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# SPIN-LATTICE RELAXATION EFFECTS IN CRITICAL SOUND PROPAGATION

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The nonasymptotic critical properties of sound propagation are studied in compressible Ising system above $T_C$. In the present paper we analyse a model where in addition to the coupling to two order-parameter fluctuations the sound mode couples linearly to the fluctuations of spin-energy and lattice-energy densities. Both subsystems exchange energy with the rate determined by the bare spin-lattice relaxation time. The total energy may be conserved or not. The crossover between insulator-like behaviour $\sim t^{-2\alpha}$ and metal-like behaviour $\sim t^{-(z\nu+\alpha)}$ in ultrasonic attenuation is investigated according to the value of ultrasonic frequency, the reduced temperature $t$, bare relaxation times and various coupling constants.

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In typical ultrasonic experiments near the magnetic phase transition temperature we observe strong anomalies of sound attenuation in magnetic metals such as some rare-earth metals, whereas in magnetic insulators only very weak anomaly is observed [1, 2]. Many theories have been proposed to describe the strong anomaly in various substances [3–6]. They assume that the sound mode is coupled to two spin fluctuations above $T_C$. On the other hand, the weak anomalies have been qualitatively explained by postulating the dominance of the linear coupling to the spin-energy density [7]. In our recent study we have investigated a model where both couplings were present [8]. In this paper we present more extensive discussion of the problem including also the lattice energy mode and transfer of energy between spin and lattice subsystems. We consider the acoustic phonon $Q$ coupled to the scalar spin $S$ and to the fluctuations of energy of spin and lattice subsystems. The Hamiltonian of the elastically isotropic system may be specified as

$$H=\frac{1}{2}\int \mathrm{d}^d x\left[rS^2+(\nabla S)^2+uS^4+C_{12}\left(\sum_\alpha e_{\alpha\alpha}\right)^2+2C_{44}\sum_{\alpha,\beta}e_{\alpha\beta}^2\right.$$

$$\left.+g\sum_\alpha e_{\alpha\alpha}S^2+fe_\mathrm{S}S^2+w(e_\mathrm{S}+ae_\mathrm{L})\sum_\alpha e_{\alpha\alpha}+\frac{e^2{}_\mathrm{S}}{C_\mathrm{S}}+\frac{e^2{}_\mathrm{L}}{C_\mathrm{L}}\right], \qquad (1)$$

where $e_{\alpha\beta}(x)$ denotes the strain tensor and $e_S(x)$ and $e_L(x)$ are the spin- and lattice-energy densities, respectively. The symbols $C_{\alpha\beta}$ stand for the bare elastic constants; $g$, $w$, $aw$, $f$ are the bare coupling constants, and $C_S$ and $C_L$ are the spin and lattice specific heats, respectively.

The dynamics of the system is described by the coupled Langevin equations

$$\dot{S} = -\Gamma\frac{\delta H}{\delta S} + \xi, \tag{2}$$

$$\ddot{Q} = -\frac{\delta H}{\delta Q} - \Gamma_Q\dot{Q} + \eta, \tag{3}$$

$$\dot{e}_S = -(\gamma_S - \lambda_S\nabla^2)\frac{\delta H}{\delta e_S} + \gamma\frac{\delta H}{\delta e_L} + \varphi, \tag{4}$$

$$\dot{e}_L = -(\gamma_L - \lambda_S\nabla^2)\frac{\delta H}{\delta e_L} + \gamma\frac{\delta H}{\delta e_S} + \psi, \tag{5}$$

where $\xi, \eta, \varphi$ and $\psi$ are Gaussian white noises with variances related to the bare damping terms $\Gamma, \Gamma_Q, (\gamma_S - \lambda_S\nabla^2)$ and $(\gamma_L - \lambda_L\nabla^2)$ by the Einstein relations.

It is convenient to represent Eqs. (2)–(4) in functional form [9] with Lagrangian given by

$$\begin{aligned} L\left(Q,\tilde{Q},S,\tilde{S},e_S,\tilde{e}_S,e_L,\tilde{e}_L\right) = \int_\omega \sum_k \Big\{ &\Gamma\tilde{S}_{k,\omega}\tilde{S}_{-k,-\omega} + \Gamma_Q(k)\tilde{Q}_{k,\omega}\tilde{Q}_{-k,-\omega} \\ &+\Gamma_S(k)\tilde{e}_{k,\omega}{}^S\tilde{e}_{k,\omega}{}^S + \Gamma_L(k)\tilde{e}_{k,\omega}{}^L\tilde{e}_{-k,-\omega}{}^L - 2\gamma\tilde{e}^S{}_{k,\omega}\tilde{e}^L{}_{-k,-\omega} \\ &-\tilde{Q}_{k,\omega}\left[(-\omega^2+\mathrm{i}\omega)Q_{-k,-\omega} + \frac{\partial H}{\partial Q_{k,\omega}}\right] - \tilde{S}_{k,\omega}\left(\mathrm{i}\omega S_{-k,-\omega} + \Gamma\frac{\partial H}{\partial S_{k,\omega}}\right) \\ &-\tilde{e}^S{}_{k,\omega}\left[\mathrm{i}\omega e^S{}_{-k,-\omega} + \Gamma_S(k)\frac{\partial H}{\partial e^S{}_{k,\omega}} - \gamma\frac{\partial H}{\partial e^L{}_{k,\omega}}\right] \\ &-\tilde{e}^L{}_{k,\omega}\left[\mathrm{i}\omega e^L_{-k,-\omega} + \Gamma_L(k)\frac{\partial H}{\partial e^L{}_{k,\omega}} - \gamma\frac{\partial H}{\partial e^S{}_{k,\omega}}\right]\Big\}, \end{aligned} \tag{6}$$

where $\Gamma_i(k) = \gamma_i + \lambda_i k^2$ for $i =$ S, L and $\tilde{S}, \tilde{Q}, \tilde{e}_S$ and $\tilde{e}_L$ are auxiliary "response" fields. With the Lagrangian all correlation and response functions can be computed as path integrals weighted with density $\exp L$.

Next we apply series of Gaussian transformations

$$e_L \to e_L + Ae_S + B\tilde{e}_S, \qquad \tilde{e}_L \to \tilde{e}_L + C\tilde{e}_S, \tag{7}$$

which decouples the energy density modes

$$Q \to Q + DS^2 + Ee_S + Fe_L + G\widetilde{S^2} + H\tilde{e}_S + J\tilde{e}_L,$$

$$\tilde{Q} \to \tilde{Q} + K\widetilde{S^2} + L\tilde{e}_S + M\tilde{e}_L, \tag{8}$$

decoupling the sound mode from the energy density modes,

$$e_L \to e_L + NS^2 + Pe_S + R\widetilde{S^2} + T\tilde{e}_S,$$

$$\tilde{e}_L \to \tilde{e}_L + U\widetilde{S^2} + V\tilde{e}_S, \tag{9}$$

and

$$e_S \to e_S + WS^2 + X\widetilde{S^2},$$

$$\tilde{e}_S \to \tilde{e}_S + Y\widetilde{S^2}, \tag{10}$$

decoupling $e_S$, $e_L$ and $S^2$, where $A$, $B$, $C$... are frequency and wave-vector dependent coefficients.

We then obtain expression for the acoustic self-energy

$$\Sigma^{AC} = \frac{k^2\left[g^2(\hat{\omega}) - i\tilde{\omega}b(\hat{\omega})g^2\right]\langle\Gamma\widetilde{S^2}S^2\rangle_{L^{eff}}}{1 - i\tilde{\omega}b(\hat{\omega})\left(1 + v\langle\Gamma\widetilde{S^2}S^2\rangle_{L^{eff}}\right)}, \tag{11}$$

where

$$b(\hat{\omega}) = \frac{1 - i\hat{\omega}}{m - i\hat{\omega}}, \quad v = f^2 C_S, \quad m = 1 - \gamma^2/\Gamma_S\Gamma_L,$$

$$\tilde{\omega} = \omega C_S/\Gamma_S, \quad \hat{\omega} = \omega C_L/\Gamma_L$$

and

$$g(\hat{\omega}) = g - wfC_S\frac{m - i\hat{\omega}(1 - a\gamma/\Gamma_S)}{m - i\hat{\omega}}$$

is a frequency-dependent effective spin–phonon coupling constant. The effective spin Lagrangian used for calculation of the expectation value in (11) contains strain- and energy density-mediated four-spin non-local interactions as a result of transformations (7)–(10). From the acoustic self-energy the sound attenuation coefficient can be easily found. Near the phase transition it can be written as

$$\frac{\alpha(\omega,t)}{k^2} \propto \left\{\frac{1}{2y\Gamma}\mathrm{Re}(g^2(\hat{\omega}))t^{-z\nu-\alpha}\mathrm{Im}\Phi(y)\right.$$

$$\left. + \frac{1}{2y\Gamma}g^2\left|b(\hat{\omega})\right|^2\tilde{\omega}^2 t^{-z\nu-\alpha}\mathrm{Im}\Phi(y) + \frac{C_S}{\Gamma_S}\mathrm{Re}\left[g^2(\hat{\omega})\overline{b(\hat{\omega})}\right]t^{-2\alpha}\left|\Phi(y)\right|^2\right\}$$

$$\times\left|1 - i\tilde{\omega}b(\hat{\omega})t^{-\alpha}\Phi(y)\right|^{-2}, \tag{12}$$

where Re and Im denote real and imaginary part of a function, $t = \frac{T - T_C}{T_C}$ is the reduced temperature and $y = \frac{\omega}{2\Gamma}t^{-z\nu}$ is the reduced frequency with $\alpha, \nu$ and $z$ being the critical exponents. The scaling function $\Phi(y)$ in the "weak-coupling" limit [5] can be written as

$$\Phi(y) = \left[1 + (y/2)^2\right]^{-\alpha/4\nu}\left\{1 + \frac{i\alpha}{y\nu}\left[i\left(1 - iy/2\right)\arctan(y/2) - \frac{1}{2}\ln\left(1 + (y/2)^2\right)\right]\right\}.$$

Depending on the relative size of $t$, $\Gamma_S/C_S$, $\Gamma$, $\Gamma_L/C_L$ and $\omega$, Eq. (12) shows many different regimes. Asymptotically i.e. for $\omega, t \to 0$ strong singularity term always dominates [3, 10]

$$\alpha(\omega, t) \propto \omega^2 t^{-(z\nu+\alpha)} \mathrm{Im}\Phi(y)/y.$$

It is rather unexpected that a similar singularity dominates also for high frequencies, $\tilde{\omega} \gg 1$, where

$$\alpha(\omega, t) \propto \omega^2 t^{-(z\nu-\alpha)} \mathrm{Im}\Phi^{-1}(y)/y.$$

This new behaviour can be obtained from the asymptotic behaviour by simple replacement $\alpha \to -\alpha$ and $\Phi \to \Phi^{-1}$. The weak singularity term,

$$\alpha(\omega, t) \propto \omega^2 t^{-2\alpha} |\Phi(y)|^2 ,$$

can be dominant for $t > t_{\mathrm{cross}}$ only if there exists a frequency window $\omega \ll \Gamma_S/C_S \ll \Gamma$, with the crossover reduced temperature $t_c\mathrm{ross} \propto (\frac{\alpha m \gamma_S}{C_S \Gamma})^{1/(z\nu-\alpha)}$. This regime is believed to take place in magnetic insulators [1, 2].

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