PHYSICA POLONICA A No. 1–2

ON DARBOUX–BÄCKLUND TRANSFORMATION AND POSITON TYPE SOLUTION OF COUPLED NONLINEAR OPTICAL WAVES

D.C. SEN AND A.R. CHOWDHURY
High Energy Physics Division, Department of Physics, Jadavpur University
Calcutta-700032, India

(Received April 8, 1999; revised version September 23, 1999; in final form February 18, 2000)

A Darboux–Bäcklund transformation is used to obtain a positon type solution of the nonlinear equations describing the propagation of coupled nonlinear optical pulses.This form of the positon solution is then compared with that obtained by the special limiting procedure applied to a two-soliton solution. It is observed that though the algebraic form of the two solutions is different yet both of these have singularities and the position of the singularities remains on the similar curve in the (x, t) plane. We also depict the form of these solutions graphically. Finally, it may be added that the method of Darboux–Bäcklund transformation is convenient for generating more than one-positon solution.

PACS numbers: 02.30.+g, 52.35.Sb, 52.35.Mw

1. Introduction

A study of the optical pulse propagation is one of the most important use of solitary waves found in the domain of integrable system [1]. On the other hand nonlinear waves other than the type of solitons have been derived recently for many integrable systems [2]. Here we have derived a new type of solutions for the equations of coupled nonlinear optical pulses by using both Darboux–Bäcklund transformation and a specialized limiting procedure on the usual two-soliton solution — these are called positons for nonlinear optics. In the first part of our paper we describe the approach of Darboux–Bäcklund transformation. In the next part we have rederived the positon solution by starting with the two-soliton form and adopting a specialized limiting procedure. It is observed that though the two forms differ in their algebraic form yet their singularity structures are very similar. Before proceeding to the formulation we may summarize the basic properties of a positon type solution:

(a) Positons are weakly localised and in one dimension possess a singularity;
(b) The properties of the corresponding Lax eigenvalue problem is special;
(c) Positons remain unchanged after mutual interaction.

2. Formulation

Coupled nonlinear optical pulses are described by equations of the following form:

$$\mathrm{i}q_x = q_{tt} + 2rq^2, \quad \mathrm{i}r_x = -r_{tt} - 2qr^2, \tag{1}$$

the corresponding Lax pair is written as

$$\Psi_t = M\Psi, \quad \Psi_x = N\Psi \tag{2}$$

$$M = \begin{pmatrix} \mathrm{i}\lambda & \mathrm{i}q \\ \mathrm{i}r & -\mathrm{i}\lambda \end{pmatrix} \tag{3a}$$

$$N = \lambda^2 \begin{pmatrix} 2\mathrm{i} & 0 \\ 0 & -2\mathrm{i} \end{pmatrix} + \lambda \begin{pmatrix} 0 & 2\mathrm{i}q \\ 2\mathrm{i}r & 0 \end{pmatrix} + \begin{pmatrix} -\mathrm{i}qr & q_t \\ -r_t & \mathrm{i}qr \end{pmatrix}$$

$$= \sum_{j=0}^{2} \lambda^j \begin{pmatrix} a_j & b_j \\ c_j & -a_j \end{pmatrix} \text{ (say)}. \tag{3b}$$

The Lax eigenfunction can be written in general as

$$\Psi = (\, \psi_{11} \ \psi_{12}\psi_{21} \ \psi_{22} \,) \tag{4}$$

and the Darboux–Bäcklund (DB) transformation can be written as [3]

$$\widetilde{\Psi} = T\Psi \quad \text{with} \quad \widetilde{\Psi} = (\, \widetilde{\psi}_{11} \ \widetilde{\psi}_{12}\widetilde{\psi}_{21} \ \widetilde{\psi}_{22} \,) \tag{5}$$

and

$$T = \lambda \begin{pmatrix} 2\mathrm{i} & 0 \\ 0 & 2\mathrm{i} \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix}. \tag{6}$$

Since we demand that the transformed equation must be of the same form, we must have

$$\widetilde{\Psi}_t = \widetilde{M}\widetilde{\Psi}, \tag{7a}$$

$$\widetilde{\Psi}_x = \widetilde{N}\widetilde{\Psi}, \tag{7b}$$

where instead of (r, q) the variable $(\widetilde{r}, \widetilde{q})$ occur in $(\widetilde{M}, \widetilde{N})$. Consistency of (5) and (7a) demands

$$T_t = \widetilde{M}T - TM, \tag{8}$$

which leads to

$$a_t = \mathrm{i}(\widetilde{q}c - rb), \quad d_t = -\mathrm{i}(qc - \widetilde{r}b),$$

$$b_t = \mathrm{i}(\widetilde{q}d - qa), \quad c_t = \mathrm{i}(\widetilde{r}a - rd) \tag{9a}$$

along with

$$q + \widetilde{q} = -\mathrm{i}b, \quad r + \widetilde{r} = -\mathrm{i}c, \tag{9b}$$

obviously

$$\det\, T(\lambda) = 4\lambda^2 + 2\mathrm{i}(d-a)\lambda + (ad - bc)$$

and if λ_1, λ_2 be two zeros of det $T(\lambda)$ then we have

$$\det\, T(\lambda) = 4(\lambda - \lambda_1)(\lambda - \lambda_2). \tag{10}$$

Now since $M, \widetilde{M}, N, \widetilde{N}$ are traceless it follows that

$$(\det\, \Psi)_x = (\det\, \Psi)_t = 0. \tag{11}$$

The same being true for det $\widetilde{\Psi}$, whence λ_1, λ_2 are independent of x and t. Now $\widetilde{\Psi}$ is collinear that is

$$\begin{pmatrix} \widetilde{\psi}_{11}(\lambda_j) \\ \widetilde{\psi}_{21}(\lambda_j) \end{pmatrix} + k_j \begin{pmatrix} \widetilde{\psi}_{12}(\lambda_j) \\ \widetilde{\psi}_{22}(\lambda_j) \end{pmatrix} = 0 \tag{12}$$

for some constants k_j. Thus $\widetilde{\Psi} = T\Psi$ is also collinear at $\lambda = \lambda_1$ and $\lambda = \lambda_2$, leading to

$$(2\mathrm{i}\lambda_1 + a)\psi_1 + b\psi_2 = 0, \quad (-2\mathrm{i}\lambda_1 + d)\psi_2 + c\psi_1 = 0,$$

$$(2\mathrm{i}\lambda_2 + a)\phi_1 + b\phi_2 = 0, \quad (-2\mathrm{i}\lambda_2 + d)\phi_2 + c\phi_1 = 0, \tag{13}$$

where

$$\psi_1(\lambda_1) = \psi_{11}(\lambda_1) + k_1\psi_{12}(\lambda_1), \quad \psi_2(\lambda_1) = \psi_{21}(\lambda_1) + k_1\psi_{22}(\lambda_1),$$

$$\phi_1(\lambda_2) = \psi_{11}(\lambda_2) + k_2\psi_{12}(\lambda_2), \quad \phi_2(\lambda_2) = \psi_{21}(\lambda_2) + k_2\psi_{22}(\lambda_2). \tag{14}$$

Solving these linear equations we get

$$a = 2\mathrm{i}(\lambda_2\phi_1\psi_2 - \lambda_1\psi_1\phi_2)/\Delta, \quad b = 2\mathrm{i}(\lambda_1 - \lambda_2)\psi_1\phi_1/\Delta,$$

$$c = 2\mathrm{i}(\lambda_1 - \lambda_2)\psi_2\phi_2/\Delta, \quad d = 2\mathrm{i}(\lambda_2\phi_2\psi_1 - \lambda_1\psi_2\phi_1)/\Delta \tag{15}$$

with $\Delta = \psi_1\phi_2 - \psi_2\phi_1$. Substituting b and c from Eq. (15) into Eq. (9b) we get

$$\widetilde{q} = -q + 2(\lambda_1 - \lambda_2)\psi_1\phi_1/\Delta, \quad \widetilde{r} = -r + 2(\lambda_1 - \lambda_2)\psi_2\phi_2/\Delta. \tag{16}$$

Before proceeding further one must check the consistency of the space part of the old and new Lax operators with the DB transformation. From Eqs. (7) and (5) we get [4]

$$\widetilde{N} = (\partial_x T + TN)T^{-1}. \tag{17}$$

If we write

$$\widetilde{N} = \sum_{j=0}^{2} \lambda^j \begin{pmatrix} \widetilde{a}_j & \widetilde{b}_j \\ \widetilde{c}_j & -\widetilde{a}_j \end{pmatrix}, \tag{18}$$

one gets from Eq. (7b) $\widetilde{N}T = T_x + TN$. On substitution we get

$$\begin{pmatrix} a_x & b_x \\ c_x & d_x \end{pmatrix} = \widetilde{N} \begin{pmatrix} 2\mathrm{i}\lambda + a & b \\ c & -2\mathrm{i}\lambda + d \end{pmatrix} - N \begin{pmatrix} 2\mathrm{i}\lambda + a & b \\ c & -2\mathrm{i}\lambda + d \end{pmatrix}. \tag{19}$$

Equating various coefficients of λ^k, we get

$$\begin{array}{llllll}\tilde{a}_2=2\mathrm{i}, & a_2=2\mathrm{i}, & \tilde{a}_1=0, & a_1=0, & \tilde{a}_0=-\mathrm{i}\tilde{q}\tilde{r}, & a_0=-\mathrm{i}qr,\\ \tilde{b}_2=0, & b_2=0, & \tilde{b}_1=2\mathrm{i}\tilde{q}, & b_1=-2\mathrm{i}q, & \tilde{b}_0=?, & b_0=q_t,\\ \tilde{c}_2=0, & c_2=0, & \tilde{c}_1=2\mathrm{i}\tilde{r}, & c_1=2\mathrm{i}r, & \tilde{c}_0=?, & c_0=-r_t.\end{array} \tag{20}$$

So that $\tilde{N}$ is of the same form as N with (q,r) changed to $(\tilde{q},\tilde{r})$ which proves that the DB transformation is compatible and Eq. (7b) can be used to generate new solutions of the set (1) by starting with any seed solution.

3. Positon solution

Let us now note that a trivial solution of Eq. (1) is $q=r=0$. For this values of (q,r), the Lax eigenfunctions are found to be [5]

$$\Psi^0=\begin{pmatrix}\tilde{\psi}^0_{11} & \tilde{\psi}^0_{12}\\ \tilde{\psi}^0_{21} & \tilde{\psi}^0_{22}\end{pmatrix}=\begin{pmatrix}\exp[g(x,t,\lambda)] & 0\\ 0 & \exp[-g(x,t,\lambda)]\end{pmatrix} \tag{21}$$

with $g(x,t,\lambda)=\mathrm{i}(\lambda t+2\lambda^2 x)$.

For simplicity we choose $\lambda_1=\sigma$, $\lambda_2=2\sigma$ so that the wave function at $\lambda=\lambda_1$ is

$$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}=\begin{pmatrix}\exp[\mathrm{i}(8\sigma^2x+2\sigma t)]\\ \exp[-\mathrm{i}(8\sigma^2x+2\sigma t)]\end{pmatrix} \tag{22}$$

and that at $\lambda=\lambda_2$ it turns out to be

$$\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}=\begin{pmatrix}\exp[\mathrm{i}(2\sigma^2x+\sigma t)]\\ \exp[-\mathrm{i}(2\sigma^2x+\sigma t)]\end{pmatrix}, \tag{23}$$

so that the exact values of the coefficients (a,b,c,d) of Eq. (15) can be explicitly obtained. Finally, we get using Eq. (16)

$$\tilde{q}=\frac{-\mathrm{i}\sigma\exp[\mathrm{i}(10\sigma^2x+3\sigma t)]}{\sin(6\sigma^2x+\sigma t)},\quad \tilde{r}=\frac{-\mathrm{i}\sigma\exp[-\mathrm{i}(10\sigma^2x+3\sigma t)]}{\sin(6\sigma^2x+\sigma t)}. \tag{24}$$

We can now obtain the new eigenfunction $\tilde{\Psi}$, written as

$$\tilde{\Psi}=$$

$$\begin{pmatrix}[2\mathrm{i}\lambda-2\mathrm{i}\sigma-\sigma\exp(\mathrm{i}\theta)/\sin\theta]\mathrm{e}^g & (\sigma/\sin\theta)\exp[\mathrm{i}(3\theta-8\sigma^2x)-g]\\ (\sigma/\sin\theta)\exp[-\mathrm{i}(3\theta-8\sigma^2x)+g] & [-2\mathrm{i}\lambda+2\mathrm{i}\sigma-\sigma\exp(-\mathrm{i}\theta)/\sin\theta]\mathrm{e}^{-g}\end{pmatrix}, \tag{25}$$

where $g=\mathrm{i}(2\lambda^2x+\lambda t)$ and $\theta=(6\sigma^2x+\sigma t)$.

With this form of the eigenfunction at hand we can now repeat the above procedure again and construct the second DB transformation. For this purpose we choose $\tilde{\lambda}_1=-2\sigma$, $\tilde{\lambda}_2=-\sigma$ to get

$$\tilde{\tilde{q}}=-\tilde{q}-2\sigma\Upsilon_q(x,t)/\tilde{\Delta}, \tag{26}$$

$$\tilde{\tilde{r}}=-\tilde{r}-2\sigma\Upsilon_r(x,t)/\tilde{\Delta}, \tag{27}$$

where $\Upsilon_q(x,t)$ and $\Upsilon_r(x,t)$ are given as

$$\Upsilon_q(x,t) = -24\sigma^2 \exp\left[\mathrm{i}(10\sigma^2 x - 3\sigma t)\right] + \frac{\mathrm{i}\sigma^2}{\sin(6\sigma^2 x + \sigma t)}$$
$$\times\Big\{10\exp\left[\mathrm{i}(16\sigma^2 x - 2\sigma t)\right] - 6\exp[\mathrm{i}(16\sigma^2 x + 2\sigma t)] - 4\exp[[\mathrm{i}(4\sigma^2 x + 4\sigma t)]\Big\}$$
$$+\frac{\sigma^2}{\sin^2(6\sigma^2 x + \sigma t)}\Big\{\exp\left[\mathrm{i}(22\sigma^2 x - \sigma t)\right] - \exp\left[\mathrm{i}(22\sigma^2 x + 3\sigma t)\right]$$
$$+\exp\left[\mathrm{i}(10\sigma^2 x + 9\sigma t)\right] - \exp[\mathrm{i}(10\sigma^2 x + 5\sigma t)]\Big\}, \tag{28}$$

$$\Upsilon_r(x,t) = -24\sigma^2 \exp-\left[\mathrm{i}(10\sigma^2 x - 3\sigma t)\right] - \frac{\mathrm{i}\sigma^2}{\sin(6\sigma^2 x + \sigma t)}$$
$$\times\Big\{[10\exp\left[-\mathrm{i}(16\sigma^2 x - 2\sigma t)\right] - 6\exp[-\mathrm{i}(16\sigma^2 x + 2\sigma t)] - 4\exp[-\mathrm{i}(4\sigma^2 x + 4\sigma t)]\Big\}$$
$$+\frac{\sigma^2}{\sin^2(6\sigma^2 x + \sigma t)}\Big\{\exp[-\mathrm{i}(22\sigma^2 x - \sigma t)] - \exp[-\mathrm{i}(22\sigma^2 x + 3\sigma t)]$$
$$+\exp[-\mathrm{i}(10\sigma^2 x + 9\sigma t)] - \exp[-\mathrm{i}(10\sigma^2 x + 5\sigma t)]\Big\}, \tag{29}$$

$$\widetilde{\Delta} = \frac{8\mathrm{i}\sigma^2}{\sin(6\sigma^2 x + \sigma t)}[9\cos(2\sigma t) - \cos(6\sigma t) - 8\cos(12\sigma^2 x)]. \tag{30}$$

4. Positon from two-soliton solution

There is another very useful approach due to Jaworski et al. [6] which starts from a two-soliton solution of the equation under consideration and then taking a specialised limit to construct the positon solution. The two-soliton solution can easily be constructed in the bilinear formalism of Hirota. One sets

$$q = G/F; \; r = H/F \tag{31}$$

in Eq. (1) to get

$$(\mathrm{i}D_x - D_t^2 - 2g_0h_0)G\cdot F, \; (\mathrm{i}D_x + D_t^2 + 2g_0h_0)H\cdot F, \; (D_t^2 + 2g_0h_0)F\cdot F. \tag{32}$$

Here D is the Hirota operator and

$$F = 1 + \exp(\eta_1) + \exp(\eta_2) + a_{12}\exp(\eta_1 + \eta_2),$$

$$G = g_0[1 + b_1\exp(\eta_1) + b_2\exp(\eta_2) + b_{12}\exp(\eta_1 + \eta_2)],$$

$$G = h_0[1 + c_1\exp(\eta_1) + c_2\exp(\eta_2) + c_{12}\exp(\eta_1 + \eta_2)] \tag{33}$$

with

$$\eta_i = p_i x - \Omega_i t + \eta_i^0, \; c_i = 1/b_i, \; b_i = -\exp(2\mathrm{i}\xi_n) = -\frac{\Omega_n^2 + \mathrm{i}p_n}{\Omega_n^2 - \mathrm{i}p_n},$$

$$p_n^2 = -\Omega_n^2(\Omega_n^2 + 4\rho_0^2), \; g_0 = \rho_o\exp(-2\mathrm{i}\rho_0^2 x), \; h_0 = \rho_0\exp(2\mathrm{i}\rho_0^2 x). \tag{34}$$

The coefficients $a_{12}, \; b_{12}, \; c_{12}$ are given as

$$a_{ij} = N_a/D_a, \quad b_{ij} = N_b/D_b, \quad c_{ij} = N_c/D_c, \tag{35}$$

where

$$N_a = 2\rho_0^2[(\widetilde{\omega}_{ij}^2 + \widetilde{P}_{ij})\cos(2\xi_i) + (\widetilde{\omega}_{ij}^2 - \widetilde{P}_{ij})\cos(2\xi_j)$$
$$+2(2p_i\Omega_i\Omega_j - p_j\omega_{ij})\sin(2\xi_i) + 2(2p_j\Omega_i\Omega_j - p_i\omega_{ij})\sin(2\xi_j)$$
$$-4(\Omega_{ij}^4 + p_{ij}^2)\sin^2(2\xi_j - 2\xi_i)] - \widetilde{\Omega}_{ij}^2(\Omega_{ij}^4 + p_{ij}^2),$$
$$D_a = 4\rho_0^2\Omega_{ij}^4 + \omega_{ij}^6 + \Omega_{ij}^2 p_{ij}^2, \tag{35a}$$
$$N_b = -a_{ij}(\Omega_{ij}^2 + \mathrm{i}p_{ij})^2 + \widetilde{\omega}_{ij}^2[\exp(2\mathrm{i}\xi_i) + \exp(2\mathrm{i}\xi_j)] + \widetilde{P}_{ij}[\exp(2\mathrm{i}\xi_i) - \exp(2\mathrm{i}\xi_j)]$$
$$+\mathrm{i}[\exp(2\mathrm{i}\xi_i)(2p_j\omega_{ij} - 4p_i\Omega_i\Omega_j) + \exp(2\mathrm{i}\xi_j)(2p_i\omega_{ij} - 4p_j\Omega_i\Omega_j)], \tag{35b}$$
$$N_c = -a_{ij}(\Omega_{ij}^2 - \mathrm{i}p_{ij})^2 + \widetilde{\omega}_{ij}^2[\exp(-2\mathrm{i}\xi_i) + \exp(-2\mathrm{i}\xi_j)]$$
$$+\widetilde{P}_{ij}[\exp(-2\mathrm{i}\xi_i) - \exp(-2\mathrm{i}\xi_j)] - \mathrm{i}[\exp(-2\mathrm{i}\xi_i)(2p_j\omega_{ij} - 4p_i\Omega_i\Omega_j)$$
$$+\exp(-2\mathrm{i}\xi_j)(2p_i\omega_{ij} - 4p_j\Omega_i\Omega_j)], \tag{35c}$$
$$D_b = D_c = \Omega_{ij}^4 + p_{ij}^2 \tag{35d}$$

with

$$\Omega_{ij} = (\Omega_i + \Omega_j),\ \widetilde{\Omega}_{ij} = (\Omega_i - \Omega_j),\ \omega_{ij} = (\Omega_i^2 + \Omega_j^2),\ \widetilde{\omega}_{ij} = (\Omega_i^2 - \Omega_j^2),$$
$$p_{ij} = (p_i + p_j),\ \widetilde{p}_{ij} = (p_i - p_j),\ P_{ij} = (p_i^2 + p_j^2),\ \widetilde{P}_{ij} = (p_i^2 - p_j^2). \tag{35e}$$

We now impose the condition $p_1 = p = -p_2$. As a consequence we get

$$\xi_2 = \xi_1 = \xi \text{ (say)}, \quad \Omega_1^2 = \Omega_2^2 = \Omega^2 \text{ (say)}$$

and

$$a_{12} = \frac{\rho_0^2\sin(2\xi)}{2\Omega^2(\rho_0^2 + \Omega^2)}[p - \Omega^2\sin(2\xi)],$$
$$b_{12} = c_{12} = \frac{\sin(2\xi)}{8\Omega^4(\rho_0^2 + \Omega^2)}[p(8\rho_0^2\Omega^2 - \rho_0^2 - 8\Omega^4) + \rho_0^2\Omega^2\sin(2\xi)]. \tag{36}$$

along with the condition

$$\Omega^4 + 4\Omega^2\rho_0^2 + p^2 = 0 \tag{37}$$

leading to

$$\Omega_\pm^2 = -2\rho_0^2 \pm 2\rho_0^2(1 - p^2/4\rho_0^2)^{1/2}. \tag{38}$$

Now in the limit $p \to 0$, these two roots behave as

$$\Omega_+^2 \sim -p^2/8\rho_0^4, \quad \Omega_-^2 \sim -4\rho_0^2. \tag{39}$$

Furthermore one should note that for the situation $\Omega_-^2 \sim -4\rho_0^2$, i.e. $\Omega_- \sim 2\mathrm{i}\rho_0^2$,

$$a_{12} \sim \frac{-p^2}{48\rho_0^4}, \quad b_{12} = c_{12} \sim \frac{-p^2(1 + 96\rho_0^4)}{768\rho_0^2} = Q \text{ (say)}. \tag{40}$$

Let us now remember that in the definition of the wave fronts η_1 and η_2 we had two arbitrary constants η_1^0 and η_2^0. We choose

$$\exp(\eta_1^0) = \sqrt{48}\rho_0^2/p, \quad \exp(\eta_2^0) = -\sqrt{48}\rho_0^2/p, \tag{41}$$

so that as $p \to 0$ terms like $a_{ij}\exp(\eta_i + \eta_j)$ are finite.

Finally we get

$$\lim_{p\to 0} F = [1 + 8\sqrt{3}\rho_0^2 \exp(-\Omega t) + \exp(-2\Omega t)],$$

$$\lim_{p\to 0} G = \rho_0 \exp(-2\mathrm{i}\rho_0^2 x)[1 - 4\sqrt{3}(2\rho_0^2 x - \mathrm{i}) \exp(-\Omega t) + Q \exp(-2\Omega t)],$$

$$\lim_{p\to 0} H = \rho_0 \exp(2\mathrm{i}\rho_0^2 x)[1 - 4\sqrt{3}(2\rho_0^2 x + \mathrm{i}) \exp(-\Omega t) + Q \exp(-2\Omega t)]. \tag{42}$$

So we get the final form of the solution as

$$q = \lim_{p\to 0} \frac{G}{F}, \quad r = \lim_{p\to 0} \frac{H}{F}. \tag{43}$$

Note that these q and r are singular at

$$x = \frac{\cos(2\rho_0 t)}{4\sqrt{3}\rho_0^2}, \tag{44}$$

whereas the two-positon obtained through the DB transformation is seen to be singular at

$$x = \cos^{-1}\left[\frac{3}{2}\cos(2\sigma t) - \frac{1}{2}\cos^3(2\sigma t)\right]. \tag{45}$$

Apparently the forms of the two curves given by Eqs. (44) and (45) are totally different, but as shown in Figs. 1 and 2 they show almost a similar variation.

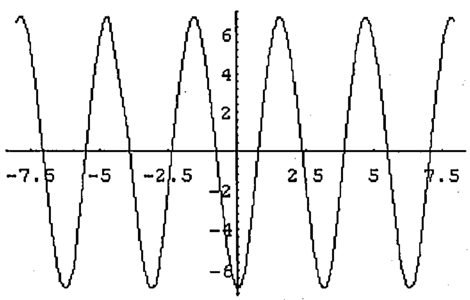

Fig. 1

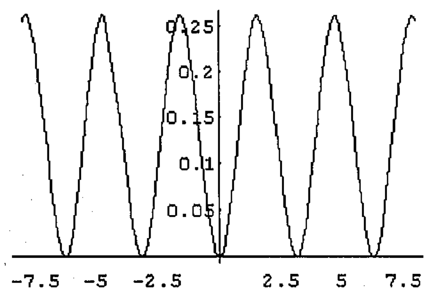

Fig. 2

Fig. 1. Position of poles, Eq. (44) of positon solution obtained as a limit of a 2-soliton expression, x vs. t, $\sigma = 1$.

Fig. 2. Position of poles, Eq. (45) of positon solution obtained by DB transformation method, x vs. t, $\sigma = 1$.

5. Conclusions and discussions

In our above analysis we have obtained one- and two-positon solutions of the nonlinear integrable system which describes coupled optical pulse propagation. This has been done in two ways. One with the help of Darboux–Bäcklund transformation and the other with the help of a limiting procedure on multisoliton states. We have exhibited the space-time variation of such solutions in Figs. 3 and 4 for the one-positon obtained through two different methods, while the two--positon is depicted in Fig. 5. Note that to construct a two-positon by the limiting procedure one has to start from a four-soliton state which we have not done in this communication. In this respect there is some advantage of the DB transformation over the latter approach.

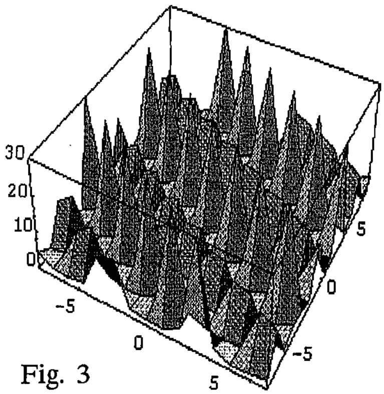

Fig. 3

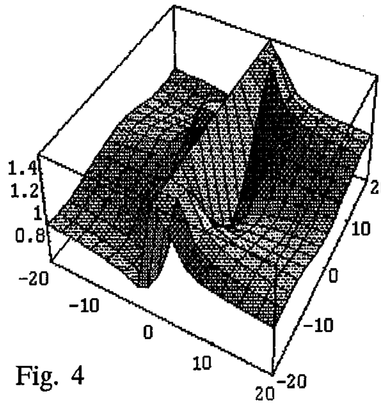

Fig. 4

Fig. 3. One-positon, Eq. (24), obtained by DB transformation, mod q vs. x and t, $\sigma = 1$.

Fig. 4 One-positon, Eq. (43), obtained as limiting procedure on 2-soliton state, mod q vs. x and t, $\rho = 1$.

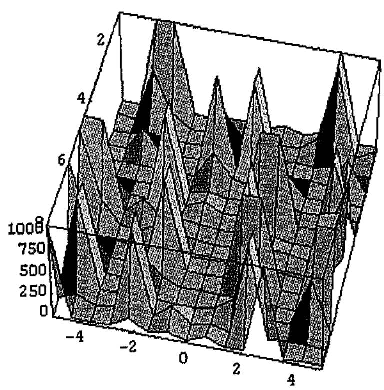

Fig. 5. Two-positon, Eq. (26), obtained by DB transformation, mod q vs. x and t, $\sigma = 1$.

One of the authors (D.C.S.) is grateful to Govt. of India for a Research Associateship (C.S.I.R) which made this work possible. Moreover the authors are grateful to the referee for his valuable suggestions.

References

[1] A. Hasegawa, *Optical Solitons in Fibers*, Springer-Verlag, Berlin 1989; A. Hasegawa, F. Tappert, *Appl. Phys. Lett.* **23**, 703 (1973).

[2] A.A. Sthaofen, *Ann. der Phys.* **128**, 33 (1994).

[3] M.A. Salle, V.B. Matveev, *Darboux Transformation and Solitons*, Springer-Verlag, Berlin 1991.

[4] *Bäcklund Transformation*, Eds. R. Eckmann, R.K. Dodd, in series *Lecture Notes in Mathematics*, Vol. 515, Springer-Verlag, Berlin 1980.

[5] K.W. Chow, W.C. Lai, C.K. Shek, K. Tso, *Chaos, Solitons, Fractals* **9**, 1901 (1998).

[6] M. Jaworski, J. Zagrodziński, *Chaos, Solitons, Fractals* **5**, 2229 (1995).