ON THE MESOSCOPIC GATE

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The possibility of constructing of mesoscopic gate controlled by external electric field is shown.

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1. Introduction

The problem of artificial neural network construction is now widely discussed. The first step to the creation of neurocomputer is a construction of the corresponding elements (transistor, trigger, etc.) of nano-scale [1–3]. The physical systems, where the electron phase coherence is preserved at a scale much larger than the atomic dimensions, are often referred to as mesoscopic systems. The problem of electron transmission through a mesoscopic device may be reduced, usually, to the investigation of wave propagation in a wave guide. This approach has allowed one to describe some phenomena in nanoelectronic systems [3–9]. The aim of this paper is to suggest a possible construction of a mesoscopic gate. Namely, we shall consider the process of "opening" of a quantum wave guide under the action of an external transversal electric field due to the shift of eigenvalues. This effect may be used for a construction of the nanoelectronic device having two states: "current exists" – "current is equal to zero" in accordance with a situation whether the quantum wave guide is "open" or "closed".

2. Results and discussion

Let us consider a two-dimensional quantum wave guide (with the Dirichlet boundary condition) in a transversal uniform electric field. The description of the electron transport reduces to the investigation of the boundary problem

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}+\frac{2m}{\hbar^2}(E+Fx)f=0, \quad f(0,y)=f(L,y)=0. \tag{1}$$

Here x, y are the Cartesian coordinates of a point, m, E are the mass and energy of an electron, respectively, $\hbar$ is the Planck constant, the parameter F is related with the strength of the external electric field, L is the width of the wave guide. Using the separation of the variables, one obtains the solution in the form

$$f(x,y) = \sum_{n=1}^{\infty} \varphi_n(x) \exp(\mathrm{i}\sqrt{E-\lambda_n}y),$$

where $\{\lambda_n\}$ is the set of eigenvalues of the operator

$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2} - Fx$$

with the zero condition at the points $x=0$, $x=L$. Thus, the problem reduces to the following one:

$$\varphi'' + 2m\hbar^{-2}(\lambda + Fx)\varphi = 0, \qquad (2)$$
$$\varphi(0) = \varphi(L) = 0.$$

Let us replace the variable

$$z = (x + \lambda/F)(2mF/\hbar^2)^{1/3}.$$

Then Eq. (2) takes the form $\varphi'' + z\varphi = 0$. The basic solutions of the equation are $Ai(-z)$, $Bi(-z)$, where $Ai(z)$, $Bi(z)$ are the Airy functions. Taking into account the boundary conditions, one obtains

$$\varphi(z) = C\left(Bi\left(-2\lambda m(F\hbar)^{-2}\right)Ai(-z) - Ai\left(-2\lambda m(F\hbar)^{-2}\right)Bi(-z)\right), \qquad (3)$$

where C is a constant, λ (eigenvalue) is a root of the equation

$$Bi\left(-2\lambda m(F\hbar)^{-2}\right)Ai\left(-(L+\lambda/F)\left(2m(F\hbar)^{-2}\right)^{1/3}\right)$$
$$-Ai(-2\lambda m(F\hbar)^{-2})Bi(-(L+\lambda/F)(2m(F\hbar)^{-2})^{1/3}) = 0. \qquad (4)$$

It is known that the wave guide "is closed" if the electron energy E is smaller than the minimal eigenvalue λ_1 because there are no travelling waves, and it "is open" in the opposite ($E > \lambda_1$) case (travelling waves exist). One can see that the situation depends on the value of F, i.e., on the strength of the external electric field. For real physical systems one has $LF/E \ll 1$. Let us consider the shift of the first eigenvalue λ_1 under the influence of the electric field. Let the difference between E and the first eigenvalue Λ_1 for the case of absence of the electric field ($\Lambda_1 = \pi^2\hbar^2(2mL^2)^{-1}$) is small ($(E-\Lambda_1)/\Lambda_1 \ll 1$). Then for $LF/\Lambda_1 \ll 1$ the Airy functions in Eq. (3) may be replaced by its asymptotic forms [10]

$$\varphi(z) \approx C\left(\cos\left(\frac{2}{3}\left(\lambda/F(2mF\hbar^{-2})^{1/3}\right)^{3/2} + \frac{\pi}{4}\right)z^{-1/4}\sin\left(\frac{2}{3}z^{3/2} + \frac{\pi}{4}\right)\right.$$
$$\left. - \sin\left(\frac{2}{3}\left(\lambda/F(2mF\hbar^{-2})^{1/3}\right)^{3/2} + \frac{\pi}{4}\right)z^{-1/4}\cos\left(\frac{2}{3}z^{3/2} + \frac{\pi}{4}\right)\right) =$$
$$= Cz^{-1/4}\sin\left(\frac{2}{3}\left(z^{3/2} - \lambda^{3/2}(2m\hbar^{-2})^{1/2}/F\right)\right).$$

Consequently, Eq. (4) simplifies considerably

$$\sin\left(\frac{2}{3}\left((L+\lambda/F)^{3/2}(2mF\hbar^{-2})^{1/2} - \lambda^{3/2}(2m\hbar^{-2})^{1/2}/F\right)\right) = 0.$$

Hence,

$$\pi\hbar(2m)^{-1/2} = \frac{2}{3}\lambda_1^{3/2}\frac{((1+LF/\lambda_1)^{3/2}-1)}{F} \approx \lambda_1^{1/2}L + 4^{-1}L^2F/\lambda^{1/2},$$

i.e., the first eigenvalue is a root of the following equation:

$$\lambda_1 - \Lambda_1^{1/2}\lambda_1^{1/2} + 4^{-1}LF = 0.$$

The shift of the eigenvalue is given by the formula

$$\lambda_1^{1/2} \approx \Lambda_1 - 4^{-1}LF\Lambda_1^{-1/2}. \tag{5}$$

The analogous calculations can be made also for the case of parabolic confinement potential (instead of the Dirichlet boundary condition). In this situation we have the following equation:

$$\varphi'' + 2m\hbar^{-2}(-\omega^2x^2/2 + \lambda + Fx)\varphi = 0$$

instead of Eq. (2). The shift of the first eigenvalue in this case is given by the known formula

$$\lambda_1 = \Lambda_1 - F^2(2m\omega^2)^{-1}.$$

3. Conclusion

The relation between a transversal electric field and the existence of a travelling wave in a quantum wave guide is considered. The shift of the eigenvalues is found. The effect of "opening" of the wave guide under the action of the electric field may be used for the construction of a gate. Let the external electric field be absent and let the energy of the electron be slightly smaller than Λ_1. In this case the transmission coefficient is zero because there are no travelling waves. Let us switch on the electric field of such strength that the inequality $\lambda_1 < E$ is valid. Then the transmission coefficient differs from zero, because there is a spreading mode (travelling wave) in the wave guide. Thus, we have a mesoscopic gate (switch) which is controlled by the electric field. In the case of the parabolic confinement potential the analogous effect of "opening of the wave guide" takes place.

To make the incoming electron energy close to Λ_1, one can use a double-barrier structure, a system of wave guides and resonators [4, 7, 11], etc., i.e., the systems for which the dependence of the transmission coefficient on the electron energy (see Fig. 1) has a resonant character (and, consequently, such devices can play a role of an energy filter for the electron). The width of the first transmission coefficient peak should be sufficiently narrow (in real double-barrier structures it is about 0.1 meV [12]).

The scheme of a possible construction is shown in Fig. 2. One can note that the physical principle for the "opening–closing" process of the wave guide is different than the principles for analogous constructions (see Ref. [8]). In our construction "the opening of the wave guide" is caused by the shift of eigenvalues of differential operator for the transversal cross-section of the wave guide due to the appearance of an electric field. Other constructions [8] use MOS structures, heterojunctions, i.e., the devices are based on different physical principles. Moreover, the effect is opposite. Namely, in our construction an increase in the transversal

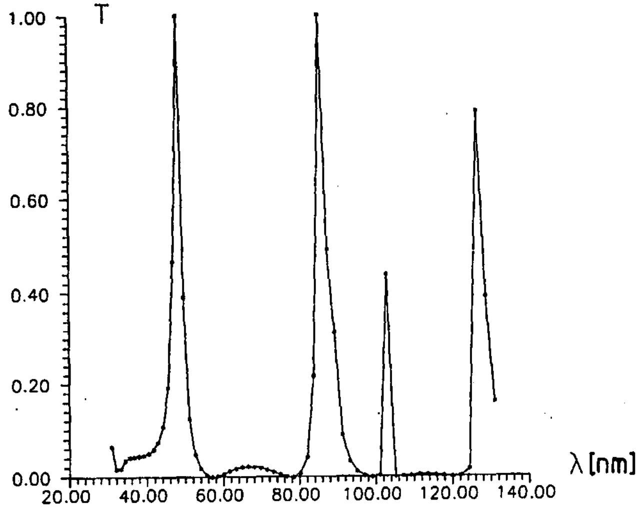

Fig. 1. Characteristics of the energy filter: dependence of transmission coefficient on the electron energy (after Ref. [14]) for a double-barrier structure. The width of the wave guide is 100 nm, the widths of the barriers are 5 nm, the heights of the barriers are 0.3 eV.

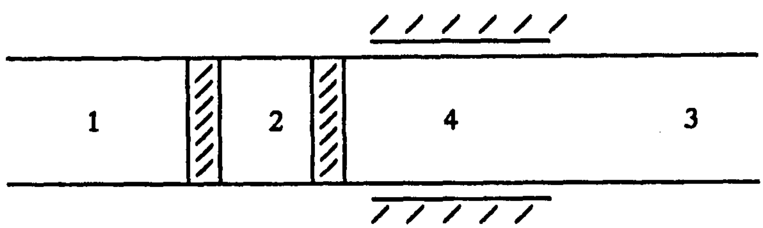

Fig. 2. Configuration of the system: 1, 3 — wave guides, 2 — filter (a double-barrier structure or a quantum resonator), 4 — gate.

electric field leads to "the opening of the wave guide" because the shift of the eigenvalues is negative (4). In other constructions an increase in the gate voltage leads to the "closing of the wave guide" because, formally, the width of the wave guide decreases.

Finally, let us note that the Schrödinger equation in an accelerated (in "x" direction) system (see, for example, Ref. [13]) has a form (1), where $F = ma$, a, is the acceleration of the system. Hence, hypothetically, the device may be used as an "accelerator detector".

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