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FORMATION OF CAVITY SOLITONS VIA VECTORIAL SECOND HARMONIC GENERATION

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We demonstrate that symmetry breaking in intra-cavity vectorial second harmonic generation may cause the formation of different types of localized structures. Moving as well as resting cavity solitons are observed and the interaction between them is studied.

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Recently intra-cavity scalar and vectorial second-harmonic generation have attracted renewed interest due to their potential applications for all-optical information processing and storage. In the scalar or degenerate case it was found that stable patterns [1] as well as localized structures (cavity solitons) [2] may be formed in the course of the up-conversion process. If two photons of orthogonally polarized fundamental (FH) waves $A_{1/2}$ generate a second harmonic (SH) B an even richer diversity of dynamical effects can be expected. For a balanced (or symmetric) input the scalar effects are recovered, but in addition symmetry breaking occurs. This may be the basis of striking spatio-temporal effects which are the subject of this work.

The scaled evolution equations in a high-finesse Fabry–Pérot resonator are

$$\left[\mathrm{i}\frac{\partial}{\partial T}+\frac{\partial^2}{\partial X^2}+\frac{\partial^2}{\partial Y^2}+\Delta_A+\mathrm{i}\right]A_1+A_2^*B=E,$$

$$\left[\mathrm{i}\frac{\partial}{\partial T}+\frac{\partial^2}{\partial X^2}+\frac{\partial^2}{\partial Y^2}+\Delta_A+\mathrm{i}\right]A_2+A_1^*B=E,$$

$$\left[\mathrm{i}\frac{\partial}{\partial T}+\frac{1}{2}\left(\frac{\partial^2}{\partial X^2}+\frac{\partial^2}{\partial Y^2}\right)+\Delta_B+\mathrm{i}\gamma\right]B+A_1A_2=0, \tag{1}$$

where X and Y denote the coordinates parallel to the interfaces scaled with the inverse linewidth of the FH resonance. The time T is measured in units of the FH photon lifetime. The only parameters left are the detunings $\Delta_{A/B}$ from the corresponding resonances and the ratio γ between the radiation losses of FH and

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SH waves. Here we assume equal driving fields E for both FH components (the input polarization is 45 degrees).

Considering the stationary plane wave solutions it turns out that above a certain threshold the symmetric branch with $A_1 = A_2$ destabilizes (pitchfork bifurcation) and two asymmetric branches emanate. The intensities of the asymmetric FH components are

$$|A_{1/2}|^2 = \frac{|E|^2}{2(\Delta_A^2+1)} + \Delta_A\Delta_B - \gamma$$
$$\pm\sqrt{\left[\frac{|E|^2}{2(\Delta_A^2+1)} + \Delta_A\Delta_B - \gamma\right]^2 - (\Delta_A^2+1)(\Delta_B^2+\gamma^2)} \qquad (2)$$

while the SH intensity is locked to $|B|^2 = \Delta_A^2 + 1$ which is independent of the driving field.

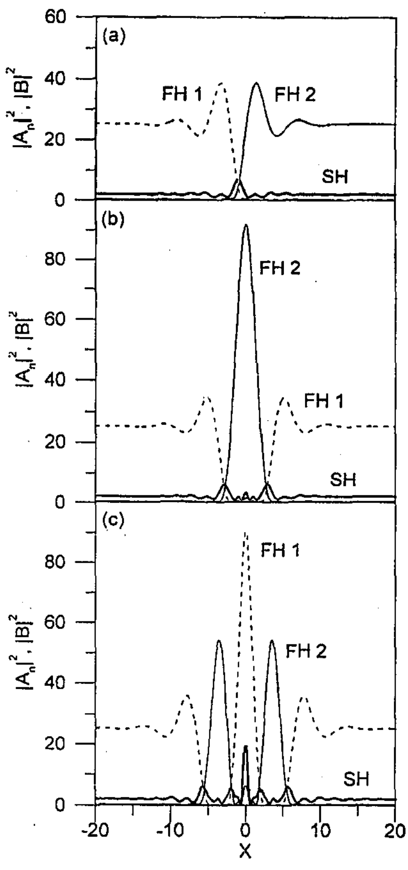

Fig. 1. Intensity distribution of cavity solitons for $\Delta_A = 1,\ \Delta_B = 1.5,\ \gamma = 1,\ E = 7$, (a) one-dimensional topological soliton, (b) cross-section of a first order two-dimensional soliton, (c) cross-section of a higher-order two-dimensional soliton.

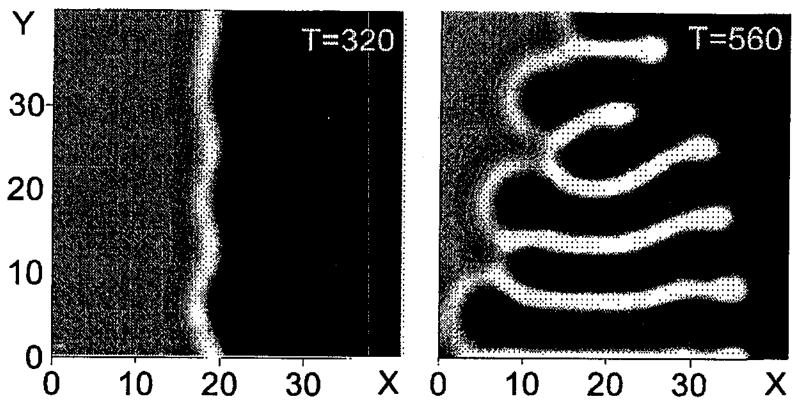

Fig. 2. Decay of a one-dimensional topological soliton due to modulational instability, the same parameters as in Fig. 1.

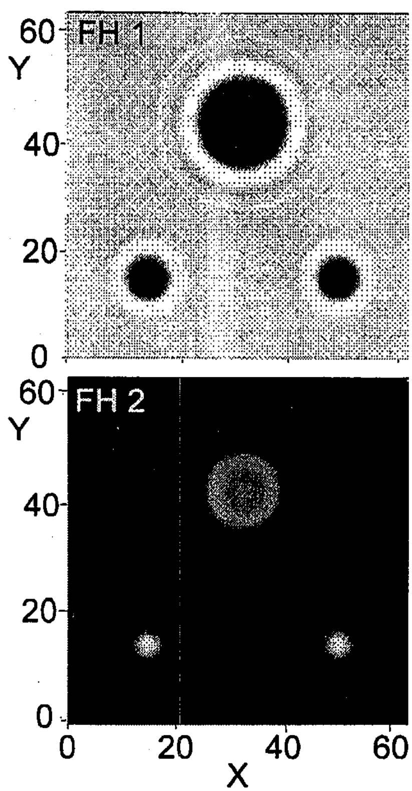

Fig. 3. Creation of multistable solitons of different orders by additional excitation pulses in one FH wave, the same parameters as in Fig. 1, diameters of super Gaussian beams: $d = 2$, $d = 6$.

There are large domains in parameter space where the stationary plane wave solutions, corresponding to two different polarization states, are modulationally stable. These two states can form a new collective state representing a polarization front or a one-dimensional *topological cavity soliton* (see Fig. 1a) similar to that recently observed numerically in the down-conversion process [3]. But we found that this one-dimensional cavity soliton is modulationally unstable and decays into a number of stripes (see Fig. 2) if the second transverse coordinate is taken into account. However, this instability appears to be rather weak. The polarization front can be stabilized by superimposing the oscillating evanescent tails (see Fig. 1a). If the polarization front is bent to form a circle this may lead to the formation of a stable localized structure. Only for certain discrete diameters a constructive interference of these oscillating tails can be achieved. Two-dimensional cavity solitons of different order are displayed in Fig. 1b,c. They can be understood as the equilibrium of the expansion force due to the modulational instability and the force arising from the oscillating tails. Their excitation becomes progressively difficult for higher-order solitons. Solitons of different orders can be excited to form arbitrary patterns on a plane wave background (see Fig. 3). This may be the basis of the implementation of a multiple valued all-optical storage scheme.

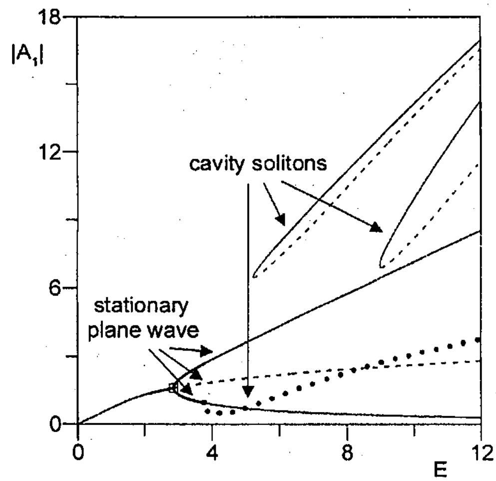

Fig. 4. Stationary plane wave solutions and branches of cavity solitons (amplitude at the centre). Dashed lines mark unstable solutions, bold lines — modulationally unstable plane wave solutions, dots — the branch of unstable cavity solitons emanating from the point where the modulational instability terminates and the square marks the symmetry breaking bifurcation. The parameters are as in Fig. 1.

After having identified different kinds of cavity solitons we consider their bifurcation behaviour (see Fig. 4). The cavity solitons emanate unstably from the critical point on the branches of asymmetric stationary plane wave solutions (dotted). Branches of higher order cavity solitons are isolated and extend usually to infinity. This is due to the existence of stable asymmetric plane wave solutions

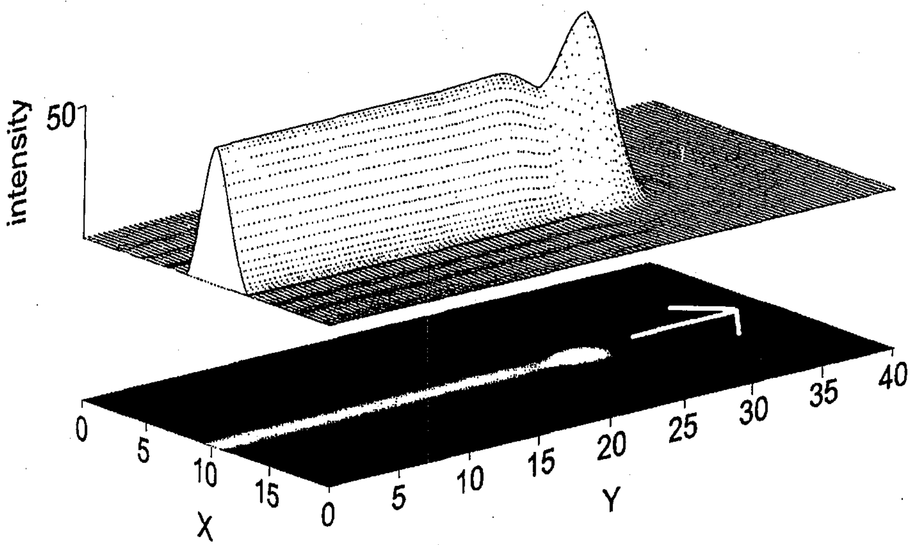

Fig. 5. Moving cavity soliton (the same parameters as in Fig. 1).

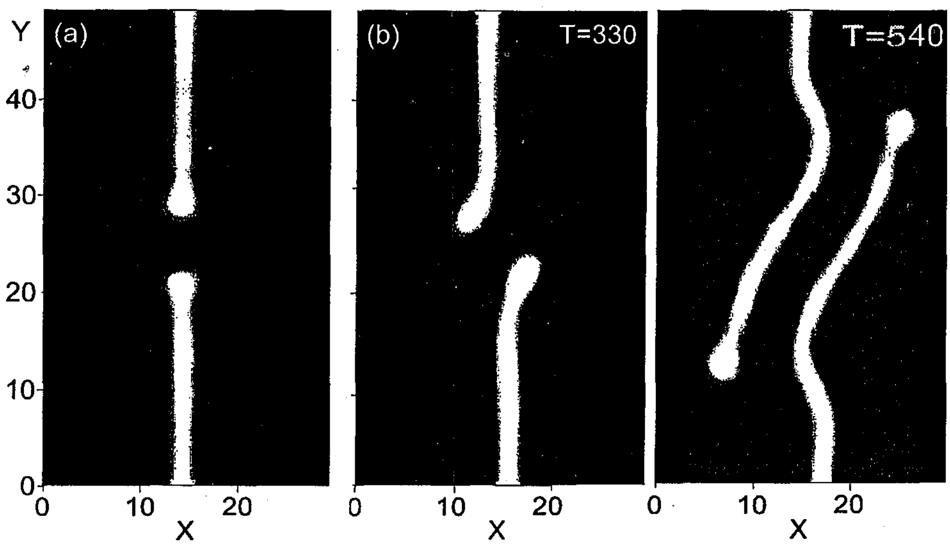

Fig. 6. Collision of two moving cavity solitons (the same parameters as in Fig. 1), (a) central collision, (b) non-central collision.

for all values of the input intensity $|E|^2$ above the critical point, at least for the parameter values considered here.

If the equilibrium of the above-mentioned forces is disturbed by an elliptic exciting beam moving cavity solitons result (see Fig. 5). They consist of circular two-dimensional solitons at either end connected by a one-dimensional

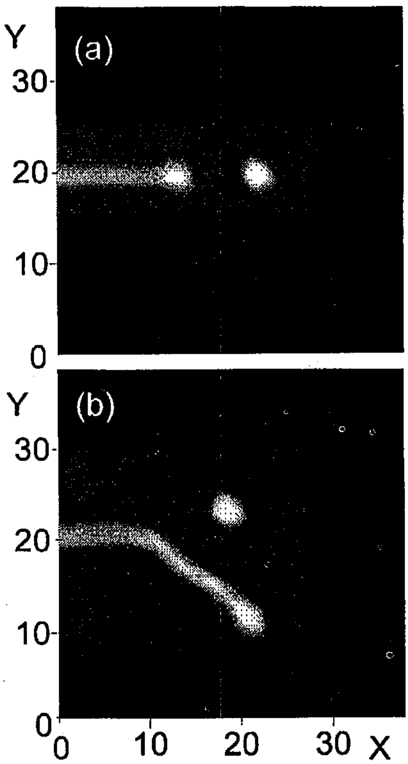

Fig. 7. Collision of a moving cavity soliton with a resting, radial symmetric soliton (the same parameters as in Fig. 1), (a) central collision, (b) non-central collision.

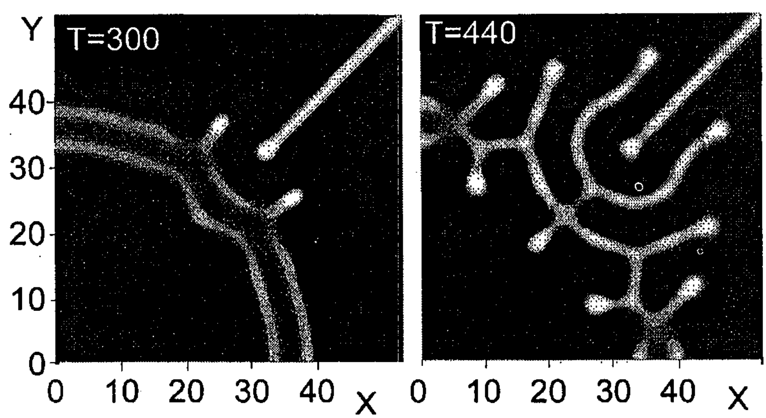

Fig. 8. Collision of a moving cavity soliton with a one-dimensional second order soliton at rest (the same parameters as in Fig. 1).

stripe-soliton. Here the motion of the cavity soliton results from an uncompensated force due to the modulational instability. Moving cavity solitons were also found for input fields exhibiting phase gradients, i.e. in this case the motion is caused by external forces [4].

If solitons move their interaction and collision behaviour becomes a critical issue. This is of particular interest because in our optical system the momentum is not conserved. After a central collision of two moving cavity solitons a stationary state results (see Fig. 6a). If the collision is not central the moving cavity solitons are deformed and pass by each other (see Fig. 6b).

If a radially symmetric and a moving cavity soliton collide centrally the velocity of the latter is decreased. The previously resting soliton is slightly deformed and pushed forward very slowly by the dynamical structure (see Fig. 7a). In the case of a noncentral collision the moving solution is scattered at the resting structure (see Fig. 7b). Figure 8 displays a decay of a higher order radially symmetric cavity soliton due to a collision with a moving soliton.

In conclusion, we identified various types of one- and two-dimensional localized structures due to symmetry breaking in intra-cavity vectorial second harmonic generation. In the case of an elliptic excitation moving structures occur. We studied the collision behaviour between two moving cavity solitons and a resting and a moving structure.

References

[1] C. Etrich, U. Peschel, F. Lederer, *Phys. Rev. E* **56**, 4803 (1997).

[2] C. Etrich, U. Peschel, F. Lederer, *Phys. Rev. Lett.* **79**, 2454 (1997).

[3] S. Trillo, M. Haelterman, A. Sheppard, *Opt. Lett.* **22**, 970 (1997).

[4] N.N. Rosanov, in: *Progress in Optics*, Ed. E. Wolf, Elsevier, Amsterdam 1994, Vol. XXXIII, p. 203.