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NON-LINEAR PROCESSES IN 1D QUANTUM SPIN ORDERED SYSTEM UNDER STRONG MAGNETIC FIELD PULSE

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Investigation of an exactly integrable quantum model of 1D magnetic (which is equivalent to a massive fermion system) is presented. It has been shown that under certain conditions a spin system will not absorb power from the strong homogeneous magnetic field pulse.

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Self-induced 1D magnetic transparency similar to the well-known optical self-induced transparency in two-level molecular gas has been predicted in our previous paper [1]. A resonant soliton caused by interaction between 1D quantum spin system and a pulse of a strong magnetic field was a result of exact solution of the problem. The effect was shown to take place in a system with a fixed configuration of parameters. In particular, we have found that self-induced transparency may take place in an anisotropic spin system with excitations of the Fermi type and with a linear dispersion law. This condition was found to be of great importance in Ref. [1].

In the present paper we analyze the role of dispersion law linearity for the above-mentioned effects. We investigate an exactly solvable 1D spin model, similar to the one referred in the paper [1], but with a gap in the dispersion law caused by the external constant magnetic field h_0. We have found a variable magnetic field time profile $h(t)$ for which the spin system has zero absorption energy. As it was shown in Refs. [1, 2], the existance of such a profile means that the proper resonant soliton exist. The relevant profile $h(t)$ has been constructed as a function of two free parameters.

The above-mentioned problem for the case of an anisotropic spin XY-chain under an external magnetic field $h(t) = h_0 + h_t$ is briefly represented below. The model is described by the following Hamiltonian:

$$H = H_0 - \mu h(t) \sum_n S_n^z, \quad H_0 = -\sum_n (J_x S_n^x S_{n+1}^x + J_y S_n^y S_{n+1}^y). \tag{1}$$

Here S_n is a spin operator at the site n. We investigated the XY-model with a specific relation between the nearest neighbour exchange constants ($J_x = -J_y = J$). In the model under consideration, the dispersion law of excitations has a gap caused by a constant h_0 of the magnetic field $h(t)$. The pulse duration of the variable external field h_t is assumed to be much smaller than the relaxation times in the system under consideration.

The Wigner transformation (see, e.g. Ref. [3]) enables us to describe this magnetic system in terms of the Fermi creation and annihilation operators a_n^+ and a_n. According to the paper [1] it is convenient to consider a two-sublattice system with the Fermi operators on even ($c(n) \equiv a_{2n}$) and odd ($d(n) \equiv a_{2n+1}$) sites of the chain. Using a transition from the discrete index n to the continual variable value x and from the Fermi operators $c(n)$ and $d(n)$ to the Fermi fields $c(x)$ and $d(x)$ one may rewrite the Hamiltonian (1) in continuous limit as follows:

$$H = \int \mathrm{d}x(\Phi^+(x), \widetilde{H}\Phi(x)), \quad \widetilde{H} = V\sigma_3 p + \mu\sigma_2 h(t), \tag{2}$$

$$\Phi(x) \equiv 2^{-1/2}\begin{pmatrix} c(x)-\mathrm{i}d(x) \\ \mathrm{i}c(x) - d(x) \end{pmatrix}, \quad V = 2Ja, \quad p = \mathrm{i}\frac{\partial}{\partial x},$$

where σ_i is the Pauli matrix, $\Phi(x)$ — the two-component Fermi field operator, $\hbar = c = 1$.

At $t \to -\infty$ and $h_0 = 0$ the density matrix $\rho(t)$ of the system is defined by the Liouville equation $\mathrm{i}\frac{\partial}{\partial t}\rho(t) = [H, \rho]$ and has a form of an equilibri wum density matrix $\rho_0 = \exp(-\beta H_0)$, ($\beta = 1/T$, T is temperature). The expression

$$\rho(t) = \sum_{\kappa} \omega_\kappa |\kappa t\rangle\langle\kappa t|$$

satisfies the Liouville equation only if time dynamics of a ket-vector $|\kappa t\rangle$ is defined by the Schrödinger equation with the Hamiltonian H. Here ω_κ is the Boltzmann weight. We seek an expression for the ket-vector in the following form:

$$|\kappa t\rangle = \int \mathrm{d}x \Psi_\kappa(x,t) a^+(x)|0\rangle, \quad a(x)|0\rangle = 0. \tag{3}$$

In this case for the function $\Psi_\kappa(x,t)$ we have a non-stationary Schrödinger equation

$$\mathrm{i}\frac{\partial}{\partial t}\Psi_\kappa(x,t) = \widetilde{H}\Psi_\kappa(x,t).$$

The operator $\widetilde{H}$ is determined by Eq. (2). The boundary condition are

$$\Psi_\kappa(x, t \to +\infty) = \exp(\mathrm{i}\alpha_\kappa)\Psi_\kappa(x, t \to -\infty),$$

$$\Psi_\kappa(x, t \to -\infty) = \exp(\mathrm{i}E_\kappa t)\Psi_\kappa(x).$$

As the external field is homogeneous, it is natural to represent $\Psi_\kappa(x,t)$ as

$$\Psi_\kappa(x,t) = \exp(\mathrm{i}kx)\Psi_\kappa(t).$$

At this stage the problem can be formulated as a search for a reflectionless potential $h(t)$ analogous to the one in the spectral problem in the framework of the inverse scattering method

$$\mathrm{i}\frac{\partial}{\partial t}\Psi_\kappa(t) = [k\sigma_3 + h(t)\sigma_2]\Psi_\kappa(t). \tag{4}$$

At $h_0 = 0$ and $h_t(|t| \to \infty) = 0$ an elementary reflectionless one-parametrical potential takes the form

$$h(t;\gamma) = \mathrm{const}/\cosh(\gamma t), \tag{5}$$

where γ is a free parameter.

Formally, introduction of a constant magnetic field h_0 corresponds to the substitution $h(t) \to h_0 + h_t$ only, and all the above-mentioned expressions are valid. However, in Eq. (4) we are interested in the only $h(t)$ that are equal not to zero at $|h| \to \infty$ but to a constant value u_0, defined by the value of an external magnetic field. The corresponding spectral problem has been considered for the non-linear Schrödinger equation with repulsion (see, for example, Ref. [4])

$$\mathrm{i}\frac{\partial}{\partial t}u = -\frac{\partial^2}{\partial x^2}u - |u|^2 u. \tag{6}$$

But there are two essential circumstances in Eq. (6) that are not satisfactory for our case: (i) the equation (6) is complex, so the quantity u is of a complex value, (ii) the boundary conditions for the equation (6) have the following form:

$$u = \begin{cases} u_0, & x \to -\infty, \\ u_0 \exp \mathrm{i}\eta, & x \to +\infty, \end{cases}$$

where the quantity η is defined by scattering data. For the problem under consideration the corresponding value u should be real, and the condition $\eta = 0$ is necessary to search the reflectionless potentials u. Under the above conditions we have found the exact solution of the equation (4) for the inverse scattering problem by the Mikhailov's method [5]. The required reflectionless potential is a two-parametrical one and can be written as

$$h(t;\gamma_1,\gamma_2) = h_0 - \frac{\partial}{\partial t}\Omega(t;\gamma_1,\gamma_2), \tag{7}$$

$$\tan(\Omega/2) = \frac{\delta \sin\gamma_1 \sinh\theta - \epsilon\cosh\gamma_2 \sin\nu}{\delta\cos\gamma_1\cosh\theta - \epsilon\sinh\gamma_2\cos\nu}.$$

The free parameters γ_1 and γ_2 define the following values in the expression (7):

$$\delta = h_0\frac{\sinh 2\gamma_2}{\cosh 2\gamma_2 - \cos 2\gamma_1}, \quad \theta = 2\epsilon t + \mathrm{const},$$

$$\epsilon = h_0\frac{\sin 2\gamma_1}{\cosh 2\gamma_2 - \cos 2\gamma_1}, \quad \nu = 2\delta t + \mathrm{const}.$$

Thus, in the framework of the inverse scattering method, the variable magnetic field profile (potential) has been constructed as a function of two free parameters. They vary amplitude and duration of magnetic field pulse $h(t) = h_0 + h_t$. The effect of self-induced transparency was shown to take place in Fermi systems with the non-linear dispersion law of excitations as well. An important remark should be made at this stage. At $h_0 \to 0$, the transition from the two-parametrical reflectionless potential (at $h_0 \neq 0$) to the one-parametrical potential (at $h_0 = 0$) is absent. It means, the obtained exact solutions relate to different classes of functions (see Eqs. (5), (7)), though both of the solutions have a common nature based on the nonlinear interaction between the external magnetic field pulse and the 1D quantum spin system.

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