Vol. 89 (1996) ACTA PHYSICA POLONICA A No. 4

CHAOTIC DYNAMICS OF PERIODICALLY DRIVEN DOMAIN WALLS IN MAGNETIC GARNET FILMS

R.A. Kosiński[a,b] and A. Sukiennicki[a,c]

[a]Institute of Physics, Warsaw University of Technology
Koszykowa 75, 00-662 Warszawa, Poland
[b]Central Institute for Labor Protection
Czerniakowska 16, 00-701 Warszawa, Poland
[c]Institute of Physics, Polish Academy of Sciences
Al. Lotników 32/46, 02-668 Warszawa, Poland

(Received December 13, 1995)

The motion of domain walls in thin garnet films was investigated numerically using Slonczewski's equations of wall motion for the case of periodic drive field. The type of the wall motion was analyzed by observation of phase trajectories and spatio-temporal diagrams. It was found that depending on the period and amplitude of the drive field the motion of the wall is periodic or chaotic, reflecting the character of the dynamical processes connected with horizontal Bloch lines in the wall.

PACS numbers: 05.45.+b, 75.10.Hk, 75.60.Ch

1. Introduction

Studies of chaotic dynamics in magnetic systems are concerned mainly on investigation of the non-linear ferromagnetic resonance in yttrium iron garnet (YIG) films (see e.g. Ref. [1]). Only a limited number of papers deals with the deterministic chaos in nonlinear dynamics of domain walls (see e.g. [2, 3]). Some years ago it was shown theoretically that for drive fields larger than the critical field in infinite material a structural transition occurs in the wall and a chaotic attractor appears in a properly chosen phase space [4]. On the other hand, recently, it has been found theoretically that the chaotic structure can appear in domain walls of magnetic bubble garnet films [5]. The so-called diffuse domain walls observed experimentally in such materials by Vural and Humphrey [6] and analyzed numerically by Kosiński [7] can be regarded as a proof of the existence of chaotic structure of domain walls. Moreover, the saturation velocity of domain walls in bubble domain films can also be treated in terms of chaotic motion [8]. In view of the fact that the chaotic motion of magnetic domain walls is a very inconvenient feature in various technological applications, it is clear that a better understanding of chaotic wall dynamics is highly required. In most of applications ac drive fields have been applied, while in the papers [4, 5] the drive fields constant in time have been used. Therefore, in this paper the periodically driven domain walls are investigated.

2. Equations of motion

The dynamics of a domain wall in the uniaxial magnetic bubble garnet film is described by a pair of nonlinear partial differential equations, which have been derived by Malozemoff and Slonczewski [9] from the Landau–Lifshitz equation with Gilbert's damping term included

$$\dot{q}/\Delta = 2\pi M\gamma \sin 2\psi - (2A\gamma/M)\psi_{zz} - (\gamma\pi/2)H_{sy}\cos\psi + \alpha\dot{\psi}, \tag{1}$$

$$\dot{\psi} = \gamma H_z + (2\gamma A/M\Delta)q_{zz} - (\alpha/\Delta)\dot{q}, \tag{2}$$

where the stray fields H_{sy} from the surfaces of the film can be taken in the form proposed by Hubert [10]

$$H_{sy} = 4M \ln \frac{h/2 - [z - 2\Delta \sinh(z/2\Delta)/\cosh(h/4\Delta)]}{h/2 + [z - 2\Delta \sinh(z/2\Delta)/\cosh(h/4\Delta)]}. \tag{3}$$

In Eqs. (1)–(3) all symbols have the same meaning as in Ref. [5], with the exception of the drive field, which was constant in [2], but here is assumed in the form dependent periodically on time

$$H_z = H\cos\omega t. \tag{4}$$

Here, A is the exchange constant, γ is the gyromagnetic ratio, M is the saturation magnetization, α is the Gilbert damping constant, $\Delta = (A/K)^{1/2}$ is the wall thickness parameter (K is the perpendicular anisotropy constant) and h is the film thickness. The variable $q(z,t)$ describes the local position of the Bloch surface of the wall and the variable $\psi(z,t)$ is the azimuthal angle of the local magnetic moment of the wall with respect to $+0x$ axis. The coordinate z is perpendicular to the film and parallel to the easy axis of the uniaxial anisotropy of the film. A dot over a symbol denotes the time derivative, the subscript zz — the second derivative with respect to the z coordinate.

3. Method of analysis

Equations of motion (1) and (2) were solved numerically by means of a full implicit scheme (described in Ref. [11]) for $N = 51$ numerical points located uniformly along the film thickness. Force-free boundary conditions were applied [9]. The initial conditions were $q(z,0) = 0$, $\psi(z,0) = \psi_s(z)$, where $\psi_s(z)$ is the static distribution resulting from the solution of Eqs. (1) and (2) with the time derivatives equal to zero.

The material parameters were taken the same as in Ref. [5]. Namely, $A = 0.81 \times 10^{-7}$ erg/cm, $4\pi M = 140$ G, $\gamma = 1.75 \times 10^7$ s^{-1} Oe^{-1}, $\Delta = 2.9 \times 10^{-6}$ cm, $\alpha = 0.156$ and $h = 1.4 \times 10^{-4}$ cm.

To analyze the type of wall motion, the trajectory $\tilde{q}(\tilde{\psi})$ of the mid-point of the wall was calculated at each step of the integration procedure. The tilde denotes that the averaged (over the thickness of the film) values $\overline{q}$ and $\overline{\psi}$ were subtracted from the instantaneous values of q and ψ, respectively, because in this paper we are interested in the wall structure during the motion but not in the overall oscillating motion of the wall.

In many cases the trajectory of the mid-point of the wall is so complicated that it is difficult to decide whether a considered motion is periodic or chaotic. To solve this problem, the so-called spatio-temporal diagrams are constructed. This technique was proposed earlier for the analysis of a spatially extended dynamical systems [12, 13]. For this purpose each numerical point is, for the time ΔT, represented by a cell with the sizes $(\Delta z, \Delta T)$, where $\Delta z = h/N$ and ΔT was chosen depending on $T = 2\pi/\omega$. The colour of each cell is black or white depending on the value of $\psi_i(n\Delta T)$ where $n = 1, 2, 3, 4 \ldots$ and $1 \leq i \leq N$. For $\psi_i \geq \overline{\psi}$ the cell is black, otherwise it is white. Thus, the stripe of black/white cells for all numerical points codes wall dynamics in the time $t = n\Delta T$. Whole spatio-temporal diagram is constructed as the sequence of such stripes for a chosen time window $t_{\mathrm{w}} = k\Delta T$. Here k was chosen depending on $T = 2\pi/\omega$.

Only asymptotical trajectories are analyzed, i.e. such for which all transients are ceased.

Our discussion will concern only this range of drive fields in which a transition to chaos occurs (for smaller drives the wall motion is regular and will not be discussed here).

4. Results and discussion

First of all, it is reasonable to remind the main result obtained for the drive field constant in time [5]: chaos appears in the structure of the wall via intermittency in the narrow range of the drive field about 26.6 Oe.

If the wall is driven periodically, the results are much more complicated. There are two control parameters: the amplitude and the period of oscillation of the drive field.

For large values of the oscillation period $T = 2\pi/\omega$ and the amplitudes H of the drive field below a certain critical value the trajectory $\tilde{q}(\tilde{\psi})$ is periodic, although more complicated than in the case of constant drive fields. Figure 1 depicts the results for $T = 150$ ns and $H = 51$ Oe. Part (a) shows the trajectory and part (b) — the spatio-temporal diagram. The picture of the trajectory reflects the fact that during one part of the oscillation period the drive field is positive and during another part — negative. Therefore certain traces of symmetry are observed. The fact that the drive field amplitude changes continuously causes that the trajectory is very complicated. However, an important feature of this trajectory is that it represents the closed and periodic curve (!). It can be easily seen at part (b) of Fig. 1, where the period observed in the picture corresponds to the period of the drive field, exactly $T = 150$ ns $= 6\Delta T$. This result shows that for such a drive amplitude the changes of the field are slow enough that the structure of the wall is able to change periodically with the same period: the horizontal Bloch lines are created in a twisted structure of the wall near surfaces of the film, they move to opposite surfaces, they are annihilated by punching through to that surfaces, next other Bloch lines are created, and so on. All these phenomena appear in a way

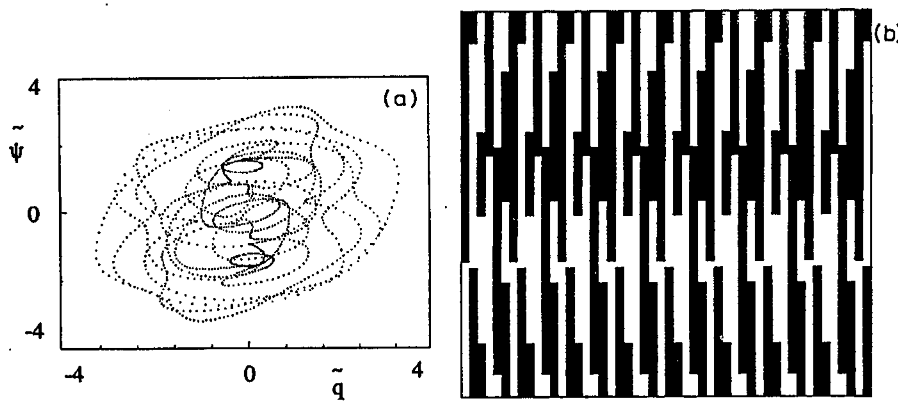

Fig. 1. Periodic motion of domain wall for H_z = 51 Oe and T = 150 ns. (a) Phase trajectory; (b) spatio-temporal diagram. Vertical size of the diagram corresponds to grid points distributed uniformly along film thickness h. Time axis is horizontal, with $\Delta T = 25$ ns and $t_w = k\Delta T = 1250$ ns (explanation in the text).

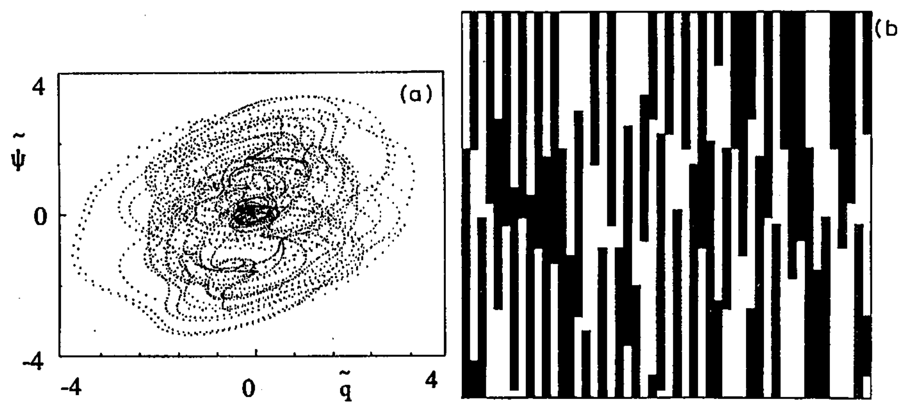

Fig. 2. Chaotic motion of the wall for $H_z = 52$ Oe and $T = 150$ ns. (a) Phase trajectory; (b) spatio-temporal diagram with the same orientation and parameters as in Fig. 1.

known very well from papers published many years ago (see e.g. Ref. [9]). What is important, however, for our present investigation is the fact that in the case considered it happens periodically.

Such a periodic motion is no longer possible if the drive field amplitude is higher than a certain critical value. It can be seen from Fig. 2 that for the same value of the period, $T = 150$ ns, but slightly higher value of the amplitude, $H = 52$ Oe, the trajectory $\tilde{q}(\tilde{\psi})$ is chaotic (part (a)). The spatio-temporal diagram (part (b)) depicts in this case a behaviour which is not periodic even after time much longer than all transients observed in periodic cases. This result reflects the fact that the behaviour of the Bloch lines in the wall is not periodic in this case. All phenomena appear in a similar way as previously with exception that now the evolution of the structure of the wall is no longer able to follow the changes of the drive field.

If the period of the drive field is changed, the critical drive field amplitude for a transition from the periodic to the chaotic behaviour changes also, however a dependence is not monotonic, e.g. for $T = 130$ ns, $H_{\text{crit}} = 55$ Oe, but for $T = 100$ ns, $H_{\text{crit}} = 39$ Oe.

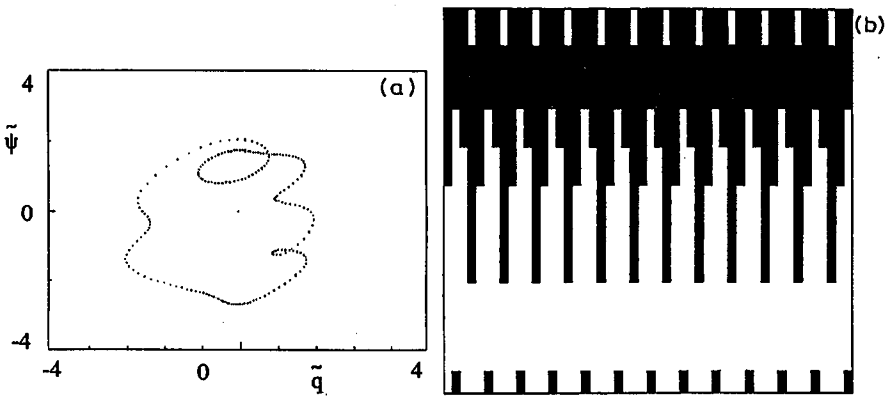

Fig. 3. Periodic motion of domain wall for $H_z = 53$ Oe and $T = 20$ ns. (a) Phase trajectory; (b) spatio-temporal diagram with $\Delta T = 5$ ns and $t_w = k\Delta T = 250$ ns (orientation of the diagram as in Fig. 1).

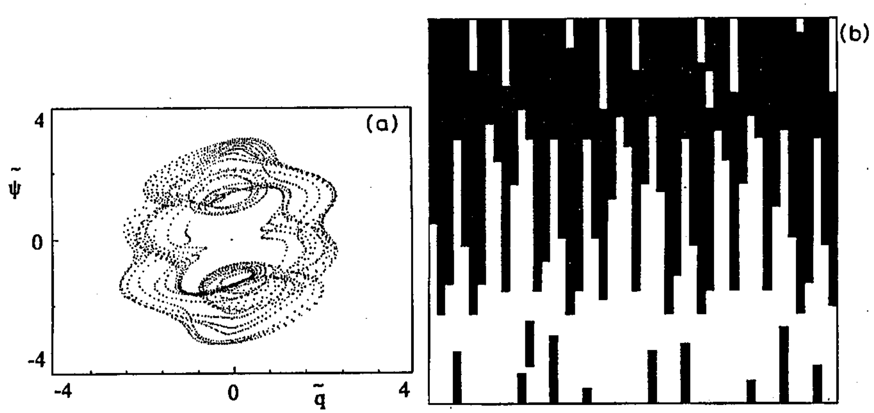

Fig. 4. Chaotic motion of the wall for $H_z = 54$ Oe and $T = 20$ ns. (a) Phase trajectory; (b) spatio-temporal diagram with the same orientation and parameters as in Fig. 3.

Now, we present the results obtained in the range of small values of T. Figure 3a and 3b depicts the trajectory and the spatio-temporal diagram, respectively, obtained for $T = 20$ ns and $H = 53$ Oe. Both pictures show clearly that the behaviour is periodic. The behaviour of the Bloch line in the wall is also periodic.

For the same value of the period, $T = 20$ ns, but slightly higher value of the drive field amplitude, $H = 54$ Oe, the chaotic behaviour is found (Fig. 4a and 4b). It is interesting that the behaviour of Bloch lines in the wall in both these cases is to some extent similar as for the case of long periods. The difference is only in the fact that now the Bloch lines are created at only one surface of the film, they move to the opposite surface, they are annihilated without punching through to the surface, other Bloch lines are created at the same surface as before, and so on. Again, all these phenomena are periodic if the drive field amplitude is smaller than a certain critical value but they are chaotic for amplitudes above this value.

For low values of the period T of the drive field, the dependence of the critical value of the field on T is very irregular: some resonance peaks appear. Such peaks occur e.g. at $T = 15$ ns for which the periodic attractor is obtained even at $H = 193$ Oe (while at $T = 10$ ns the chaotic attractor appears already for $H = 134$ Oe) and at $T = 35$ ns — for which the periodic attractor is observed also even for $H = 193$ Oe. The existence of these peaks shows that for the corresponding value of the drive field period there is a resonance of the drive field with some changes in the structure of the wall, which causes that the critical value for the appearance of the chaotic motion is so high.

5. Conclusions

It is found that the motion of domain wall in a periodic drive field can be periodic or chaotic. The main cause which defines the type of wall motion for a specified value of the amplitude of the drive field is the relation between characteristic times for the wall (time of generation and annihilation of horizontal Bloch line and time of shifting a single line from one to the opposite film surface) and the drive field period. The exact relation is very complex, however it can be stated that if these values are, in a certain sense, in a resonance — then even for high values of the drive field amplitude — the wall motion can be periodic.

It is interesting that the periodic motion of the wall can be observed even in the cases for which the domain wall contains Bloch lines. In the earlier papers describing the dynamics of domain walls for the case in which the vertical Bloch lines can be generated it was observed that the motion of domain walls may be periodic only if the Bloch lines are absent in the wall. When the vertical Bloch lines appear in the wall its motion is always chaotic [14]. For the wall in which the horizontal Bloch lines can be generated — which is the case investigated in the present paper — the periodic wall motion can be still observed even when the Bloch lines are generated.

Acknowledgments

The work was supported by the Committee for Scientific Research (Republic of Poland) under grant No. 2 P03B 042 08.

References

[1] P.E. Wigen, M. Ye, D.W. Peterman, *J. Magn. Magn. Mater.* **140-144**, 2074 (1995).
[2] H. Suhl, X.Y. Zhang, *J. Appl. Phys.* 61, 4216 (1987).

[3] H. Okuno, Y. Sugitani, K. Hirata, *J. Magn. Magn. Mater.* **140-144**, 1879 (1995).

[4] J.J. Żebrowski, A. Sukiennicki, *Acta Phys. Pol. A* **72**, 299 (1987).

[5] A. Sukiennicki, R.A. Kosiński, *J. Magn. Magn. Mater.* **129**, 213 (1994).

[6] K. Vural, F.B. Humphrey, *J. Appl. Phys.* **51**, 549 (1980).

[7] R.A. Kosiński, *Phys. Rev. B* **50**, 6751 (1994).

[8] E.E. Kotova, V.M. Chetverikov, *Fiz. Tverd. Tela* **32**, 1269 (1990) [*Sov. Phys. Solid State* **32**, 748 (1990)].

[9] A.P. Malozemoff, J.C. Slonczewski, *Magnetic Domain Walls in Bubble Materials*, American Press, New York 1979.

[10] A. Hubert, *J. Appl. Phys.* **46**, 2276 (1975).

[11] R.A. Kosiński, J.J. Żebrowski, A. Sukiennicki, *J. Phys. D* **22**, 451 (1989).

[12] K. Kaneko, in: *Formation Dynamics and Statistics of Patterns*, Eds. K. Kawasaki, M. Suzuki, A. Onuki, World Sci., Singapore 1989, p. 1.

[13] R.A. Kosiński, *Phys. Lett. A* **169**, 263 (1992).

[14] R.A. Kosiński, *IEEE Trans. Magn.* **30**, 1233 (1994).