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ON AHARONOV–BOHM EFFECT IN MULTICONNECTED SUPERCONDUCTOR

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The system of co-axial cylinders: superconductor–ferromagnet– superconductor is considered. The temperature of the outer superconductor transition to the normal state, derived from the Ginzburg–Landau energy functional, depends on the state of the inner one. The quantity of heat is evaluated, which is liberated at the inner cylinder pushing out of core.

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1. Introduction

The phenomenon of a charged particle interaction with the field of magnetic vector potential $\boldsymbol{A}(\boldsymbol{r})$, provided that the magnetic field

$$\boldsymbol{H}(\boldsymbol{r}) = \mathrm{rot}\boldsymbol{A}(\boldsymbol{r}) = 0 \tag{1}$$

is zero in the whole particle localization region $\{P\}$, is known as the Aharonov–Bohm effect. The AB-effect is the consequence of the main axiom of quantum mechanics, asserting the single-valuedness of the particle wave function $\Psi(\boldsymbol{r})$ when $\boldsymbol{r}$ rounds a closed curve in $\{P\}$ [1]. Numerous experiments were realized during the period 1960–1980 to reveal the AB-effect because of its big importance in physics foundations understanding. In most of these experiments the electron microscopy technique was used. Meanwhile, it has been already noted in [1] that superconductors are the objects in which the AB-effect leads to non-trivial consequences. In fact, the effect was observed, when one placed a long narrow solenoid into the coil of SQUID and varied the current I_s through the solenoid. As the Josephson current I_c of SQUID depends on the phase shift $\Delta\theta$ between electron waves, passed through the upper and the lower halves of coil and $\Delta\theta$ is determined by the vector potential of solenoid, the increase in I_s produces the periodical change of $I_c(I_s)$, although $H = 0$ for any I_s [2]. One can abandon, in principle, current feed wires of the coil and observe I_c oscillations, measuring its magnetic field H_{ind} alterations. Namely, it was shown in [3] that the sum of the coil flux Φ_{ind} and the solenoid flux

Φ_{ext} must be equal to the integer number (n) of the flux quantum $\Phi_0 = 20$ Oe μm^2 in certain conditions. According to [3],

$$\Phi_{\text{ind}} = \frac{\mu}{1+\mu}\left(n\Phi_0 - \Phi_{\text{ext}}\right), \tag{2}$$

where $\mu = Rd/2\lambda^2$, R is the coil radius, d is its thickness, λ is the field penetration depth into the superconductor. It is shown beneath that the small modification of this system leads to some unexpected phenomena, which may be of interest for the solid state physics.

2. Calculation section

Let the thin film of a ferromagnetic metal of thickness l be deposited onto the inner surface of the hollow cylinder with the inner radius R and the thickness d. The metal magnetic anisotropy will be considered as sufficiently high, so that its magnetization $\boldsymbol{M}$ is always directed along cylinder's axis Oz. The segment of cylinder $|z| < L_2/2$ is made of the 1-type superconducting material (SC-2), while other segments are dielectric (their role is to form the substrate for film). Another superconductor (SC-1) having the form of taper with very small apex angle γ may slide along Oz in the hole of the outer one (Fig. 1). The presence of

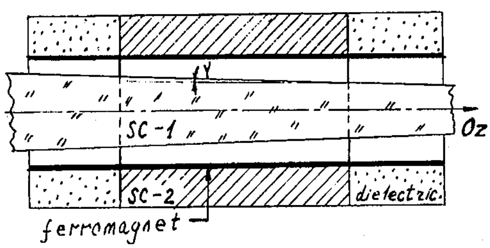

Fig. 1. The system geometry. The dielectric layer between the magnet and the SC-2 is not shown. Proportions are not observed.

isolators is implied between the film and both superconductors and these isolators are rather thin (≈ 30 Å) to disregard it afterwards. The lengths of magnet L and of superconductors L_i ($i = 1$ corresponds to the inner superconductor SC-1), comply with the condition

$$l \ll d \ll R \ll L_2 \ll L_1 \ll L, \tag{3}$$

so that the influence of magnet's ends (the regions of magnetic field concentration) may be ignored [4] and the AB-effect condition (1) is fulfilled for Cooper's pairs in SC-2. It is evident, further, that the greater the quantity L_2/R is, the smaller is the relative contribution of the SC-2 ends areas to the overall energy. Therefore if they approximate the taper as the cylinder with the average radius X along SC-2, then the system geometry may be considered as a two-dimensional one with the polar coordinates ρ, ϕ, where $\rho = 0$ corresponds to the common axis Oz of all cylinders. Let the critical field of SC-1 comply with the condition $H_1^c(T) \gg H_2^c(T)$.

If SC-1 is in the normal state (its temperature may be maintained higher than T of environment) and SC-2 is in the superconducting one, the latter holds some current, creating a flux Φ_{ind} inside it according to (2). When the SC-1 is cooled to the environment temperature and it transforms into the superconducting state, this flux will be ejected outside the SC-1 core. Doing so, one can evaluate the SC-1 energy in the context of London theory ([5], §15.5) as

$$W_1 = \left[n_{\text{s}}\frac{m}{2}\overline{v^2} + \frac{1}{2}\frac{1}{8\pi}\boldsymbol{H}^2(\rho = X)\right] 2\pi X L_2 \lambda_1, \tag{4}$$

where v is the velocity of superconducting electrons, rotating on the cylinder periphery, and n_{s} is its concentration. As in London's theory

$$\lambda = \left(\frac{mc^2}{4\pi n_{\text{s}} e^2}\right)^{1/2} \tag{5}$$

it may be calculated easily that

$$W_1 = \frac{3}{2}\frac{\boldsymbol{H}^2(X)}{8\pi} 2\pi X L_2 \lambda_1. \tag{6a}$$

Analogously

$$W_2 = \left[\frac{1}{2} + \left(\frac{\lambda_2}{d}\right)^2\right] \frac{\boldsymbol{H}^2(X)}{8\pi} 2\pi R L_2 d \tag{6b}$$

for SC-2 at $d < \lambda_2$. Taking $d/\lambda_2 = 0.5$ (see §3), one can obtain that when $d \gg \lambda_1$, then $W_1 \ll W_2$. One can say that in such a situation the inside area $0 < \rho < X$ does not contribute to the system energy. For the evaluation of this energy it is necessary to pass from London theory to the theory of Ginzburg and Landau (GL) ([5], §17.1). According to [3] they introduce the wave function of the superconducting electrons in SC-2:

$$\Psi(r) = \left(\frac{n_{\text{s}}}{2}\right)^{1/2} \exp(in\phi) = |\Psi| \exp(in\phi) \tag{7}$$

and besides, one can consider $\mathrm{d}n_{\text{s}}/\mathrm{d}\rho = 0$ if $d \ll \xi_0$, where ξ_0 is the Cooper pair diameter. The free energy of the system may be written in the form

$$\frac{\mathcal{F} - \mathcal{F}_n}{2\pi L_2} = \int_R^{R+d} \left[-a|\Psi|^2 + \frac{b}{2}|\Psi|^4 + \frac{|\Psi|^2}{4m}\left(\frac{n\hbar}{\rho} - \frac{2e}{c}A\right)^2\right] \rho \mathrm{d}\rho$$
$$+ \int_X^{R+d} \frac{[H - H_0(\rho)]^2}{8\pi} \rho \mathrm{d}\rho, \tag{8}$$

where $a = \alpha(T_0 - T)/T_0$, α, b are the constants of a material, T_0 is the temperature of a one-connected specimen transition to the normal state at absence of H_{ext}, $\mathcal{F}_n$ is the energy of normal electrons, $\boldsymbol{H}_0$ is the magnetic induction inside the magnet: $\boldsymbol{H}_0 = 4\pi M \boldsymbol{e}_z$ ($H_0 = 0$ outside it). It is a typical expression of the GL-theory. Only the last summand deserves some comments. According to Ginzburg and Landau, both $\mathcal{F}$ and $\mathcal{F}_n$ comprise the energy of magnetic field in this (superconducting or normal) state. This energy, by definition, is equal to work which must be expended for the field creation. Thus the formula for $(\mathcal{F} - \mathcal{F}_n)$ includes the work A on the current field H_{ind} creation at fixed (i.e. created already) magnet's field: $A = \int (H_{\text{ind}}^2/8\pi)\mathrm{d}V$. The overall field in the point ρ, ϕ is $H(\rho) = H_{\text{ind}} + H_0(\rho)$ and the form of Eq. (8) becomes evident. The boundary conditions for this variational problem on arguments $A(\rho)$ and $|\Psi|$ are as follows:

a) $A(\rho)$ is the continuous function at $\rho \geq 0$.

b) $A(\rho < X) = 0$. (Indeed,

$$A(X) = \frac{1}{2\pi X}\oint \boldsymbol{A}\mathrm{d}\boldsymbol{l} = \frac{1}{2\pi X}\int \mathrm{rot}\boldsymbol{A}\mathrm{d}\boldsymbol{S} \approx H(X)\lambda_1 = H(R)\lambda_1 \ll A(R)$$

— see below the formula (13b).)

c) $H = 0$ at $\rho > R + d$.

d) $\lim\limits_{\varepsilon\to 0}[H(R-\varepsilon) - H(R+\varepsilon)] = H_0$.

Varying Eq. (8) over $A(\rho)$ and having regards to (a)–(d), one can obtain ($\psi = \Psi(b/a)^{1/2} = N^{1/2} \leq 1$ is the unitless wave function of the SC electrons, Ξ is the Heaviside function)

$$\frac{\mathrm{d}}{\mathrm{d}\rho}\frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho}(A\rho) = \Xi(\rho - R)\left(\frac{\psi}{\lambda}\right)^2\left(A - \frac{n\Phi_0}{2\pi\rho}\right). \tag{9}$$

The solution of this equation is

$$\begin{aligned} A &= P(\rho^2 - X^2)/\rho, && X < \rho < R - l, && (10a)\\ A &= P_1\rho + P_2/\rho, && R - l < \rho < R, && (10b)\\ A &= n\Phi_0/2\pi\rho + G(\rho), && R < \rho < R + d, && (10c)\end{aligned}$$

where P_i are still unknown constants, $G(\rho) = P_3 I_1(\psi\rho/\lambda) + P_4 K_1(\psi\rho/\lambda)$ is the linear combination of the modified Bessel functions [6]. Simple calculations show that its Taylor power series in $x = (R + d - \rho)$ near $x = 0$ may be approximated as

$$G(x) = G(0)\left[1 + \frac{\psi^2}{2\lambda^2}x^2 + \frac{\psi^2}{6\lambda^2}\frac{x^3}{R} + \left(\frac{\psi^4}{\lambda^4} - \frac{\psi^2}{\lambda^2 R^2}\right)\frac{x^4}{24} + \ldots\right]. \tag{11}$$

It follows from (11) that at $d \ll R$ and $\mu \sim 1$ (but not at $\mu \gg 1$!) one can confine oneself to the first two items in it. Then the conditions (a) and (c) give

$$G(\rho) = \frac{(z-n)\Phi_0}{2\pi\rho}\left[1 + \frac{\psi^2}{2\lambda^2}(R + d - \rho)^2\right]\Delta + \mathrm{O}\left(\frac{d^2}{R^2}\right), \tag{12}$$

$$\frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho}(A\rho) - H_0 = -\frac{(z-n)\Phi_0}{2\pi R}\frac{Nd}{\lambda^2}\Delta, \qquad R - l < \rho < R, \tag{13a}$$

$$A = \frac{n\Phi_0}{2\pi\rho} + \frac{(z-n)\Phi_0}{2\pi\rho}\left[1 + \frac{N}{2\lambda^2}(R + d - \rho)^2\right]\Delta, \qquad R < \rho < R + d, \tag{13b}$$

where $z = H_0\,(2\pi Rd)/\Phi_0 = \Phi_{\text{ext}}/\Phi_0$ and

$$\Delta = \left[1 + \frac{d(R^2 - X^2)}{2\lambda^2 R}N\right]^{-1}. \tag{14}$$

Integrating (8) by parts, inserting (9) into (8), and taking into account (10a) and (13), one transforms the formulae forthe free energy F in reduced variables, minimized on $A(\rho)$, to the following form ($F = \mathcal{F}4\pi/(H^c)^2V$, where $V = 2\pi RdL_2$ is the SC-2 volume, $H^c = a(4\pi/b)^{1/2}$ is the critical field of a massive specimen):

$$F(T, X, N) = -N + \frac{1}{2}N^2 + NQ\Delta. \tag{15}$$

Here the parameter $Q = (\xi|z-n|/R)^2$, $\xi = \hbar/(4ma)^{1/2}$ is the coherence length ($\xi(T) = \lambda(T)/\chi$, χ is the known GL-parameter of a material, not depending on temperature in the first approximation). It is evident that in the limits of this approximation the product μQ is a constant also: $\mathrm{d}(\mu Q)/\mathrm{d}T = 0$. One can consider without losing physical generality that $\mu Q = 1$. Let $X = R - l$ at the beginning and the SC-2 is heated from very low temperature. By virtue of (14) and (3), at $\mu \sim 1$:

$$F(T, X = R - l) = -(1-Q)N + \left(\frac{1}{2} - \frac{2l}{R}\right)N^2 + \mathrm{O}\left(\frac{l^2}{R^2}\right). \tag{16}$$

From this formula it results that on heating, the outer layer (SC-2) turns into the normal state when $Q = 1$ (and consequently $\mu = 1$) i.e. at the Little–Parks temperature ([5], §15.5)

$$T_1 = T_0\left[1 - \frac{\hbar^2}{4m\alpha R^2}(z-n)^2\right]. \tag{17}$$

But if $X = 0$, then at $T = T_1$

$$F(T_1, X = 0) = N^2\left(\frac{1}{2} - \frac{1}{1+N}\right). \tag{18}$$

These function decreases at $0 \le N \le 0.62$ and increases at $0.62 \le N \le 1$ i.e. the free energy minimum corresponds to the superconducting state. Thus, further heating is necessary to transfer SC-2 to the normal state. With the increase in temperature, the potential well depth of the function $F(X = 0, N)$ decreases and vanishes at some temperature T_2. This temperature is defined by the solution of system ([6]):

$$\partial F/\partial N|_{N_0} = 0, \qquad \partial^2 F/\partial N^2|_{N_0} = 0, \tag{19}$$

where N_0 is the point of minimum of the function $F(N)$ at T and $X = \mathrm{const}$. The implicit solution of (19) for $X = 0$ is

$$\mu(T_2) = 1.89\,(\mu Q)^{1/3} - 1. \tag{20}$$

If $\mu Q = 1$, then $\mu(T_2) = 0.89 < 1$ and so, there exist the temperature interval $[T_1,\ T_2]$ in which one can exert control over the layer SC-2 superconductivity, changing the state of the area $\rho < R - l$. Such change may be realized by the SC-1 transformation along Oz, so that $X = 0$ at the beginning ($\mathcal{F}_a$) and $X = R - l$ at the end of process ($\mathcal{F}_b$). (The above mentioned variant with the SC-1 heating and cooling seems to be more difficult to perform experimentally.) The isothermal SC-1 movement is accompanied by the SC-2 heat absorption Q_f from environment. This process takes place not only at $T_1 \le T \le T_2$, but also at $T \le T_1$, where, however, such absorption does not lead to the SC-2 phase transition. The analysis of formula (15) shows that the curve $F(N)$ has only one minimum at $T \le T_1$, so the heat process is reversible inside this temperature interval, i.e. the SC-1 movement in the opposite direction (out of the SC-2 hole to infinity) leads to the heat liberation. If the taper velocity is low, the quantity $N_0(t)$ is determined by minimum of the function $F(N)$ with the time-dependent parameter $X(t)$ at any time moment t ([5], §19.6, formulae 19.47–48). Therefore, the quantity Q_f may be calculated using the usual formula of thermodynamics $Q_f = T\Delta S$, where S is the system entropy. By the entropy definition

$$Q_f(T) = -T\left(\frac{\partial\overline{\overline{\mathcal{F}}}_b}{\partial T} - \frac{\partial\overline{\overline{\mathcal{F}}}_a}{\partial T}\right) = T\left[\frac{\partial}{\partial T}(\overline{\overline{\mathcal{F}}}_a - \mathcal{F}_n) - \frac{\partial}{\partial T}(\overline{\overline{\mathcal{F}}}_b - \mathcal{F}_n)\right], \tag{21}$$

where the symbol "=" signifies the free energy functional, minimized over N. For the calculation of (21), one can use the formula of the micro-theory of superconductivity ([5], §19.6):

$$\lambda = \lambda_0\left[\frac{T_0}{2(T_0 - T)}\right]^{1/2}, \tag{22}$$

which is correct at $T - T_0 \ll T_0$. Then

$$\frac{\partial(\overline{\overline{\mathcal{F}}} - \mathcal{F}_n)}{\partial T} = \left.\frac{\partial(\mathcal{F} - \mathcal{F}_n)}{\partial T}\right|_{N_0} = -N_0\frac{\partial}{\partial T}\left[\frac{(H^c)^2 V}{8\pi}\right]. \tag{23}$$

At $T = T_1$ they may take simple $N_0(X = R - l) = 0$, $N_0(X = 0) = 0.62$, whence it follows that

$$Q_f = -0.62T_1\frac{\partial}{\partial T}\left[\frac{(H^c)^2 V}{8\pi}\right] > 0. \tag{24}$$

As it is seen, the quantity $Q_f(T_1)$ has the same order as this heat which is taken up by the volume V inside the massive superconductor, when the field H^c turns on ([5], §15.2). The natural question arises how the quantity Q_f depends on temperature. One can calculate without difficulty that

$$\left(\frac{\mathrm{d}Q_f}{\mathrm{d}T}\right)_{T_1} < 0. \tag{25}$$

On the other hand, $Q_f \to 0$ at $T \to 0$ by force of the Nernst theorem. It means that there is an optimal temperature T_x for the thermomechanical effect observation, where Q_f passes through maximum. However, it cannot be estimated in the context of this work approximations. First, the assumption $\mu \sim 1$, which was considered above as true, loses its validity out of the sharp interval near T_1, which complicates the mathematics highly. Second, the GL-theory itself is applicable only near T_0. Therefore, the micro-theory of superconductivity must be used for the T_x and $(Q_f)_{\max}$ determination which demands a specific treatment.

3. Final remarks

To be specific, one can dwell on the following values of the parameters:

a) the material for SC-1 is Pb with $\lambda_0 = 380$ Å, $H^c(T = 0) = 800$ Oe, $T_0 = 7$ K.

b) the material for SC-2 is Cd with $\lambda_0 = 1350$ Å, $H^c(0) = 30$ Oe, $T_0 = 0.5$ K and $\chi = 0.1$ [7].

Under this value of χ, the equalities $Q = 1$ and $\mu = 1$ may occur simultaneously if $z = 0.49$ (consequently $n = 0$) and $d/R = 0.08$. The condition $d \ll \xi_0$ ($\xi_0 = \lambda_0/\chi$) is fulfilled satisfactorily in this case. The magnitude R itself may be chosen so that the temperatures T_1, T_2 lie not far from the formal boundaries of the GL-theory feasibility $T_0 - T \ll \chi^2 T_0$. By application of (22) it is easy to obtain that the value $R_0 = 4.85$ μm gives:

$$T_1 = 0.990T_0, \qquad T_2 = 0.991T_0, \qquad \lambda(T_1) = 9700 \text{ Å}.$$

The decrease in z from $1/2$ entails the T_1 shift in the direction to T_0 which spoils the effect. As for the condition $l \ll d$, it will be always true, because the monoatomic layer of Ni placed onto the surface of cylinder of the radius R_0 creates flux $z > 1/2$ already. To fit the value z to $1/2$ in that case, one can consider ferromagnet in the form of tube with the longitudinal section.

By this means the AB-effect leads to the existence of peculiar thermomechanical phenomena in the system given above. The physical sense of these phenomena is rather simple: the exclusion of flux out of the tube core to its pre-surface region leads to the increase in the electrical current density

$$|j| = \frac{e^2}{mc} n_s A(R) \tag{26}$$

and, consequently, in the SC-2 superconducting electrons kinetic energy. The transition to the normal state appeared to be favourable energetically. The Cooper pairs breaking at $T > 0$ stimulates the system entropy increase, which is supplied externally. The existence of such effect extends, in principle, the number of methods of the superconductor cooling apart from its magnetization in the field H^c.

References

[1] Y. Aharonov, D. Bohm, *Phys. Rev.* **123**, 1077 (1961).

[2] R. Feynman, R.B. Leighton, M. Sands, *The Feynman Lectures on Physics*, Vol. 3, Addison-Wesley, London 1963, Ch. 19.

[3] G. Kunstatter, M. Revzen, L. Trainor, *Ann. Phys. (USA)* **145**, 329 (1983).

[4] H. Lipkin, *Phys. Rev. D* **23**, 1467 (1981).

[5] A.A. Abrikosov, *Fundamentals of Theory of Metals*, Nauka, Moskva 1987 (in Russian).

[6] R.M. Arutunian, G.F. Zharkov, *Zh. Eksp. Teor. Fiz.* **78**, 1530 (1980) (in Russian).

[7] M.S. Haikin, *Zh. Eksp. Teor. Fiz.* **34**, 1389 (1958).