Vol. 97 (2000) *ACTA PHYSICA POLONICA A* No. 5

Proceedings of the European Conference "Physics of Magnetism '99", Poznań 1999

SYNCHRONIZATION AMONG SPIN-WAVE AMPLITUDES IN CHAOTIC NONLINEAR FERROMAGNETIC RESONANCE

A. Krawiecki

Institute of Physics, Warsaw University of Technology
Koszykowa 75, 00-662 Warsaw, Poland

A model of chaos in high-power ferromagnetic resonance in coincidence regime, based on three-magnon interactions of the uniform mode with a group of pairs of parametric spin waves, is investigated numerically. The results are interpreted from the point of view of chaotic synchronization theory. If all spin waves are identical, all of them are excited above the first-order Suhl instability threshold and in the chaotic regime their amplitudes show marginal synchronization, i.e. they differ only by a multiplicative factor, constant in time. If spin waves have slightly different instability thresholds, only one or few of them are excited. In the latter case, addition of weak thermal noise changes the results qualitatively. For low rf field amplitude, but above the threshold for chaos, still only few spin-wave pairs are excited above the thermal level. For higher rf field amplitude all spin waves in the group are excited, and their amplitudes are not synchronized. These results suggest that low correlation dimension of chaotic attractors, observed often in nonlinear ferromagnetic resonance, can be connected with chaotic synchronization among spin-wave amplitudes, in particular just above the threshold for chaos.

PACS numbers: 76.50.+g, 75.30.Ds, 05.45.+b

Synchronization between two chaotic systems or subsystems x_1, x_2 occurs if $x_1(t) = x_2(t)$ after all transients die out, independently of the initial conditions [1]. An extension of this concept is marginal synchronization (MS): one of its forms occurs when $x_1(t) = Ax_2(t)$, where A is a constant which may depend on the initial conditions, but not on time [2, 3]. In chaotic nonlinear ferromagnetic resonance (FMR) only synchronization between external chaotic modulation of the dc field and absorption in the sample was reported [4]. In this paper synchronization among chaotic amplitudes of spin waves (SW) excited in the first-order Suhl instability process [5] is considered.

We consider a model of high-power FMR in coincidence regime, i.e. when the uniform mode with frequency ω_0 is driven by the rf field with frequency ω close to ω_0 and amplitude h_T. If h_T exceeds the first-order Suhl instability threshold

$h_T^{\rm thr}$, the uniform mode decays into pairs of SW with opposite wave vectors and frequencies $\omega_n \approx \omega/2$ [5]. The equations of motion for the slowly varying in time parts of the complex amplitudes of SW pairs a_n and of the uniform mode amplitude a_0 can be written as [6]

$$\begin{aligned}\dot{a}_0 &= |\eta_0 + {\rm i}\Delta\omega_0|\,|\eta_1 + {\rm i}\Delta\omega_1|\,\varepsilon - (\eta_0 + {\rm i}\Delta\omega_0)\,a_0 - {\rm i}\sum_n V_{0n} a_n^2 + \xi_{\rm th},\\ \dot{a}_n &= -\,(\eta_n + {\rm i}\Delta\omega_n)\,a_n - {\rm i}V_{0n}^* a_n^* a_0 + \xi_{\rm th},\end{aligned} \tag{1}$$

where $n = 1, 2 \ldots N$ labels the SW pairs, $\varepsilon = h_T/h_T^{\rm thr}$, $\eta_{0,n}$ is phenomenological damping, $\Delta\omega_0 = \omega_0 - \omega$, $\Delta\omega_n = \omega_n - \omega/2$, V_{0n} are coefficients of three-magnon interactions between the SW pair n and the uniform mode, $\xi_{\rm th}$ is the level of thermal excitation of SW, and the dot denotes derivative with respect to rescaled time $t' = \eta_1 t$. In Eq. (1) all dampings, detunings and SW amplitudes are normalized to η_1 and V_{0n} to V_{01}, so all these quantities are dimensionless. In real samples in liquid helium temperature $\eta_1 \propto 10^6$ s^{-1} and $\xi_{\rm th} \propto 10^{-10}$. We solved (1) numerically with $N = 12$, $\eta_0 = 1.25$ and $\Delta\omega_0 = -1.5$.

First we consider N identical SW with all $\Delta\omega_n = 3.0$ and $\eta_n = V_{0n} = 1.0$. For these parameters the system (1) is chaotic for $\varepsilon > 1.95$ (Fig. 1) [6] and all SW are excited. Nevertheless, for $\xi_{\rm th} = 0$ the SW amplitudes show MS, i.e. for any m, n we have $|a_m| = A_{m,n}\,|a_n|$, where $A_{m,n}$ is a constant depending on the initial conditions (Fig. 2a). Setting $\xi_{\rm th} = 10^{-10}$ causes that A_{mn} changes very slowly with time, but this effect is difficult to observe even for long time intervals.

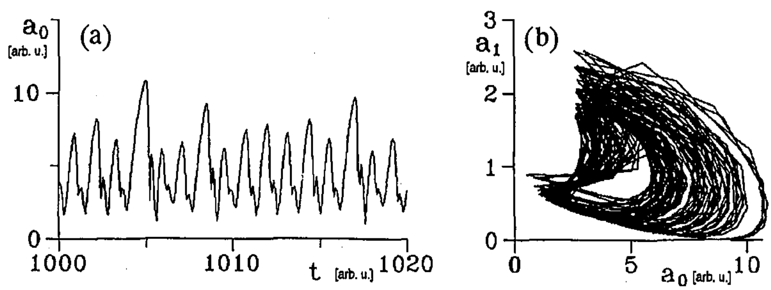

Fig. 1. (a) Time series of $|a_0|$ and (b) chaotic attractor at $\varepsilon = 3.5$ for the case of identical SW and $\xi_{\rm th} = 0$.

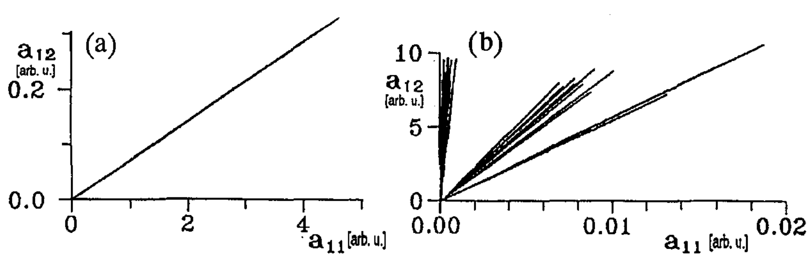

Fig. 2. (a) MS of two identical SW and (b) lack of MS of two slightly different SW; $\varepsilon = 3.5$, $\xi_{\rm th} = 0$.

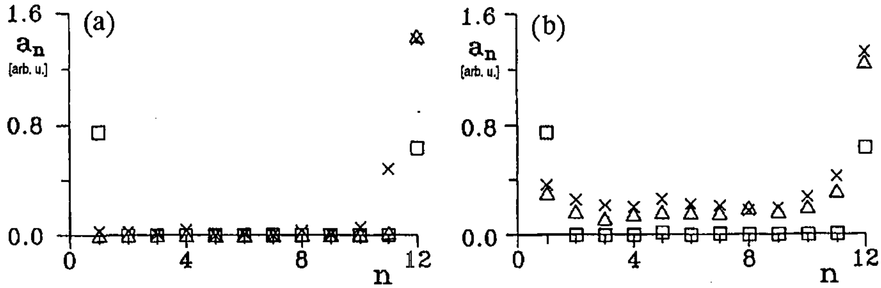

Fig. 3. Mean values of SW amplitudes a_n for the case of slightly different SW: (a) $\xi_{th} = 0$, (b) $\xi_{th} = 10^{-10}$; $\varepsilon = 2.0$ (squares), 3.0 (triangles), 3.5 (crosses).

Second we consider N slightly different SW with $\Delta\omega_n = 3.0 + 0.01n$, $\eta_n = 1.0 + 0.01n$ and $V_{0n} = 1.0 - 0.01n$. In this case MS among SW amplitudes is lost (Fig. 2b). The absolute values of amplitudes of individual SW, averaged over a long time interval, are shown in Fig. 3 for different values of ε. If $\xi_{\mathrm{th}} = 0$, then for a wide range of ε above the threshold for chaos only separate SW are excited (Fig. 3a). The number of excited SW may even decrease with increasing rf field amplitude (two with $n = 1$ and $n = 12$ for $\varepsilon = 2.0$, one with $n = 12$ for $\varepsilon = 3.0$). Only for $\varepsilon = 3.5$ two neighbouring SW with $n = 11$ and $n = 12$ form a narrow packet of excited modes. If $\xi_{\mathrm{th}} = 10^{-10}$, then separate SW are excited only just above the threshold for chaos, but for increasing rf field amplitude all SW become quickly excited (Fig. 3b). In the latter case these SW, which are also most easily excited in the case without thermal noise, have the highest mean amplitudes (SW with $n = 1$ and $n = 12$). The mean amplitudes of other SW are an order of magnitude lower. It can be observed that the time series of $|a_n|$ with $1 < n < 12$ consist of long intervals during which the amplitudes are almost equal to zero, and chaotic bursts, during which $|a_n|$ are on the order of 1.0 (not shown). This resembles on-off intermittency [7], predicted so far only for two SW pairs with significantly different parameters [6].

In the models of chaos in high-power FMR in coincidence regime considered so far [8, 6], interactions among a number of significantly different SW pairs and the uniform mode have usually been taken into account. In this paper the interacting SW are either identical or only slightly different, which enables one to study synchronization among SW amplitudes. The properties of the system (1) are typical of a class of systems with MS [9]. Such systems consist of many subsystems described by equations of motion whose form is identical for all subsystems, but whose parameters can be different. In such systems groups of identical subsystems can be separated so that the subsystems within each group, but not the ones belonging to different groups, show MS. In the present case such subsystems are SW pairs. If the subsystems belonging to two groups have significantly different parameters, the variables connected with all subsystems can be chaotic and assume non-zero values. On the other hand, if the two groups are almost identical, it can happen that only variables connected with subsystems belonging to one group are chaotic. In the latter case, the variables connected with subsystems of the second group decay to zero or even diverge to infinity [9]. The results of this paper suggest that chaotic SW dynamics can be used to study the properties of MS in an experimental system.

It should be noted that the phenomena similar to the ones considered in this paper for the case of chaotic SW dynamics are well known in the stationary state of high-power FMR in parallel pumping [10] and main resonance [11]. In these cases, if the SW excited above the instability threshold are identical, their individual amplitudes can be arbitrary, and only sum of them is a given function of the rf field amplitude. Small differences between SW pairs cause that only one of them is excited. The results presented here may also, to some extent, explain low values of the correlation dimension of chaotic attractors observed experimentally, in particular just above the threshold for chaos, in high-power FMR [12]. This dimension is close to the number of degrees of freedom of the chaotic system. The amplitudes of identical SW in chaotic high-power FMR show MS and thus they are not independent variables, while from a set of slightly different SW pairs only separate pairs are strongly excited. The latter hypothesis must be verified by investigating more complicated, high-dimensional models of chaos in high-power FMR.

References

[1] L.M. Pecora, T.L. Carroll, G.A. Johnson, D.J. Mar, J.F. Heagy, *Chaos* **7**, 520 (1997).

[2] J.M. González-Miranda, *Phys. Rev. E* **53**, R5 (1996).

[3] M.A. Matías, J. Güémez, C. Martín, *Phys. Lett. A* **226**, 264 (1997).

[4] D.W. Peterman, M. Ye, P.E. Wigen, *Phys. Rev. Lett.* **74**, 1740 (1995).

[5] H. Suhl, *J. Phys. Chem. Solids* **1**, 209 (1957).

[6] A. Krawiecki, A. Sukiennicki, *Acta Phys. Pol. A* **88**, 269 (1995).

[7] J.F. Heagy, N. Platt, S.M. Hammel, *Phys. Rev. E* **49**, 1140 (1994).

[8] G. Wiese, H. Benner, *Z. Phys. B, Condens. Matter* **79**, 119 (1990).

[9] A. Krawiecki, A. Sukiennicki, *Chaos, Solitons & Fractals*, to be published.

[10] V. Zakharov, V. Lvov, S.S. Starobinets, *Usp. Fiz. Nauk* **114**, 609 (1974) [*Sov. Phys.-Usp.* **17**, 869 (1975)].

[11] R.D. McMichael, P.E. Wigen, *Phys. Rev. B* **42**, 6723 (1990).

[12] S. Mitsudo, M. Mino, H. Yamazaki, *J. Magn. Magn. Mater.* **104-107**, 1057 (1992).