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ELECTRON–OPTICAL PHONON COUPLING IN DOUBLE-BARRIER QUANTUM WELL STRUCTURE

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The electron–LO phonon coupling constant for double-barrier quantum well structure was calculated using the Frohlich model of the electron–phonon interaction and assuming coupling of the confined electron with bulk LO phonon mode. Magnitude of the Huang–Rhys factor g and possibility of detecting of phonon replicas in the resonant tunneling current are discussed for GaAs–$Al_xGa_{1-x}As$ and CdTe based structures.

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It is well known that in heterostructures and quantum wells there is no translational symmetry in the direction perpendicular to the interface. Thus, the momentum-conserving selection rule breaks down, which indicates a significant difference between the electron–phonon coupling in these systems and that in the corresponding bulk materials. This is why the electron–phonon interaction in 2D systems was extensively studied for the last nearly 20 years. Here, we shall concentrate on the role played by the electron–phonon coupling in resonant tunneling in the double-barrier quantum well (DBQW) structures. The theoretical description of the phonon assisted resonant tunneling, given by Jonson [1], is very similar to that of absorption of light in F-centers in the presence of phonons [2]. Generally, the problem can be solved exactly [3] only for a localized electron for which the impact of the interaction with phonons on its state is negligible. Unfortunately, the above formalism applied to the interacting electron–bulk LO phonon system in heterostructures leads to divergent self-energies and cannot be directly adopted. This divergence can be avoided by taking into account either the screening [4] or the interaction with confined LO phonon modes [5]. However, Mori and Ando [6] have shown that for single and double heterostructures the sum of contributions of all kinds of phonon modes (interface and confined) is exactly equal to that of the bulk phonons if the coupling constants are independent of modes. Here we propose another way of elusion of such divergence, namely an approximate diagonalization of the electron–phonon Hamiltonian by means of the second order perturbation theory [7].

Resonant tunneling occurs when the electron energy in the emitter matches the energy of the quasi-localized state in the well (called the resonant state). It is well known that during the resonant tunneling the charge is stored in the well. Carriers are in the two-dimensional subband, where their motion parallel to the interfaces is free whereas their motion in the direction normal to the interface is confined. The electron state $\phi_{n,\boldsymbol{k}}$ is characterized by the subband number n and 2-dimensional wave vector $\boldsymbol{k}$. Here we shall limit our considerations to the DBQW structures for which the subband separations are much larger than the phonon energy. We assume that the electron in the resonant state couples with the bulk LO phonon mode and the electron–phonon interaction is described by the Frohlich model [7]. For simplicity, we consider a system at $T = 0$, in which initially there are no phonons present. As a result of interaction in the lowest order of the perturbation theory, transitions will be induced between the state $\phi_{n,\boldsymbol{k}}|0\rangle$ (where $|0\rangle$ represents the phonon vacuum state) and the states of the system $\phi_{n,\boldsymbol{k}-\boldsymbol{q}_\parallel} a_{\boldsymbol{q}}^{+}|0\rangle$ in which one has a single phonon with the momentum $\hbar\boldsymbol{q}$ present. The electron momentum is equal to $\hbar\boldsymbol{k} - \hbar\boldsymbol{q}_\parallel$. Here $\boldsymbol{q}$ is a phonon momentum and $\boldsymbol{q}_\parallel$ is its component parallel to the interfaces. In the following, all the considered transitions are virtual.

The energy change Δ_n due to the lattice deformation caused by tunneling electron is

$$\Delta_n = -\sum_{q} \frac{|V_q|^2 |f_{nn}(q_\perp)|^2}{\hbar\omega_{\mathrm{LO}} + E(q_\parallel)} \tag{1}$$

Here $E(q_\parallel) = \hbar^2 q_\parallel^2 / 2m^*$ and

$$V_q = -\mathrm{i}\hbar\omega_{\mathrm{LO}} \frac{(\hbar/2m^*\omega_{\mathrm{LO}})^{1/4}(4\pi\alpha/V)^{1/2}}{q}.$$

The form factor $f_{nn}(q_\perp)$ is defined as

$$f_{nn}(q_\perp) = \int_b^{b+w} \mathrm{d}z |\xi_n(z)|^2 \exp(\mathrm{i}q_\perp z), \tag{2}$$

where $\xi_n(z)$ is the envelope wave function for the n-th subband in the well, b and w are the barrier and the well widths, respectively. We use the wave function calculated within the WKB approximation, which has the advantage of having an analytical form. For symmetric DBQW structure the ground subband envelope function is

$$|\xi_0(z)|^2 = N_0(E)\left\{\cos^2[\alpha(z)+\beta] - 2\mathrm{e}^{-2\kappa b}\cos(2\beta)\cos[\alpha(z)-\beta]\cos[\alpha(z)+\beta] + \mathrm{e}^{-4\kappa b}\cos^2[\alpha(z)-\beta]\right\}, \tag{3}$$

where

$$\begin{aligned} &\alpha(z) = k(z-b-w), && \beta = \arctan\sqrt{\kappa/k} \\ &k = \sqrt{2m^*E/\hbar^2}, && \kappa = \sqrt{2m^*(V-E)/\hbar^2}. \end{aligned} \tag{3a}$$

Here V is the height of the barrier. The factor $N_0(E)$ is the normalization factor depending on the transmission coefficient for the DBQW structure. For the resonant state it reads

$$N_0(E_{\mathrm{R}}) = (w/2 + 1/\kappa)^{-1}. \tag{3b}$$

Having defined all expressions involved in Eq. (1) one can calculate the energy change Δ due to the interaction of the confined electron with the bulk LO phonon. This can be done numerically only. Usually the coupling constant is defined as $\alpha = \Delta/\hbar\omega_{LO}$ [7]. In our case the coupling constants $\alpha_n = \Delta_n/\hbar\omega_{LO}$ are different for different subbands and are dependent on the DBQW structure parameters.

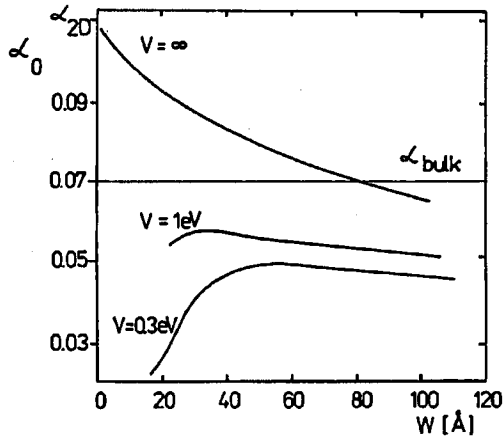

Fig. 1. The coupling constant α_0 for the ground subband in GaAs–$Al_xGa_{1-x}As$ DBQW structure as a function of the well width.

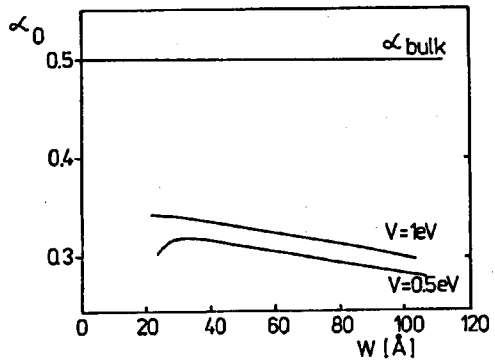

Fig. 2. The coupling constant α_0 for the ground subband in CdTe based DBQW structure as a function of the well width.

The coupling constant α_0 for the ground subband in GaAs–$Al_xGa_{1-x}As$ DBQW structure is plotted in Fig. 1 for several values of barrier height V as a function of the well width w. For comparison coupling constants for the bulk material and for the infinitely high barrier are also shown. We note that in Fig. 1 the two cases of infinite and finite barrier differ by the character of the dependence of α_0 on the width w. This is due to the fact that in the former case the carrier is totally localized in the well, whereas in the latter the electron wave function

spreads out over the barriers. The coupling constant in DBQW structures based on CdTe is shown in Fig. 2. The electron–LO phonon coupling for both types of DBQW structures is smaller than that in corresponding bulk materials, even for unscreened interaction. This result is consistent with the similar calculation done by Das Sarma [8] for electron–LO phonon coupling in GaAs inversion layers.

The strengths of phonon replicas in resonant tunneling current are characterized by the Huang–Rhys factor g [1]. This factor depends on the difference between the electron–phonon interaction matrix elements for the emitter and for the well [3]. If the electron–phonon coupling in the emitter material is neglected, which seems to be a justified approximation for weakly ionic material as GaAs ($\alpha_{\text{bulk}} = 0.07$), the factor g would be simply equal to α_n. For GaAs–$Al_xGa_{1-x}As$ DBQW structures g is much smaller than unity and consequently it is hard to expect that the phonon replicas will be observable in the tunneling current. On the other hand, taking into account much stronger electron–phonon coupling in bulk CdTe ($\alpha = 0.5$) this material seems to be much more suitable for phonon replicas observation. More detailed discussion of a possibility of detecting phonon replicas in tunneling current for CdTe based DBQW structures will be subject of another paper.

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