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## ON MAGNETIC IMPURITIES IN GAPLESS FERMI SYSTEMS

L.S. BORKOWSKI

Institute of Physics, A. Mickiewicz University  
Matejki 48/49, 60-769 Poznań, Poland

In ordinary metals, antiferromagnetic exchange between conduction electrons and a magnetic impurity leads to screening of the impurity spin below the Kondo temperature,  $T_K$ . In systems such as semimetals, small-gap semiconductors and unconventional superconductors, a reduction in available conduction states near the chemical potential can greatly depress  $T_K$ . The behavior of an Anderson impurity in a model with a power-law density of states,  $\rho(\epsilon) \propto |\epsilon|^r$ ,  $r > 0$ , for  $|\epsilon| < \Delta$ , where  $\Delta$  is small compared to the bandwidth, is studied using the non-crossing approximation. The transition from the Kondo singlet to the magnetic ground state can be seen in the behavior of the impurity magnetic susceptibility  $\chi$ . The product  $T\chi$  saturates at a finite value at low temperature for coupling smaller than the critical one. For sufficiently large coupling  $T\chi \rightarrow 0$ , as  $T \rightarrow 0$ , indicating a complete screening of the impurity spin.

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### 1. Introduction

In a number of Fermi systems the density of states  $\rho(\epsilon)$  vanishes at the Fermi surface  $E_F$  and varies linearly or quadratically for  $|\epsilon|/D \equiv |E - E_F|/D \ll 1$ , where  $D$  is the energy scale associated with the conduction electron bandwidth. This situation may arise, e.g. in heavy-fermion or cuprate superconductors and anisotropic heavy-fermion semiconductors [1]. Also exotic phases of the Hubbard model may possess  $\rho(\epsilon) \propto |\epsilon|$  in two dimensions [2].

In normal metals dilute impurities coupled antiferromagnetically to the conduction band lead to a low-temperature reduction of the Curie term in the impurity magnetic susceptibility and an increase in the resistivity. This is known as a Kondo effect. The formation of the spin-singlet state favored by the antiferromagnetic coupling depends on the availability of electronic states at low energies.

Earlier studies by poor-man's scaling and large- $N$  method, where  $N$  is impurity orbital degeneracy, showed that the Kondo effect survives if the coupling between electrons and the impurity  $J$  is larger than a critical value  $J_c$  [3]. In a gapless system with  $\rho(\epsilon) \propto |\epsilon|^r$ ,  $J_c$  scales linearly with  $r$  for  $r \ll 1$ . A large- $N$  approach to

magnetic impurities in superconductors [4, 5] leads to similar results for  $J_c$ . However, for  $r \leq 1$  or  $N = 2$ , any finite impurity concentration was found to result in  $J_c = 0$ . Numerical renormalization group calculations [6, 7] and third-order scaling [8] show that the Kondo effect does not occur for  $r > 1/2$  in the particle-hole symmetric problem. Breaking this symmetry, e.g. by potential scattering or band asymmetry, helps the screening of the impurity moment. The critical coupling  $J_c$  was found to be strongly dependent on the magnitude of the potential scattering term [6]. Earlier calculations for the case of a full gap,  $\rho(\epsilon) = 0$  for  $|\epsilon| < \Delta \ll D$ , also found a finite  $J_c$  away from the particle-hole symmetry [9, 10].

In this work the  $SU(N)$  Anderson model is studied in the non-crossing approximation (NCA). In the limit of large Coulomb repulsion  $U$  on the impurity site and for temperatures  $T \ll U$ , the model has the following form:

$$H = \sum_{k,m} \epsilon_k c_{km}^\dagger c_{km} + E_f \sum_m f_m^\dagger f_m + V \sum_{k,m} (c_{km}^\dagger f_m b^\dagger + \text{h.c.}) + \lambda \left( \sum_m f_m^\dagger f_m + b^\dagger b - 1 \right), \quad (1)$$

where  $E_f$  is the position of the bare impurity level,  $f$  and  $b$  are the impurity fermion and the slave boson operators, respectively. The last term in the Hamiltonian follows from the restriction of the Hilbert space to a singly occupied impurity site,  $\sum_m f_m^\dagger f_m + b^\dagger b = 1$ ,  $m = 1, \dots, N$ . The self-energies of the slave boson and the impurity fermion Green's functions are given by

$$\Sigma_0(\omega + i0^+) = NV^2 \int_{-\infty}^{\infty} d\epsilon f(\epsilon) \rho(\epsilon) G_m(\omega + \epsilon + i0^+) \quad (2)$$

and

$$\Sigma_m(\omega + i0^+) = V^2 \int_{-\infty}^{\infty} d\epsilon (1 - f(\epsilon)) \rho(\epsilon) G_0(\omega - \epsilon + i0^+). \quad (3)$$

The density of states of the conduction band is assumed to be of the form  $\rho(\epsilon) = C|\epsilon/\Delta|^r \exp[-(\epsilon/D)^2]$  for  $0 < |\epsilon| < \Delta/D$ , and  $C \exp[-(\epsilon/D)^2]$  otherwise, and  $C$  is a normalization constant. The exponential part of  $\rho(\epsilon)$  does not influence the low-energy physics in any important way, while it is convenient in solving the integral equations (2) and (3).

## 2. Numerical results

Here we focus on the non-degenerate case,  $N = 2$ . Results for static spin susceptibility are shown in Fig. 1 for  $\Delta/D = 10^{-5}$ ,  $E_f/D = -0.67$ , and  $r = 1$  and  $r = 2$ . For larger  $\Gamma \equiv \pi N_0 V^2$ ,  $T\chi$  decreases to zero at low temperature, which is associated with the screening of the impurity spin. For  $\Gamma$  smaller than a certain critical coupling  $\Gamma_c$ ,  $T\chi$  remains finite, as  $T \rightarrow 0$ , indicating that impurity is not screened. The critical coupling for the data sets presented in Fig. 1 is  $\Gamma_c/D \simeq 0.108$  for  $r = 1$  and  $\Gamma_c/D \simeq 0.115$  for  $r = 2$ . A qualitatively similar behavior of the impurity susceptibility was found by numerical renormalization group calculations [6, 7].

The transition from the spin-singlet ground state to the unscreened moment is also reflected in the impurity density of states. The Abrikosov-Suhl resonance

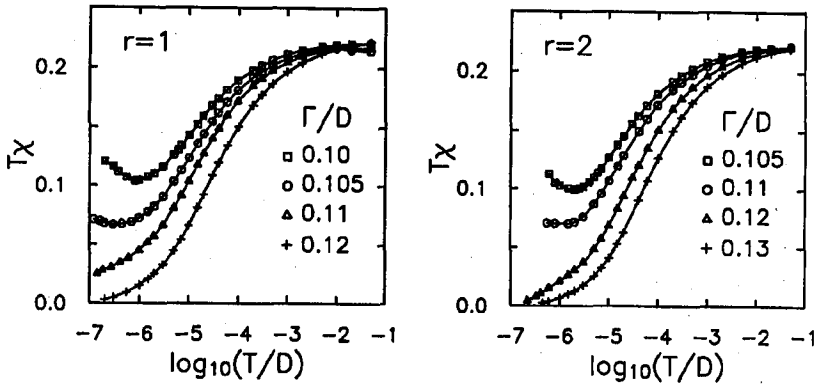


Fig. 1. Impurity spin susceptibility  $T\chi$  as a function of  $\log(T/D)$  for  $r = 1$  and  $r = 2$ . The magnitude of the pseudogap is  $\Delta/D = 10^{-5}$  and the bare impurity level is  $E_f/D = -0.67$ .

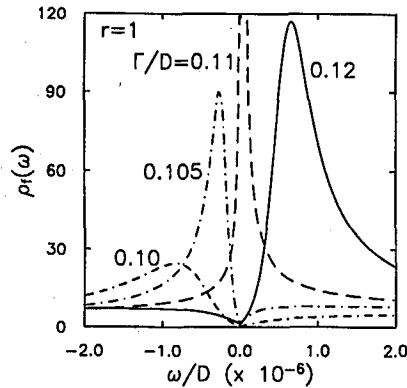


Fig. 2. The low-energy part of the impurity density of states  $\rho_f(\omega)$  for  $r = 1$  and the same data set as in Fig. 1, evaluated at  $T/D = 2 \times 10^{-7}$ ,  $1.2 \times 10^{-7}$ ,  $1.4 \times 10^{-7}$ , and  $2 \times 10^{-7}$  for  $\Gamma/D = 0.10$ ,  $0.105$ ,  $0.11$ , and  $0.12$ , respectively.

approaches the Fermi level when  $\Gamma \rightarrow \Gamma_c$ . For  $\Gamma < \Gamma_c$  the resonance falls below  $E_F$  as illustrated in Fig. 2. The analogous behavior of  $N_f(\omega)$  was noted earlier by Ogura and Saso [11] for the case of a full gap ( $r = \infty$ ).

A preliminary analysis of the dependence of the critical coupling on  $\Delta$  in the limit  $\Gamma \ll -E_f$ , and  $\Delta/D \ll 1$ , indicates scaling  $\Gamma_c \propto D/\ln(D/\Delta)$ , independent of  $r$ , at least for  $r \geq 1$ . This can be expected on the basis of the large- $N$  mean-field results in the Kondo limit [4], where it was found that the critical exchange coupling is  $J_c \simeq 2D/\ln(2D/\Delta)$  for  $\Delta \ll D$  in a model with  $\rho(\epsilon) = \text{const}$  outside the pseudogap region.

A more detailed study, including results for  $N > 2$ , will be presented in a separate publication.

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