POSSIBLE LONG TIME TAILS IN CURRENT–CURRENT CORRELATION FUNCTION FOR TWO-DIMENSIONAL ELECTRON GAS IN RANDOM MAGNETIC FIELD

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We consider a two-dimensional degenerate electron gas in the presence of perpendicular random magnetic field. The magnetic field disorder which is assumed to be Gaussian is characterized by two parameters. The first is proportional to the amplitude of local magnetic field fluctuations. The second one characterizes the disorder on the longer scale — we call it the screening length. Using the Kubo formula for the conductivity we have found a class of diagrams which leads to the long time tails in the current–current correlation function. For short times in comparison with the diffusion time corresponding to the screening length this function behaves like logarithm of time, for longer times it decays like t^{-1} which of course brings into question the diffusive character of the behavior of the charged particle in random magnetic field widely assumed in the literature.

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The theoretical description of a two-dimensional, quantum, noninteracting, degenerate electron gas moving in a spatially random magnetic field attracts nowadays considerable attention in view of its possible relation to the theory of quantum Hall effect near the filling factor $\nu = 1/2$ proposed by Halperin and collaborators [1]. The characteristic feature of behavior of these systems is their temporal and spatial dispersion reflecting underlying microscopic motion nonlocality [2]. Progress in developing the general theory capable of describing this dispersion in degenerate electron gas is rather slow. Attempts were directed, therefore, to an analysis of relatively simpler problem, namely the particle diffusion. In the recent work Aronov, Mirlin and Wölfle [3] proposed the derivation of the zero frequency diffusion coefficient for a two-dimensional degenerate electron gas in random magnetic field. In the present paper we propose a next step in the direction of the full analysis by using the Kubo formula for the conductivity. Analyzing a particular class of diagrams we have found the long time tails in the current–current correlation function.

We consider a gas of noninteracting, charged, spinless fermions moving on a two-dimensional plane in the presence of a spatially random magnetic field $\boldsymbol{B}(r)$ perpendicular to that plane. The single particle Hamiltonian reads

$$H = \frac{1}{2m}\left(\frac{\hbar}{\mathrm{i}}\nabla - \frac{e}{c}\boldsymbol{A}\right)^2, \tag{1}$$

where $\boldsymbol{A}$ is the magnetic field vector potential.

The paramount difficulty is now the choice of statistical ensemble for the random magnetic field. The seemingly obvious, and natural, choice of Gaussian distributed magnetic field with the second moment $\langle B_z(r)B_z(r')\rangle \propto \delta(r-r')$ results in infrared divergencies in the lowest order terms in the perturbation theory [3]. This is due to the long range vector potential correlations reflecting the fact that the above-mentioned Gaussian distribution for magnetic field permits the field configurations in which arbitrary large magnetic flux is penetrating the plane. The mean square value of magnetic flux inside the region is proportional to its area, and this invalidates the perturbational approach to the problem.

To avoid this we consider a model of the magnetic field disorder in which the above-mentioned configurations are explicitly suppressed. To this effect we assume that the components of the vector potential $A_i(r)$ $(i = x, y)$ are the Gaussian distributed quantities with zero mean, and whose second order cumulant, in the Fourier space, is given as

$$\langle A_i(q)A_j(q')\rangle = (2\pi)^2\delta(q+q')\left(\frac{\hbar c}{e}\right)^2 \gamma\frac{1}{q^2+\mu^2}\delta^{\mathrm{T}}{}_{ij}(q), \tag{2}$$

where $\delta^{\mathrm{T}}{}_{ij}(q) = \delta_{ij} - q_iq_j/q^2$, and γ and μ are some constants. This ensemble is closely related to that used by Bausch, Schmitz and Turski [4] in their analysis of the classical particle diffusion in crystals with randomly distributed topological defects. It is also identical to the one proposed by Aronov, Mirlin and Wölfle [3]. In their paper γ was proportional to the fluctuations of the magnetic field and μ was introduced at the intermediate steps of calculations as a "soft cut-off" in order to avoid the above-mentioned infrared divergencies in the perturbation theory. They did not discuss the meaning of this parameter, because it drops out from their final formula for the diffusion coefficient. This is true for the calculations in the lowest order in γ, as we shall see the higher order terms contain μ in the very nontrivial way.

One can attach a clear physical meaning to the parameters γ and μ in Eq. (2) by analyzing an analogy between the above discussed model and another model of randomness in which these coefficients appear in a quite natural way. This auxiliary model has a well-defined physical interpretation and long and short wavelength behaviors of both these models are identical.

Imagine two identical, infinitely thin solenoids (Aharonov–Bohm flux tubes) piercing our plane at points separated by a distance ℓ. The magnetic flux in one solenoid is $+\Phi$ and in the second $-\Phi$. For the sake of definiteness we shall call such an object a flux dipole. On the two-dimensional plane the dipole is characterized by its position, i.e., the coordinates of its "center of mass" and the angle φ between the vector joining the flux tubes and, say, the x axis.

Now we assume that we have many such dipoles penetrating the plane and let the average dipole density be n. The position of the i-th dipole $\boldsymbol{R}_i$ may be arbitrary and the angle φ_i is homogeneously distributed in the interval $(0, 2\pi)$. Notice that our disorder ensemble, by construction, allows only for configurations in which the total magnetic flux through the plane equals zero. For such kind of the magnetic disorder we obtain the following expression for the second moments of the Fourier space components of the vector potential field $A_i(\boldsymbol{q})$:

$$\langle A_i(\boldsymbol{q})A_j(\boldsymbol{q}')\rangle = C(\boldsymbol{q},\boldsymbol{q}') = (2\pi)^2\delta(\boldsymbol{q}+\boldsymbol{q}')2n\left(\frac{\Phi}{\Phi_0}\right)^2 g(|\boldsymbol{q}|)\delta_{ij}^{\mathrm{T}}(\boldsymbol{q}), \tag{3}$$

where $g(|\boldsymbol{q}|) = [1 - J_0(|\boldsymbol{q}|\ell)]/\boldsymbol{q}^2$, $J_0(z)$ is the zero order Bessel function and $\Phi_0 = e/\hbar c$. The ensemble average denotes here

$$\langle \mathcal{O}\rangle = \prod_{i=1}^{N}\left(\frac{1}{2\pi S}\int_0^{2\pi}\mathrm{d}\varphi_i\int \mathrm{d}^2R_i\mathcal{O}\right), \tag{4}$$

where N is the number of dipoles and S is the area of the system ($n = N/S$).

We shall also assume that the magnetic flux $\Phi \ll \Phi_0$, thus we may neglect the higher order cumulants which are of the order $n\Phi^r$ with $r \geq 4$. Alternatively we may assume that the fluxes in different dipoles are mutually independent random quantities with Gaussian distribution of zero mean and the second moment equal to Φ^2. In this case the higher order cumulants also vanish.

Now let us discuss some properties of the introduced disorder. If the condition $\sqrt{n}\ell \gg 1$ is satisfied then on the length scales shorter than ℓ (we call ℓ the screening length in the following) the magnetic field is spatially uncorrelated because the magnetic fluxes in randomly placed solenoids are mutually independent. This follows also from the behavior of $C(\boldsymbol{q},\boldsymbol{q}')$ for $q\ell \gg 1$. In this limit $g(|\boldsymbol{q}|) \to 1/\boldsymbol{q}^2$, as for the Dirac's delta correlated magnetic field. In the opposite limit, $|\boldsymbol{q}|\ell \to 0$, the function $g(|\boldsymbol{q}|) \to$ const and we have the case of spatially uncorrelated vector potential field. Comparing the short and long wavelength behaviors of the models defined by Eq. (2) and Eq. (3) we see that they are indeed identical provided $\gamma = 2n(\Phi/\Phi_0)^2$ and $\mu = 2/\ell$. Whichever of the limiting behaviors of the vector potential correlation is pertinent to an actual physical system it depends on the value of the Fermi wavelength for the electron gas. In this paper we are interested in the situation when the Fermi wavelength $\lambda_{\mathrm{F}} = 2\pi/k_{\mathrm{F}}$ is much smaller than ℓ, i.e., $k_{\mathrm{F}}\ell \gg 1$.

In the zero temperature and $\omega \to 0$ limit the Kubo formula for the real part of the conductivity reads

$\mathrm{Re}\sigma_{xx}(\omega, \boldsymbol{k} = 0)$

$$= \frac{e^2\hbar^3}{2\pi m^2 S}\mathrm{Re}\left\{\mathrm{Tr}\left[\widehat{V}_x\widehat{G}_{\mathrm{R}}(E_{\mathrm{F}}+\omega)\widehat{V}_x\left(\widehat{G}_{\mathrm{A}}(E_{\mathrm{F}})-\widehat{G}_{\mathrm{R}}(E_{\mathrm{F}})\right)\right]\right\}, \quad (5)$$

where $\widehat{V}_x = (1/\mathrm{i})\partial_x - (e/\hbar c)A_x(r)$ is proportional to the x-th component of the velocity operator and $\widehat{G}_{\mathrm{R/A}}(E)$ are retarded and advanced propagators corresponding to the one-particle Hamiltonian (1). In the position representation they have the form

$$\widehat{G}_{\mathrm{R/A}}(E;r,r') = \sum_n \frac{\varphi_n(r)\varphi_n(r')}{E-\epsilon_n \pm \mathrm{i}\eta}, \quad (6)$$

where $\varphi_n(r)$ and ϵ_n are eigenfunctions and corresponding eigenenergies, respectively. In Eq. (5) Tr denotes integration over r and S is the system's area.

In our analysis we apply the standard method of the averaging over the disorder [5, 7]. Every Green's function in Eq. (5) is expanded in the power series of the components of the vector potential which we treat as a perturbation. In the second step every term of the resulting series is averaged according to the assumed properties of the magnetic field disorder. The exact summation of that series is impossible. What is usually done is the extraction and summation of certain classes of diagrams which are believed to give the most important contribution to the final result. The criterion of "importance" is based on the behavior (an index of divergence) of a corresponding analytical expression in the limit $\omega \to 0$ [5]. In the present case the most divergent diagrams may be divided in two classes. The first class leads to renormalization of the one-particle Green function (the averaged Green function). The second class consists of the ladder diagrams. These two classes when summed up give the so-called Boltzmann value of the conductivity [5], or equivalently the Boltzmann diffusion constant, if one uses the density–density response function instead of Eq. (5) and extracts its low frequency and long wavelength behavior [3, 6].

This procedure well-established and widely used in electrostatic potential disorder case, however, suffers here, from a certain difficulty. Namely, the self-energy in the averaged Green function calculated for the disorder ensemble defined by Eq. (2) is infinite in the limit $\mu \equiv 2/\ell \to 0$. The averaged Green function in the limit $k\ell \gg 1$ equals

$$\langle G^{\mathrm{R/A}}(E,k)\rangle = 1/(E - k^2/2m \pm \mathrm{i}\omega_0/2), \quad (7)$$

where $\omega_0 = n(\Phi/\Phi_0)^2 k\ell/(2m)$ and even for very small density of dipoles n or for very small amplitude of magnetic field fluctuations γ the self-energy operator becomes dominant for large ℓ. Of course we realize that the self-energy in the averaged Green function has no direct physical meaning (due to the fact that the Green function is not a gauge invariant object) thus this infinity may be of no importance. Such a point of view seems to be justified, because in the final expression for the diffusion constant (a gauge invariant quantity) calculated in the ladder diagram approximation the limit $\mu \to 0$ may be performed and the diffusion coefficient is finite.

On the other hand the methods of calculations of the diffusion coefficient (for example by solving the Dyson equation in the ladder diagram approximation [3]) depend on the assumption $\omega_0 \ll E_{\mathrm{F}}$. Thus the question arises whether the discussed difficulties, concerning the breaking up of the perturbation theory, are spurious showing up on the intermediate steps of calculation only and not influencing the final results, or that they are of a physical origin and one should pay attention to them.

We think that the simplest method to answer this question is to perform the analysis of the diagrams' divergencies similar to that presented by Kirkpatrick and Dorfman [5] for the electrostatic potential disorder problem, i.e., to analyze the divergencies with respect to ω in the given order of the expansion parameter γ. It turns out that it is enough to consider diagrams of the first order to notice some very interesting features. These diagrams are shown in Fig. 1. Here the continuous lines correspond to the unperturbed Green functions $G^0(E,\boldsymbol{k}) = 1/(E-k^2/2m\pm i\eta)$ and the broken line represents the averaging. The number of diagrams is much larger comparing to the case of electrostatic potential fluctuations case. This is due to: (1) the appearance of the second power of the vector potential in the Hamiltonian, this term must be kept in order to conserve the gauge invariance: (2) the appearance of the gauge invariant derivatives $\widehat{\nabla}$ in Eq. (5). The origin of the first five diagrams 1a–e is the expansion of the Green function in powers of the vector potential. In particular, diagrams 1d and 1e correspond to the term A^2 in the Hamiltonian. The diagrams 1f–i come from the expansion of the Green function and from covariant derivatives, and the last diagram 1j represents the contraction of two A's in two covariant derivatives in Eq. (5).

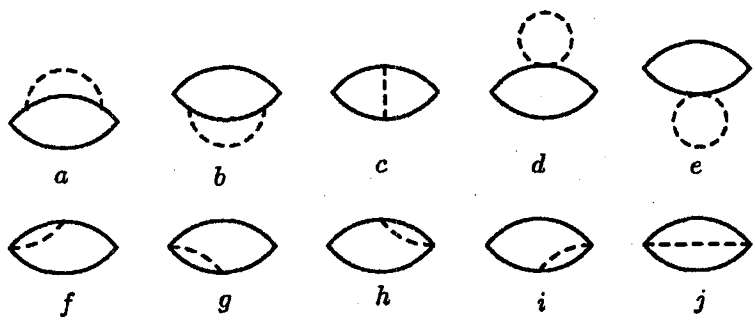

Fig. 1. Diagrams for the first order perturbation theory for the conductivity. The continuous lines correspond to the nonperturbed Green functions, the broken lines to the averaging over the disorder, respectively.

The first three diagrams are the lowest order diagrams belonging to the above discussed classes and although each of them separately is divergent in the limit $\ell \to \infty$, the sum of them has a well-defined limit and it constitutes the first order term in the density expansion for the frequency dependent conductivity. That sum behaves like γ/ω^2. The diagrams 1d and 1e do not give contribution to the real part of the conductivity. The contributions from the next four diagrams cancel. The last diagram 1j has an interesting behavior. Its analytical expression is proportional to the integral

$$\int_0^{2\pi} \mathrm{d}\theta \frac{1}{k^2+q^2-2kq\cos\theta+\mu^2}, \tag{8}$$

where $k = \sqrt{2m(E + \hbar\omega)}/\hbar$ and $q = \sqrt{2mE}/\hbar$. We see that for $\mu \neq 0$ it is finite for $\omega = 0$. However for $\mu = 0$ it behaves like γ/ω. Such a behavior suggests possible relationship between ω and μ when both of them are small.

Before proceeding further let us notice that up to now all the calculations have been performed in the Coulomb, symmetric gauge. The same result however is obtained in the Landau gauge $A_x = 0$. In that case all the diagrams originating from the covariant derivatives vanish and we are left with the first five diagrams 1a–e. Again the diagrams 1d–e do not give contribution and for $\mu = 0$ the γ/ω divergence obtained previously from the diagram 1j is contained now in the sum of the first three diagrams 1a–c. Of course this is nothing special, it should be so because the expression (5) is gauge invariant.

Fig. 2. Diagrams leading to the nonanalytical corrections to the conductivity. Here the solid lines represent the dressed (averaged) Green functions.

The natural generalization of the diagram 1j is presented in Fig. 2. Contrary to Fig. 1 the solid lines now represent the dressed Green functions. It is interesting that assuming the diffusive behavior of the electron gas we may calculate the sum of these two diagrams exactly. Namely, without the broken lines the sum of two diagrams is closely related to the density–density correlation function, more precisely it is proportional to the Kubo relaxation function [8] which for $q \to 0$ and $\omega \to 0$ behaves like

$$\Phi(q,\omega) = \frac{m/2\pi\hbar^2}{Dq^2 - \mathrm{i}\omega}. \tag{9}$$

That is why the analytical expression corresponding to the sum of the diagrams in Fig. 2 may be calculated and the final result reads

$$\mathrm{Re}\delta\sigma = \frac{e^2\gamma}{32\pi^3 m D\mu^2} \frac{\ln\left(\frac{D\mu^2}{\omega}\right) + \frac{\pi}{2}\left(\frac{\omega}{D\mu^2}\right)}{1 + \left(\frac{\omega}{D\mu^2}\right)^2}. \tag{10}$$

Equation (10) has no limit for $\omega \to 0$ ($\mathrm{Re}\delta\sigma \propto -\ln(\omega)$).

It is interesting to write the sum of diagrams from Fig. 2 in the "time representation"

$$\delta\sigma = \frac{e^2\gamma}{16\pi^3 m}\int_0^\infty \mathrm{d}t \exp(\mathrm{i}\omega t) F(t), \tag{11}$$

where

$$F(t) = \int_0^\infty \mathrm{d}q \frac{q}{q^2 + \mu^2} \exp(-Dq^2 t) \tag{12}$$

is proportional to the current–current correlation function. From the above we see that for small times in comparison with the diffusion time corresponding to the screening length ($D\mu^2 t \ll 1$) $F(t) \propto \ln(D\mu^2 t)$ and for asymptotically large times ($D\mu^2 t \gg 1$) $F(t) \propto t^{-1}$.

The obtained result is interesting and not entirely clear. First, in conjunction with the diagrammatic analysis presented earlier it suggests that the screening length or the "soft cut-off" in the terminology of Aronov, Mirlin and Wölfle has an important physical meaning determining time characteristics for the system. The power law decay of the function $F(t)$ is of the great interest. The t^{-1} behavior suggests that the long time tails are of quantum origin [5], for the classical Lorentz gas model would give t^{-2} behavior. In order to fully appreciate whether this decay is indeed of a quantum mechanical origin it requires a detailed analysis of the velocity–velocity correlation function for the classical particle in random magnetic field. To the best of our knowledge this is not available at the moment.

We should now return briefly to the discussion of the applicability of the perturbation theory. According to Eq. (7) the perturbation theory is applicable if $\hbar\omega_0 \ll E_{\rm F} \equiv \gamma\ell\lambda_{\rm F} \ll 1$. Consider now a part of the system, whose the linear dimension is $R \propto \gamma^{-1/2}$ ($R \ll \ell$). The typical magnetic field flux contained in that part of the system $\Phi \propto \Phi_0$, consequently the typical magnetic field $B \propto \Phi_0/R^2 \propto \gamma\Phi_0$. For a particle of energy $E_{\rm F}$ the corresponding cyclotron radius $r_{\rm c} = c\sqrt{2mE_{\rm F}}/eB \propto \sqrt{2mE_{\rm F}}/\hbar\gamma \propto 1/\gamma\lambda_{\rm F}$. Thus the condition $\gamma\ell\lambda_{\rm F} \ll 1$ may be rewritten as $r_{\rm c} \gg \ell$ which means that the cyclotron radius corresponding to the local magnetic field fluctuation should be much larger than the screening length.

In conclusion we have studied the behavior of the two-dimensional degenerate electron gas in the presence of the perpendicular random magnetic field. Analyzing the lowest order diagrams we have pointed out the importance of the proper choice of the magnetic field disorder ensemble. Secondly, taking into account a certain class of diagrams we have obtained the long time tails in the current–current correlation function.

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