DYNAMIC SUSCEPTIBILITY IN THIN FILMS
WITH ANTIFERROMAGNETIC COUPLING
BETWEEN LAYERS

R. ŚWIRKOWICZ

Institute of Physics, Warsaw University of Technology
Koszykowa 75, 00-662 Warsaw, Poland

(Received July 10, 1996)

The dynamic susceptibility of the system with antiferromagnetic coupling between layers is investigated within the framework of the multiband model using the equation of motion with random phase approximation. Calculations are performed in the mixed Bloch–Wannier representation and a general form for $\chi$ is found. The susceptibility can be written in terms of two-particle Green's functions expressed in the local coordinate system with the $z$ axis aligned along the local magnetization. The expression depends on an angle between the magnetization direction in a given layer and the crystal axis. Preliminary numerical calculations are performed for two systems: ultrathin Cr film and Fe/Cr multilayer structure. Imaginary part of the susceptibility corresponding to different layers is calculated and spin waves are discussed.

PACS numbers: 75.10.Lp, 75.30.Cr, 75.30.Ds, 75.50.Ee

1. Introduction

Recently, systems with antiferromagnetic coupling between layers have been widely investigated experimentally and theoretically. Especially, films and superlattices consisting of Fe and Cr, as well as Fe and transition nonmagnetic or noble metals have been studied [1–3]. The oscillations of the exchange coupling are observed in such materials. Changes in the magnetic coupling strongly influence spin waves and the temperature dependence of the magnetization [4–6]. Therefore, an investigation of the spin-wave modes characteristic of systems with antiferromagnetic coupling between layers seems to be very important. So far, the approach based on phenomenological or Heisenberg models was used [7–11]. However, in the case of transition metals calculations performed within the framework of the itinerant electron model are the most required ones.
In the band model spin waves for thin films and semi-infinite systems with ferromagnetic coupling between layers have been mainly investigated [12–15]. The dynamic susceptibility method within the random phase approximation (RPA) has been used. Calculations of the dynamic susceptibility performed in the Bloch–Wannier representation for various values of the wave vector and various values of energy allow one to determine spin-wave amplitudes, dispersion relations, local density of magnon states, temperature dependence of the magnetization [13–15]. Elaboration of the theory of dynamic susceptibility for nonuniform and finite systems with antiferromagnetic coupling seems to be a natural and significant extension of the existing approach. It would allow one to investigate spin waves in films and multilayered structures, in particular, in systems like Fe/Cr with antiferromagnetic or ferromagnetic type of the coupling. As far as uniform bulk materials are concerned, the theory of dynamic susceptibility in antiferromagnets within the framework of the multiband tight-binding model was worked out by Cade and Young [16], whereas the appropriate approach for ferromagnets was given by Cooke et al. [17]. Spin waves were investigated and remarkable consistency with experimental results was obtained. In nonuniform structures calculations of the static susceptibility based on the simple one-band model were performed [18].

The aim of the paper is to present the band approach to the theory of the dynamic susceptibility for systems in which the magnetization direction can change along the axis perpendicular to the surface and interfaces. As a special case films with antiferromagnetic coupling between layers are considered. Preliminary numerical calculations are performed. Ultrathin Cr films and Fe/Cr structures are simulated. The results obtained for the transverse susceptibility and spin waves are presented.

2. The Hamiltonian and one-electron properties

Calculations are based on the multiband Hamiltonian taken in the following form:

\[
H = \sum_{\nu j m \sigma} E_{\nu j m \sigma} d_{\nu j m \sigma}^{\dagger} d_{\nu j m \sigma} + \sum_{(\nu j, \nu j'), m m'} H_{m m'}^{\nu \nu'}(j, j') d_{\nu j m \sigma}^{\dagger} d_{\nu j' m' \sigma} + \frac{1}{2} \sum_{\nu j m', m m' m_1 m_1'} U_{m m_1 m_1 m_1'}^{\nu m'} d_{\nu j m_1 m_1'}^{\dagger} d_{\nu j m_1 m_1'}^{\dagger} d_{\nu j m_1 m_1'}^{\dagger} d_{\nu j m_1 m_1'}^{\dagger} d_{\nu j m_1 m_1'}^{\dagger} d_{\nu j m_1 m_1'}^{\dagger}
\]

(1)

\(d_{\nu j m \sigma}\) denotes an annihilation operator of an electron with spin \(\sigma\) and the orbital index \(m\) at the lattice point \(\nu j\), where \(\nu\) corresponds to the layer index and \(j\) is the radius vector in the film plane. \(H_{m m'}^{\nu \nu'}(j, j')\) denote two-center hopping integrals. \(E_{\nu}\) is the one-electron potential in the layer \(\nu\) and elements of matrix \(U\) describe the effective intraatomic interactions between electrons. The orbital representation is used for \(U\).

We consider a system which consists of \(n\) ferromagnetic layers, however, the direction of the magnetization in the layer \(\nu\) is characterized by an angle \(\vartheta_\nu\) with respect to the \(z\) axis, common to the whole system. The angle \(\vartheta_\nu\) may strongly depend on the layer index. In the case of simple antiferromagnetic coupling between
layers, \( \vartheta_{\nu} \) is equal to 0 or \( \pi \). In a general case, dependence of angle \( \vartheta_{\nu} \) on layer index can be more complex.

The magnetocrystalline anisotropy appears to be a very difficult and subtle problem in the band model approach [19]. It is not discussed in the paper. Hamiltonian (1) which constitutes the base of the considerations is invariant under rotations about the same angle with respect to any axis for the whole system. Therefore, the \( z \) axis of the crystal coordinate system can be taken in the direction perpendicular to the surface of the film. The choice seems to be the most appropriate for Fe/Cr multilayers, where the magnetization perpendicular to the surface can be expected. In other cases a rotation about an angle \( \pi/2 \) can be introduced to align the \( z \) axis in the film plane. Such a rotation does not change the Hamiltonian.

To study structures with antiferromagnetic coupling between layers, the local coordinate system is introduced. In the layer \( \nu \) the \( z' \) axis of the new coordinate system is aligned along the local direction of quantization. It corresponds to the rotation about an angle \( \vartheta_{\nu} \) in the atomic sheet with index \( \nu \). The rotation leads to the following transformation of operators \( d_{\nu jm}\sigma \):

\[
d_{\nu jm}\sigma = \cos \left( \vartheta_{\nu} \right) c_{\nu jm\sigma} - \hat{\sigma} \sin \left( \vartheta_{\nu} \right) c_{\nu jm-\sigma},
\]

where \( \hat{\sigma} = 1 \) for \( \sigma = \uparrow \) and \(-1 \) for \( \sigma = \downarrow \). Hopping terms in Hamiltonian (1) expressed by means of new operators \( c_{\nu jm\sigma} \) depend on \( \cos [(\vartheta_{\nu} - \vartheta_{\nu'})/2] \) and \( \sin [(\vartheta_{\nu} - \vartheta_{\nu'})/2] \) as well as on various spin indices \( \sigma, \sigma' \). The Hamiltonian is diagonalized in the Hartree–Fock (HF) approach with the use of the following transformation:

\[
c_{\nu jm\sigma} = \frac{1}{N} \sum_{h\alpha} T_{\nu\alpha}^{hm\sigma} e^{ihj} a_{\alpha h}.
\]

Because of the translational symmetry in the plane of the film the Fourier transform with two-dimensional wave vector \( h \) parallel to the surface is introduced. In the direction perpendicular to the surface more general transformation is used with \( T_{\nu\alpha}^{hm\sigma} \) corresponding to amplitudes of electrons which can be found numerically. In the expression (3) \( N \) denotes the number of atoms in the plane.

The local density of states is calculated according to the formula

\[
g_{\nu\sigma} = \sum_{ahm} |T_{\nu\alpha}^{hm\sigma}|^2 \delta (E - E_{\alpha h}),
\]

where \( E_{\alpha h} \) denotes the one-electron Hartree–Fock energy.

The mean magnetization in the layer \( \nu \) defined as

\[
m_{\nu} = \sum_{\sigma m} \hat{\sigma} \langle d_{\nu jm\sigma}^+ d_{\nu jm\sigma} \rangle
\]

after using transformations (2) and (3) takes the following form:

\[
m_{\nu} = \cos (\vartheta_{\nu}) \frac{1}{N} \sum_{ahm\sigma} \hat{\sigma} |T_{\nu\alpha}^{hm\sigma}|^2 f_{\alpha h} - \sin (\vartheta_{\nu}) \frac{1}{N} \sum_{ahm\sigma} (T_{\nu\alpha}^{hm\sigma})^* T_{\nu\alpha}^{hm-\sigma} f_{\alpha h},
\]

where \( f_{\alpha h} = \langle a_{\alpha h}^+ a_{\alpha h} \rangle \) denotes the Fermi–Dirac distribution function. For the simple antiferromagnetic case \( (\vartheta_{\nu} = 0 \) or \( \pi \) the magnetization calculated in the crystal coordinate system is directly expressed by the magnetization in the local system, however, with the sign dependent on the layer index.
3. The dynamic susceptibility

The dynamic susceptibility of the system is expressed by means of the spin operators in the following way:

$$\chi^{ij}(r, r', t - t') = (g \mu_B)^2 \frac{i}{\hbar} \langle [S^i(r, t - t'), S^j(r', 0)] \rangle$$

(7)

with

$$S^i(r, t) = \frac{1}{2} \psi^+(r, t) \sigma^i \psi(r, t).$$

(8)

In the above expression $\sigma^i$ is the Pauli matrix and $\psi(r, t)$ denotes the field operator

$$\psi(r, t) = \sum_{\nu j m \sigma} \phi^\nu_{jm}(r) \chi \sigma d_{\nu j m \sigma}.$$  

(9)

One-electron function $\phi^\nu_{jm}(r)$ corresponds to the state localized at the lattice point $\nu j$ with orbital index $m$. $\chi \sigma$ is a spin function. Operators $d_{\nu j m \sigma}$ can be expressed by means of the local operators $c_{\nu j m \sigma}$ and then by $a_{\alpha k}$, which diagonalize the Hamiltonian in the HF approach. It is convenient to introduce operators

$$S^{\sigma \sigma'}_{\nu q} = \sum_{\alpha \alpha' k m m'} (T^{\nu k m \sigma}_{\nu k m' \sigma'}) T^{k + q m' \sigma'}_{\nu k + q m \sigma} F^{m m'}_{\nu q}(q) a^{+}_{\alpha k} a_{\alpha' k + q},$$

(10)

where $F^{m m'}_{\nu q}(q)$ is the atomic form factor and $q$ is the two-dimensional vector parallel to the surface. The Fourier transform of the spin operator $S^\nu_{\nu q}(q, t)$ can be then expressed by means of $S^{\sigma \sigma'}_{\nu q}$. For example, in the case of a simple antiferromagnet spin operator $S^{-}_{\nu q}(q, t)$ can be written in the form

$$S^{-}_{\nu q}(q, t) = \cos^2 \left( \frac{\vartheta_{\nu}}{2} \right) S^{1\dagger}_{\nu q}(t) - \sin^2 \left( \frac{\vartheta_{\nu}}{2} \right) S^{1\dagger}_{\nu q}(t).$$

(11)

In a general case all operators $S^{\sigma \sigma'}_{\nu q}$ with various $\sigma$ and $\sigma'$ can be included.

To calculate the dynamic susceptibility the Green function defined as follows is introduced

$$G^{\sigma \sigma'}_{\mu}(\alpha k, \alpha' k + q) = \frac{i}{\hbar} \Theta(t) \langle [a^{+}_{\alpha k} a_{\alpha' k + q}, S^{\sigma \sigma'}_{\nu q}(0)] \rangle,$$

(12)

where $\Theta(t)$ is a step function. The equation of motion for $G^{\sigma \sigma'}_{\mu}$ is found using Hamiltonian (1) expressed by means of operators $a_{\alpha k}$ and it is solved within the RPA.

When the matrix $X$ related to the Green function in the way

$$X_{\nu \mu m m'}^{\sigma_1 \sigma_2 \sigma_3 \sigma_4}(q, \omega) = \frac{1}{N} \sum_{\alpha_1 \alpha_2 k} (T^{\nu k m \sigma_1}_{\nu k m' \sigma_2})^* T^{k + q m' \sigma_2}_{\nu k + q m \sigma_3} G^{\sigma_3 \sigma_4}_{\mu}(\alpha_1 k, \alpha_2 + q)$$

(13)

is introduced, the solution can be written in the form

$$X_{\nu \mu M}^{\sigma_1 \sigma_2 \sigma_3 \sigma_4}(q, \omega) = \sum_{\nu_1 M_1 \sigma_1' \sigma_2'} (I - \Gamma V)^{-1} \nu_1 M_1 M \langle \nu_1 \nu_1 M_1 M_1 (\Gamma F)_{\nu_1 \mu M_1} \rangle_{\nu_1 M_1 M_1}$$

(14)

where index $M \equiv mm'$. In the above formula $I$ denotes a unit matrix and $\Gamma$ is the non-enhanced susceptibility defined as follows:
The elements of matrix $V$ are related to elements of $U$ according to expression

$$V_{\nu\mu}^{\sigma_1\sigma_2\sigma_3\sigma_4} = \delta_{\nu\mu} U^{\nu}(\delta_{\sigma_4\sigma_4} \delta_{\sigma_2\sigma_3} - \delta_{\sigma_1\sigma_2} \delta_{\sigma_3\sigma_4}).$$

Formally, the structure of the matrix $\chi$ is similar to the one obtained in the case of thin films with ferromagnetic coupling [20]. The main difference is that in the case under consideration various spin indices are mixed and the summation over $\sigma$ appears in formula (14). The size of matrices is considerable. It depends on a number of layers in the film, a number of orbitals taken as basis functions and also on spin indices.

In the case of simple antiferromagnets the transverse and longitudinal parts of the susceptibility as well as of matrix $\chi$ are not coupled, therefore these parts can be investigated separately. In such a case, the transverse dynamic susceptibility can be written in the form

$$\chi = \begin{pmatrix} \chi^{++} & \chi^{+-} \\ \chi^{-+} & \chi^{--} \end{pmatrix}.$$  

Matrices which appear in the last formula for $\chi$ are expressed in terms of appropriate elements of $X$. For example, elements of the matrix $\chi^{-+}$ are equal to

$$\chi_{\nu\mu}^{-+}(q, \omega) = \sum_M F_M^*(q) \begin{pmatrix} \cos^2 \left(\frac{\vartheta_\nu}{2}\right) \cos^2 \left(\frac{\vartheta_\mu}{2}\right) X_{\nu\mu M}^{1111} \\
+ \sin^2 \left(\frac{\vartheta_\nu}{2}\right) \sin^2 \left(\frac{\vartheta_\mu}{2}\right) X_{\nu\mu M}^{1111} - \cos^2 \left(\frac{\vartheta_\nu}{2}\right) \sin^2 \left(\frac{\vartheta_\mu}{2}\right) X_{\nu\mu M}^{1111} \\
- \sin^2 \left(\frac{\vartheta_\nu}{2}\right) \cos^2 \left(\frac{\vartheta_\mu}{2}\right) X_{\nu\mu M}^{1111} \end{pmatrix}.$$  

The susceptibilities $\chi^{+-}$, $\chi^{--}$ and $\chi^{++}$ take similar forms.

According to formula (18) it is easy to notice that in the case of the film with ferromagnetic coupling between layers the obtained expression is reduced to

$$\chi_{\nu\mu}^{-+}(q, \omega) = \sum_M F_M^*(q) X_{\nu\mu M}^{1111},$$

because angles $\vartheta_\nu$ and $\vartheta_\mu$ are equal to zero for all layers. Matrix expression for $X$ can be also simplified in this case. The results obtained for thin ferromagnetic films are the same as in Ref. [20].

However, in a general case, when the angle $\vartheta$ varies from one layer to another in a more complex way the obtained expressions are quite complicated. In such a case because of a strong mixing the longitudinal and transverse components cannot be calculated separately.

Tedious numerical calculations are necessary in order to find susceptibility matrix $\chi$. First of all, one should find elements of $\Gamma$. Because of the size of the matrix, the calculations are time-consuming even in the case of the structure with
simple antiferromagnetic coupling between layers. To make calculations more effective one should introduce further simplifications. The approach to the problem can be similar as in thin ferromagnetic films. Possible procedures are presented and discussed in Ref. [20].

4. Numerical results

In this section some numerical results will be presented to illustrate the theory elaborated in the previous sections. Calculations are preliminary. Cr and Fe/Cr systems are simulated in a simple way.

There are some experimental evidences that the surface of Cr(001) is ordered ferromagnetically [21]. Tight-binding and \textit{ab initio} approaches to the ground state of Cr(001) films show that the magnetic moments couple ferromagnetically within the layers and antiferromagnetically with the moments at adjacent layers [22–24]. Such a type of the magnetic structure is assumed in the paper for the ground state of Cr film.

The calculations are performed within the one-band model. In such a case in all formulae presented in previous sections the orbital indices \( m \) can be dropped. It should be pointed out that in the one-band approach the size of \( \Gamma \) and \( X \) is considerably reduced.

The film of bcc structure with the surface perpendicular to the [001] direction is considered. The hopping of electrons between the nearest and the next nearest neighbors is taken into account. Calculations are performed for the one effective band, which corresponds to five \( d \) bands lumped together, therefore the Fermi energy is calculated with the condition that the number of electrons per atom in the middle layer is equal to 5.0. The integral \( U \) is assumed to be 0.5 eV and values of \( E \) are shifted in such a way that the enhancement of the magnetic moment near the surface could be obtained. Small shifts are also taken into account in other layers. The hopping integral for the first neighbors is taken to be equal to 0.72 eV. For the second neighbors it is correspondingly smaller. Calculations are performed within the HF approximation. Terms of the form \( \langle \psi_{\nu j \sigma} \psi_{\nu j -\sigma} \rangle \) non-diagonal in spin indices, which appear in the HF potential are assumed to be equal to zero at the starting point. However, all these terms are consequently taken into account in the next iteration steps. In the problem under consideration the terms appear to be relatively small.

The profile of the magnetization calculated for Cr film is presented in Fig. 1. Due to the changes in the one-electron potential the strong enhancement of the magnetic moment at the surface is obtained. Inside the film the moment is substantially lower, though similarly as in other approaches, it is still higher than in the bulk material. The calculated density of states for the surface and central layers is presented in Fig. 2. The curves in general resemble the ones calculated in Ref. [23]. Peaks corresponding to surface states can be seen.

The real and imaginary parts of non-enhanced susceptibility \( \Gamma \) calculated for the surface and central layers are presented in Fig. 3. A high peak which appears in \( \text{Im} \Gamma_{11} \) is a result of the enhanced density of states at the surface. The shift of the peak towards higher values of energy as compared to the main peak in the central plane can be related to the strong enhancement of the surface magnetization. Real
part of $\Gamma$ calculated for the surface layer is also correspondingly higher than for the central plane.

Next, the calculations are performed for the enhanced transverse susceptibility $\chi$. Elements of matrices $\chi^{--}$ and $\chi^{++}$ are small as compared to $\chi^{-+}$ and $\chi^{+-}$. Therefore it seems that in the case under consideration only matrices $\chi^{-+}$ and $\chi^{+-}$ should be discussed [9]. Imaginary part of $\chi^{-+}$ exhibits the pronounced peak in the first layer (Fig. 4a). On the other hand, according to Fig. 4b well defined peaks can be seen in $\text{Im}\chi^{+-}$ for the second and fourth layers. Peaks which appear at low energies are relatively low and broad, whereas these corresponding to higher values are well pronounced. In fact, in the scale of Fig. 4a it is difficult to observe the small peak at lower values of energy. One can expect that the calculated peaks are related to spin-wave modes, however, the low-lying branches seem to be damped. These lowest modes are optical ones and are very strongly localized

Fig. 1. The profile of the magnetic moment for the Cr film.

Fig. 2. The density of states for the surface (a) and central (b) layers in Cr film. Solid and dashed lines correspond to majority and minority spins, respectively.
near the surface. Wave functions corresponding to the modes characteristic of the system under consideration are depicted in Fig. 5. The total number of the modes is equal to $2n$. The lowest modes presented in Fig. 5a have strongly enhanced amplitudes in the first layer and can be related to the first sublattice (with the magnetization parallel to the $z$ axis). On the other hand, low-lying modes presented in Fig. 5b with amplitudes strongly enhanced at the second layer can be related to the second sublattice (with the magnetization aligned antiparallel to the $z$ axis). The fact that the low lying modes are strongly localized near the surface is a result of the boundary conditions with the magnetic moment enhanced in the surface layer. Due to the boundary conditions, layers with different magnetization directions are not fully equivalent.
As the second example Fe/Cr structure is simulated with three Fe layers separated by three and four atomic sheets of Cr. The integral $U$ equal to 0.8 eV is taken for Fe in consistency with Ref. [25]. The shift of the one-electron potential $E$ in Cr layers as compared to Fe ones is about 0.45 eV.

The ground state of Fe/Cr systems appears to be a very difficult problem. One-electron properties were investigated in many papers with the use of model and \textit{ab initio} approaches [26–29]. Investigations show that the coupling between Fe layer and the adjacent Cr plane is antiferromagnetic. Between two Fe layers separated by Cr spacer the coupling can be ferromagnetic or antiferromagnetic in dependence on a thickness of the spacer. \textit{Ab initio} calculations show that for an odd number of Cr layers ferromagnetic coupling is more favorable, whereas for an even number of Cr layers the antiferromagnetic one takes place. In this last case the spin density wave can form in the spacer [29]. The ground-state problem is not discussed in the present paper. At the start the state consistent with the results of Ref. [29] is taken and the ferromagnetic ordering of two Fe layers separated by 3 planes of Cr is assumed. The profile of the magnetization calculated in a self-consistent way is depicted in Fig. 6. Features very characteristic of the distribution of the magnetization in Fe/Cr system can be reproduced in the present approach. The magnetization of Fe is slightly suppressed for layers which are in direct contact with Cr and it is enhanced in the central Fe plane.

Dynamic susceptibility $\chi = \frac{1}{2}(\chi^{+-} + \chi^{-+})$ calculated for the system under consideration is presented in Fig. 7. The curves correspond to $\text{Im} \chi$ obtained for
surface and interface Fe layers. One can see low-energy peaks which are related to spin waves. The very high peak found for the surface plane directly shows that the mode strongly localized near the surface is obtained. Wave functions of the two spin-wave branches with the lowest energies are depicted in Fig. 8. Amplitudes of the modes are strongly enhanced in the surface and they are practically equal to zero in the central part of the film, namely in the Cr layers. The first mode propagates with the same phase in both Fe sublayers, therefore, it is the acoustic mode. The second mode appears to be out-of-phase optical. Energies of the spin waves correspond to the zero of the determinant. Values calculated for these two modes with \( q = \pi/8a(1,0) \), where \( a \) is the lattice constant, are close to each other and are equal to 13.1 meV and 14.4 meV for acoustic and optical branches, respectively.

Calculations are also performed for 3Fe/4Cr/3Fe system with antiferromagnetic coupling between Fe layers. In this case the optical branch with the lowest energy is found, whereas the acoustic mode corresponds to the higher value. Preliminary calculations show that for \( q = \pi/8a(1,0) \) the difference in energy between the two low-lying modes in the antiferromagnetic case is greater than in the ferromagnetic one. It could suggest that the absolute value of the magnetic coupling is higher in the system with four-layer spacer. Such a conclusion seems to be consistent with results of \textit{ab initio} calculations [29] as well as obtained in Ref. [30]. However, it should be pointed out that to obtain reliable values of the coupling, the high accuracy numerical calculations for \( q = 0 \) must be performed [6].

Fig. 7. Imaginary part of \( \chi \) for surface (solid line) and interface (dashed line) Fe layers in 3Fe/3Cr/3Fe system; \( q = \pi/8a(1,0) \).

Fig. 8. Wave functions for two low-lying modes in 3Fe/3Cr/3Fe system.
5. Summary and discussion

The band model approach to the problem of the dynamic susceptibility in thin films with antiferromagnetic coupling between layers is elaborated in the paper. It can be applied to any system in which the direction of the magnetization changes along the axis perpendicular to the surface. In particular, it can be used to bilayers or multilayers with antiferromagnetic coupling. Spin waves characteristic of such systems can be investigated, amplitudes and energies of the modes can be determined. Within the framework of the proposed approach it is possible to investigate conditions under which the interface modes, already found in the Heisenberg model [7] can appear. The difference in energy calculated for two lowest spin-wave modes with \( q = 0 \), namely the in-phase and out-of-phase ones, corresponds to the magnetic coupling between sublayers separated by non-magnetic spacer [6]. Therefore, the presented approach allows one to estimate the coupling in the dependence on the thickness of the spacer for systems in which oscillations take place.

Two simple examples of application of the theory are given. In general, the results are consistent with the ones which can be expected on the basis of the Heisenberg model [9, 31]. However, calculations performed for ultrathin films within the framework of the simple one-band model show that spin waves strongly localized near the surface of Cr can be damped.

It should be emphasized that numerical calculations presented in the paper are preliminary and are performed within the framework of the one-band model. The extension to the multiband model is simple, though calculations can be time-consuming. Such calculations have been already performed for Ni/Ag/Ni systems with antiferromagnetic coupling between Ni sublayers across the Ag spacer. The weakly localized near the surfaces out-of-phase and in-phase modes with close values of energy were found. The dispersion relation was determined. The detailed results will be published in a separate paper.

References