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SUPERCONDUCTING PROPERTIES OF THE CHARGED WEAKLY INTERACTING BOSE GAS ("d" SYMMETRY)

Z.M. GALASIEWICZ

Institute of Physics, Wrocław University of Technology
Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland

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A system of charged weakly interacting bosons with "d" symmetry is considered. The interaction $U^{(d)} = U^{(s)} f(\theta)$ contains the angle dependence $f(\theta) \sim |Y_{2,1}|^2$ consistent with "d" symmetry. This system resembles much more free bosons than the system with "s" symmetry (interaction $U^{(s)}$). For example at $T = 0$, $n \approx n_c^{(d)}$, where n, $n_c^{(d)}$ are the densities of particles and condensate respectively.

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Most results of the recent experiments, specially for the cuprate high temperature superconductors, are consistent with the "d-wave" symmetry [1, 2]. More precisely, important is the particular form of the "d-wave" known as the $d_{x^2-y^2}$ one, connected with $l = 2$ and $m \pm 1$.

Superconducting properties of the charged free and weakly interacting Bose gas with "s" symmetry were considered in Refs. [3–5] respectively. The ground state of the weakly interacting bosons has been considered already in 1947 by Bogolyubov [6].

For the case of "s" symmetry the interaction was described by $U^{(s)} = 2\pi\hbar^2 a/m$, where "$a$" denotes the scattering length. In case considered here "d" symmetry $U^{(d)}$ contains the angle dependence $f(\theta)$ proportional to $\frac{1}{2}\left(Y_{2,1}Y_{2,1}^{*} + Y_{2,-1}Y_{2,-1}^{*}\right)$, see e.g. [7], where $Y_{2,\pm 1} \sim \sin\theta\cos\theta\exp(\pm \mathrm{i}\phi)$, $l = 2$, $m = \pm 1$ i.e.

$$U^{(d)} = U^{(s)} f(\theta), \quad f(\theta) = (\sin\theta\cos\theta)^2, \quad \sqrt{f(\theta)} = |\sin\theta\cos\theta| > 0. \tag{1}$$

The considered now system is described by the Hamiltonian

$$\widehat{H} = E_0 + \sum_{p\neq 0} \frac{p^2}{2m} a_p^{+} a_p + U^{(s)} \frac{N}{V} \sum_{p\neq 0} f(\theta)(a_p a_{-p} + a_{-p}^{+} a_p^{+} + 2a_p^{+} a_p), \tag{2}$$

$$U^{(s)} \frac{N}{V} = \frac{mu^2}{2}, \quad u = \frac{\sqrt{\pi a N/V}\, 2\hbar}{m},$$

where "u" is the sound velocity.

After Bogolyubov [6] unitary transformation $(a^+, a) \to (b^+, b)$ the diagonalized Hamiltonian (2) has the form

$$\widehat{H} = \text{const} + \sum_{p\neq 0} \varepsilon(p,\theta) b_p^+ b_p, \quad \varepsilon(p,\theta) = \sqrt{\left(\frac{p^2}{2m}\right)^2 + (up)^2 f(\theta)}. \tag{3}$$

On the basis of the flow properties one can express the density of bosons n as a sum of the density of the normal component n_n and the density n_s of the superfluid component ($n = n_\mathrm{s} + n_\mathrm{n}$).

On the other hand, after use of the Bogolyubov transformation to the operator $a_p^+ a_p$ one can, like in [4], introduce the alternative decomposition of the density n. Namely

$$n = n_\mathrm{n}^{(d)}(\gamma, T) + n_\mathrm{s}^{(d)}(\gamma, T) = \frac{1}{V}\sum_p a_p^+ a_p$$

$$= n_\mathrm{c}^{(d)}(\gamma, T) + n_\mathrm{int}^{(d)}(u) + n_\mathrm{ex}^{(d)}(\gamma, T), \qquad \gamma = \frac{mu^2}{k_B T}. \tag{4}$$

(The dimensionless parameter γ was denoted in [4, 5] by "s".)

In Eq. (4) $n_\mathrm{c}^{(d)}(\gamma, T)$ describes the density of the Bose condensate. The density $n_\mathrm{int}^{(d)}$ presents bosons which due to interactions $U^{(d)}$ are at $T = 0$ not in condensate. In case of noninteracting bosons at $T = 0$, 100% of particles is in the Bose condensate. On the other hand, in superfluid He^4 at $T = 0$, because of strong interaction, 90% of particles is not in the condensate.

Finally, $n_\mathrm{ex}^{(d)}(\gamma, T)$ is the excitations density described by the Bose distribution function.

Now, in case of interaction $U^{(d)}(\theta)$ we have for $n_\mathrm{int}^{(d)}$ the following expression:

$$n_\mathrm{int}^{(d)}(u) = \frac{1}{2\pi^2\hbar^3}\int_0^L p^2 \mathrm{d}p \frac{1}{2}\int_0^\pi \left[\frac{p^2}{2m} + mu^2 f(\theta) - \varepsilon(p,\theta)\right] \frac{\sin\theta \mathrm{d}\theta}{2\varepsilon(p,\theta)}\bigg|_{L\to\infty}$$

$$= n_\mathrm{int}^{(s)}(u) \overset{(1)}{I^{(d)}} = 0.057 n_\mathrm{int}^{(s)}(u), \tag{5}$$

$$\overset{(1)}{I^{(d)}} = \frac{1}{2}\int_0^\pi f(\theta)\sqrt{f(\theta)}\sin\theta \mathrm{d}\theta = \frac{1}{2}\int_0^\pi \sin^4\theta |\cos^3\theta| \mathrm{d}\theta = 0.057.$$

We see that in case of "d" symmetry $n_\mathrm{int}^{(d)}$ is two orders of magnitude smaller than $n_\mathrm{int}^{(s)}$.

For $n_\mathrm{ex}(\gamma, T)$ appearing in (4), using formula for critical temperature T_c^0 for free Bose gas, we have

$$n_\mathrm{ex}^{(d)}(\gamma, T) = \frac{(mk_\mathrm{B}T)^{3/2}}{2^{1/2}\pi^2\hbar^3} J^{(d)}(\gamma) = n\left(\frac{T}{T_\mathrm{c}^0}\right)^{3/2} \frac{J^{(d)}(\gamma)}{J(0)}, \tag{6}$$

$$J(0) = \int_0^\infty \frac{x^{1/2}\mathrm{d}x}{\mathrm{e}^x - 1} = 2.315,$$

where

$$J^{(d)}(\gamma) = \int_0^\infty \frac{x\mathrm{d}x}{\mathrm{e}^x - 1} \overset{(2)}{I}(x,\gamma)$$

$$= \gamma^{3/2} \int_0^\infty \frac{z\mathrm{d}z}{\left(\sqrt{z^2+1}+1\right)^{1/2}} \frac{1}{2} \int_0^\pi \frac{f(\theta)\sqrt{f(\theta)}\sin\theta\mathrm{d}\theta}{\mathrm{e}^{z\gamma f(\theta)} - 1}, \tag{7}$$

and

$$\overset{(2)}{I}(x,\gamma) = \frac{1}{2}\int_0^\pi \frac{\sin\theta\mathrm{d}\theta}{\left[\sqrt{x^2+\gamma^2 f^2(\theta)}+\gamma f(\theta)\right]^{1/2}} = \frac{1}{\sqrt{x}} - \overset{(3)}{I}(x,\gamma), \tag{8}$$

$$\overset{(3)}{I}(x,\gamma) = \frac{\gamma}{2}\int_0^\pi \frac{\cos^2\theta(\cos^2\theta - \sin^2\theta)\sin\theta\mathrm{d}\theta}{\left[\sqrt{x^2+\gamma^2 f^2(\theta)}+\gamma f(\theta)\right]^{3/2}}$$

$$\times \left[\frac{\gamma f(\theta)}{\sqrt{x^2+\gamma^2 f^2(\theta)}} + 1\right] > 0. \tag{9}$$

Finally, we have

$$J^{(d)}(\gamma) = J(0) - \int_0^\infty \frac{x\mathrm{d}x}{\mathrm{e}^x - 1} \overset{(3)}{I}(x,\gamma). \tag{10}$$

From (8), (9) and form of $I^{(s)}(\gamma)$ (see [4]) follows

$$J(0) > J^{(d)}(\gamma) > J^{(s)}(\gamma). \tag{11}$$

The second version of $J^{(d)}(\gamma)$, formula (7), is convenient for a power series expansion in powers of $\gamma f(\theta)$.

Formula (6) shows that $n_{\mathrm{ex}}^{(d)}$ vanishes at $T = 0$ i.e.

$$n_{\mathrm{ex}}^{(d)}(\gamma, 0) = 0. \tag{12}$$

From (4), (5) and (12) follows that at $T = 0$

$$n = n_{\mathrm{c}}^{(d)}(\gamma,0) + n_{\mathrm{int}}^{(d)}(u) = n_{\mathrm{c}}^{(d)}(\gamma,0) + 0.057 n_{\mathrm{int}}^{(s)}(u) \approx n_{\mathrm{c}}^{(d)}(\gamma,0), \tag{13}$$

where $n_{\mathrm{int}}^{(s)} < n$. Therefore, at least 94% of bosons is in the Bose condensate. In comparison to the system with "s" symmetry the system with the "d" one behaves much more similar to the free bosons system.

As we mentioned above, the density of the normal component $n_{\mathrm{n}}(\gamma, T)$ is calculated from the flow properties of the excitations with energies $\varepsilon = \varepsilon(p,\theta) + pv$. In the power series development of the averaged current we consider only term proportional to v, where v denotes relative velocity between normal and superfluid component.

In order to find relation between $n_{\mathrm{ex}}^{(d)}$ and $n_{\mathrm{n}}^{(d)}$ we rewrite expression for $n_{\mathrm{ex}}^{(d)}$ in the form (see [4])

$$n_{\mathrm{ex}}^{(d)}(\gamma, T) = \frac{\sqrt{2}(mu)^3}{3\pi^2\hbar^3} J^{(d)}(\gamma), \qquad (mu)^3 = (mk_{\mathrm{B}}T)^{3/2}\gamma^{3/2}. \tag{14}$$

Here $I^{(d)}(\gamma)$ is given in terms of the second version presented in (7).

For the density of the normal component we have

$$n_{\mathrm{n}}^{(d)}(\gamma,T)=\frac{\sqrt{2}(mu)^3}{3\pi^2\hbar^3}\frac{1}{2}\gamma^{3/2}\int_0^\infty\int_0^\pi\frac{f(\theta)\sqrt{f(\theta)}\sin\theta\mathrm{d}\theta\mathrm{d}z}{\mathrm{e}^{z\gamma f(\theta)}-1}$$
$$\times\frac{\mathrm{d}}{\mathrm{d}z}\left[\frac{\left(\sqrt{z^2+1}-1\right)^{3/2}z}{\sqrt{z^2+1}}\right], \tag{15}$$

where

$$\frac{\mathrm{d}}{\mathrm{d}z}\left[\frac{\left(\sqrt{z^2+1}-1\right)^{3/2}z}{\sqrt{z^2+1}}\right]$$
$$=\frac{3}{2}\frac{z}{\left(\sqrt{z^2+1}+1\right)^{1/2}}-\frac{\left(\sqrt{z^2+1}+2\right)\left(\sqrt{z^2+1}-1\right)^{1/2}}{2\left(\sqrt{z^2+1}\right)^3}. \tag{16}$$

The first term of the right hand side of (16) appears in (7). From (7), (14), (15) and (16) follows

$$n_{\mathrm{n}}^{(d)}(\gamma,T)=n_{\mathrm{ex}}^{(d)}(\gamma,T)-\frac{2}{3}n\left(\frac{T}{T_{\mathrm{c}}^0}\right)^{3/2}\gamma^{1/2}\frac{K^{(d)}(\gamma)}{J(0)}. \tag{17}$$

The integral $K^{(d)}$ is expressed by the second term on the right hand side of (16) as follows:

$$K^{(d)}(\gamma)=\frac{\gamma}{2}\int_0^\infty\int_0^\pi\frac{f(\theta)\sqrt{f(\theta)}\sin\theta\mathrm{d}\theta\mathrm{d}z}{\mathrm{e}^{z\gamma f(\theta)}-1}\frac{\left(\sqrt{z^2+1}+2\right)\left(\sqrt{z^2+1}-1\right)^{1/2}}{2\left(\sqrt{z^2+1}\right)^3}$$
$$=\frac{\gamma^{3/2}}{2}\int_0^\infty\int_0^\pi\frac{\mathrm{d}z\sin\theta\mathrm{d}\theta}{\mathrm{e}^z-1}$$
$$\times\frac{\left[\sqrt{z^2+\gamma^2f^2(\theta)}+2\gamma f(\theta)\right]\left[\sqrt{z^2+\gamma^2f^2(\theta)}-\gamma f(\theta)\right]^{1/2}}{2\left[\sqrt{z^2+\gamma^2f^2(\theta)}\right]^3}. \tag{18}$$

Hence

$$n_{\mathrm{ex}}^{(d)}(\gamma,T)\geq n_{\mathrm{n}}^{(d)}.$$

The first expression for $K^{(d)}$ in (18) is convenient for a power series expansion.

The critical temperature T_{c} is determined from the condition that the density of the condensate $n_{\mathrm{c}}^{(d)}(\gamma,T)$ should vanish at $T=T_{\mathrm{c}}$. From (4), (5) and (6) we have (see also (11))

$$1=\frac{n_{\mathrm{int}}^{(d)}}{n}+\left(\frac{T_{\mathrm{c}}^{(d)}}{T_{\mathrm{c}}^0}\right)^{3/2}\frac{J^{(d)}\left(\gamma_{\mathrm{c}},T_{\mathrm{c}}^{(d)}\right)}{J(0)},\qquad J(0)>J^{(d)}. \tag{19}$$

Because $n_{\mathrm{int}}^{(s)}/n<1$, it follows from (5) that $n_{\mathrm{int}}^{(d)}/n=0.057n_{\mathrm{int}}^{(s)}\ll 1$ and can be neglected in (19). Equation (19) has now the form (see (11))

$$\left(\frac{T_{\mathrm{c}}^{(d)}}{T_{\mathrm{c}}^0}\right)^{3/2}=\frac{J(0)}{J^{(d)}\left(\gamma_{\mathrm{c}},T_{\mathrm{c}}^{(d)}\right)}>1,\qquad T_{\mathrm{c}}^{(d)}>T_{\mathrm{c}}^0. \tag{20}$$

For the "s" symmetry considered in [4] there are estimations of the integrals $I^{(s)}(\gamma)$, $K^{(s)}(\gamma)$ as a series expansion in powers of γ. Namely

$$J^{(s)}(\gamma) = 2.315 - 2.243\gamma^{1/2} + 1.294\gamma - 0.471\gamma^{3/2},$$
$$K^{(s)}(\gamma) = 1.122 - 0.706\gamma. \tag{21}$$

After integration over angles of suitable terms in (21) we get approximate expressions for the case of "d" symmetry. In case $J^{(s)}(\gamma) \to J^{(d)}(\gamma)$

$$\gamma^\alpha \to \frac{1}{2}\gamma^\alpha \int_0^\pi f^\alpha(\theta)\sin\theta \mathrm{d}\theta$$

and we have

$$J^{(d)}(\gamma) = 2.315 - 0.747\gamma^{1/2} + 0.172\gamma - 0.027\gamma^{3/2} < J^{(s)}(\gamma). \tag{22}$$

In case $K^{(s)}(\gamma) \to K^{(d)}(\gamma)$

$$\gamma^\alpha \to \frac{1}{2}\gamma^\alpha \int_0^\pi f^{\alpha+1/2}(\theta)\sin\theta \mathrm{d}\theta$$

and

$$K^{(d)}(\gamma) = 0.374 - 0.05\gamma < K^{(s)}(\gamma). \tag{23}$$

Now we will perform estimations of (22), (23) for γ in the interval $0 \leq \gamma \leq 0.5$. The mean values (arithmetic) $\bar{J}^{(s)}$, $\bar{J}^{(d)}$ for integrals $J^{(s)}(\gamma)$, $J^{(d)}(\gamma)$ are

$$\bar{J}^{(s)} = 1.75, \qquad \bar{J}^{(d)} = 2.1. \tag{24}$$

This leads to the relations (see (20))

$$T_{\mathrm{c}}^{(d)} = 1.065 T_{\mathrm{c}}^0 \approx T_{\mathrm{c}}^0, \qquad \mathrm{T}_{\mathrm{c}}^{(\mathrm{s})} = 1.214\mathrm{T}_{\mathrm{c}}^0. \tag{25}$$

The mean values of integrals $K^{(s)}(\gamma)$, $K^{(d)}(\gamma)$ are

$$\bar{K}^{(s)} = 0.995, \quad \bar{K}^{(d)} = 0.365 = 0.367 K^{(s)}. \tag{26}$$

From (4), (5) we have

$$n_{\mathrm{s}}^{(d)} - n_{\mathrm{c}}^{(d)} = n_{\mathrm{int}}^{(d)} + n_{\mathrm{ex}}^{(d)} - n_{\mathrm{n}}^{(d)} = 0.057 n_{\mathrm{int}}^{(s)} + n_{\mathrm{ex}}^{(d)} - n_{\mathrm{n}}^{(d)}. \tag{27}$$

From (16) and (27) follows

$$n_{\mathrm{s}}^{(d)} - n_{\mathrm{c}}^{(d)} = 0.057 n_{\mathrm{int}}^{(s)} + \frac{2}{3} n \left(\frac{T}{T_{\mathrm{c}}^0}\right)^{3/2} \gamma^{1/2} 0.367 \bar{K}^{(s)} < n_{\mathrm{s}}^{(s)} - n_{\mathrm{c}}^{(s)}. \tag{28}$$

We see that there is more Bose condensate in the density of superfluid component $n_{\mathrm{s}}^{(d)}$ than in $n_{\mathrm{s}}^{(s)}$.

As concerns electrodynamics it was shown in [8] that in case of interaction $U^{(s)} \neq 0$ in the Schafroth [3] formula for supercurrent of free bosons the density of the condensate n_{c} should be replaced by the density of superfluid component n_{s}. Namely ($U^{(s)} \neq 0$)

$$j^{(s)} = -\frac{e^2}{mc} n_{\mathrm{s}}^{(s)} \boldsymbol{A}. \tag{29}$$

From (28) we see that especially when $T \to 0$, $j^{(d)}$ ($U^{(d)} \neq 0$) can be considered in good approximation as proportional to $n_{\mathrm{c}}^{(d)}$ like in the Schafroth case for noninteracting bosons.

From (13), (25), (28) follows that considered here system of weakly interacting bosons with "*d*" symmetry ($U^{(d)} = U^{(s)} f(\theta)$) resembles much more the system of free bosons than the system with "*s*" symmetry [4, 5].

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