Vol. 85 (1994) ACTA PHYSICA POLONICA A No. 4

CLOUD OF VIRTUAL PHOTONS SURROUNDING A NONRELATIVISTIC ELECTRON

G. COMPAGNO[a], R. PASSANTE[b], F. PERSICO[c] AND G.M. SALAMONE[a]

[a] Istituto di Fisica dell'Università, via Archirafi 36, 90123, Palermo, Italy
[b]Istituto per le Applicazioni Interdisciplinari della Fisica
Consiglio Nazionale delle Ricerche
via Archirafi 36, 90123, Palermo, Italy
[c]Istituto di Fisica dell'Università
and Istituto per le Applicazioni Interdisciplinari della Fisica
via Archirafi 36, 90123, Palermo, Italy

Dedicated to Prof. Iwo Białynicki-Birula on the occasion of his 60th birthday

A review is presented of the virtual cloud of photons surrounding a nonrelativistic electron. Two cases are considered. In the first, the electron is taken as bound to a proton to form a $1s$ hydrogen atom. In the second case the electron is considered as free. It is shown that the virtual cloud, as measured by the transverse electromagnetic energy density, in the near region of the ground-state hydrogen is not influenced by the dynamic properties of the atom and in practice it coincides with that of the free electron. The influence of the atomic eigenvalue spectrum is evident in the far region of the atom where retardation effects play an important role. These results are discussed from a physical point of view.

PACS numbers: 12.20.–m

1. Virtual clouds in QED

The coupling between a quantized field and its sources has observable effects even when the source–field system is in its ground state. Consequently, these effects are present even at $T = 0$ K and they are called "vacuum" effects because they may be thought as arising from the interaction between the source and the vacuum state of the field. A particular class of vacuum effects is that of "cloud effects", since in general the ground state of the coupled (source+field) system describes a complex structure, in which a cloud of virtual quanta of the field surrounds the source. Usually such a structure is called a dressed source, in contrast with the bare one which describes the source in the absence of the interaction with the field [1]. Investigation of this structure is of current interest in quantum chromodynamics

[2], in quantum mesodynamics [3] and in quantum electrodynamics (QED), both in its nonrelativistic [4, 5] and relativistic [6] versions.

Limiting our discussion to nonrelativistic QED, the cloud of virtual photons around a source made of positive and negative electric charges arises because, due to the charge–photon interaction, the bare ground state of the source–field system is coupled to excited source states yielding transitions accompanied by the emission of one or more photons. Such a transition does not conserve the bare source+field energy, but it can take place if the lifetime of the excited state is short-lived enough, which is the reason for calling "virtual" the photons emitted in the transition. This leads to a picture of the source continuously emitting and reabsorbing the virtual photons. These are the virtual photons which contribute to the electromagnetic cloud dressing the source [7].

Some important characteristics of the virtual cloud can be derived immediately in a very general manner by considering the above pictorial model and the restrictions due to Heisenberg uncertainty principle. Indeed, a virtual transition which involves a change ΔE in the bare energy of the system is allowed to take place only if

$$\Delta E \Delta t \approx \hbar, \tag{1.1}$$

where Δt is its lifetime.

A virtual photon emitted in such a process can travel a distance Δr, which is roughly equal to c times the lifetime Δt:

$$\Delta r \approx c\Delta t \approx c/\Delta E. \tag{1.2}$$

If one is interested in local properties of the field at distances much larger than the electron Compton wavelength ($r \gg \lambda_C = \hbar/mc$), then the above relation between Δr and ΔE allows to neglect contributions coming from relativistic photons ($\Delta E \gg mc^2$) [8]. The nonrelativistic approach is justified, for example, when the source is a hydrogen atom and the field is studied at distances larger than the Bohr radius, which is much larger than Compton wavelength. Indeed, relativistic effects have been studied in detail around the hydrogen atom [6] and it has been shown that they can be neglected at large distances, as expected.

If the source is a free charged particle, it is possible to correlate its position to local observables of the field at a given distance. In this case, a nonrelativistic approach is justified only when distances larger than the Compton wavelength are taken into account [9].

On the basis of the Heisenberg uncertainty principle it is possible also to connect some properties of the spatial structure of clouds to the internal dynamics of the source. In particular, it is possible to relate the spatial behaviour of the cloud to the structure of the eigenvalue spectrum of the bare source. Let us consider, e.g., a hydrogen atom and one-photon virtual transitions, in which a photon with energy $\hbar\omega$ is emitted and the source jumps from the ground level to the n-th level. The change in bare energy is

$$\Delta E = \hbar\omega + E_n - E_1, \tag{1.3}$$

and the distance that the photon can travel is roughly

$$\Delta r \approx \frac{\hbar c}{\hbar\omega + E_n - E_1} \leq \frac{\hbar c}{E_n - E_1} = \lambda_{n1}, \tag{1.4a}$$

so that

$$\Delta r \leq \lambda_{2p,1s} \approx 1200\,\text{Å}. \tag{1.4b}$$

These considerations would roughly indicate a size of about 1200 Å for the virtual photon cloud [7]. If one keeps into account two-photon emission [10], the size of the cloud appears to be unlimited. In fact, for transitions which do not involve change of the bare atomic state, one has

$$\Delta E = \hbar\omega_1 + \hbar\omega_2, \tag{1.5a}$$

so that the distance which can be travelled, Δr, is [10]:

$$\Delta r \approx \frac{c}{\omega + \omega'}. \tag{1.5b}$$

From Eq. (1.5b) it is possible to argue that in a dressed state the local observables of the field are modified with respect to their values in the absence of sources even at very large distances from the source.

The properties of the photon cloud can be studied in detail by taking the quantum average of appropriate field operators on the dressed state. The choice of the operator is in principle arbitrary and it can be justified mainly by the particular aspect of the problem which one wishes to investigate. Different operators exist which can be used to describe the cloud, each of them being useful under some aspect. Thus in order to emphasize the shell structure of the cloud and its relation with the eigenvalue spectrum of the bare source, an appropriate operator is the so-called "coarse-grained energy operator" $W(r)$, defined as [4]:

$$W(r) = \frac{1}{4\pi}\left[\boldsymbol{E}_{\perp}^{(-)}(r)\cdot\boldsymbol{E}_{\perp}^{(+)}(r) + \boldsymbol{B}^{(-)}(r)\cdot\boldsymbol{B}^{(+)}(r)\right], \tag{1.6}$$

where $(-)$ and $(+)$ indicate respectively negative and positive frequency components of the transverse fields. The use of this quantity leads to a description of the cloud structure in terms of shells, whose size is directly related to the bare atomic eigenvalue spectrum, as indicated by Eq. (1.4a) [4].

A local observable, however, which can be easily connected to observable effects like interatomic forces or spontaneous decay [5, 11] is the electromagnetic energy density distribution. With respect to the coarse-grained energy density, it has the advantage of a direct physical meaning.

The electromagnetic energy density distribution is defined as the quantum average of the field energy density operator

$$\mathcal{H}(r) = \frac{E^2(r) + B^2(r)}{8\pi}, \tag{1.7}$$

which is quadratic in the field operators. In Eq. (1.7), $E^2(r)$ is the square of the total electric field, which can be partitioned for convenience into its transverse and longitudinal components

$$\boldsymbol{E}(r) = \boldsymbol{E}_{\perp}(r) + \boldsymbol{E}_{\parallel}(r). \tag{1.8}$$

The transverse fields are expanded in terms of Bose operators $a_{\boldsymbol{k},\sigma}$ and $a^{\dagger}_{\boldsymbol{k},\sigma}$, which obey well known commutation rules, as

$$\boldsymbol{B}(r) = \sum_{\boldsymbol{k},\sigma} \mathrm{i}\,[\kappa \times e^{\sigma}(k)]\sqrt{\frac{2\pi\hbar ck}{V}}\left(a_{\boldsymbol{k},\sigma}\mathrm{e}^{\mathrm{i}\boldsymbol{k}\boldsymbol{r}} - a^{\dagger}_{\boldsymbol{k},\sigma}\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\boldsymbol{r}}\right), \tag{1.9a}$$

$$\boldsymbol{E}_{\perp}(r) = \sum_{\boldsymbol{k},\sigma} \mathrm{i}\boldsymbol{e}^{\sigma}(\boldsymbol{k})\sqrt{\frac{2\pi\hbar ck}{V}}\left(a_{\boldsymbol{k},\sigma}\mathrm{e}^{\mathrm{i}\boldsymbol{k}r} - a^{\dagger}_{\boldsymbol{k},\sigma}\mathrm{e}^{-\mathrm{i}\boldsymbol{k}r}\right), \tag{1.9b}$$

The average value of the electric and magnetic fields, Eqs. (1.8), in the vacuum state of the field, is zero,

$$\left\langle \{0_{\boldsymbol{k},\sigma}\}|\boldsymbol{B}(r)|\{0_{\boldsymbol{k},\sigma}\}\right\rangle = \left\langle \{0_{\boldsymbol{k},\sigma}\}|\boldsymbol{E}_{\perp}(r)|\{0_{\boldsymbol{k},\sigma}\}\right\rangle = 0, \tag{1.10}$$

but the square value of these fields has a non-zero (actually infinite) average value, uniform in free space

$$\left\langle \{0_{\boldsymbol{k},\sigma}\}|B^2(r)|\{0_{\boldsymbol{k},\sigma}\}\right\rangle = \left\langle \{0_{\boldsymbol{k},\sigma}\}|E^2_{\perp}(r)|\{0_{\boldsymbol{k},\sigma}\}\right\rangle = \frac{2\hbar c}{\pi}\int \mathrm{d}k k^3. \tag{1.11}$$

Thus in the minimum energy state, with no source present, the fields average to zero at any point, but they are subjected to random fluctuations yielding a non-vanishing variance. We call "zero point" (Z.P.) terms the quantities (1.11). They cannot be observed directly, but they can be regarded as responsible for vacuum effects. We will now show that the presence of a source determines a change in these fluctuations which depends on the distance from the source.

In this paper, which we are glad to dedicate to Iwo Białynicki-Birula on the occasion of his 60th birthday, we wish to discuss the cloud effects around a nonrelativistic electron. By this we mean an electron which is both spinless and slowly moving. Thus we will consider two situations, in the first of which our electron is bound by the central potential created by a fixed and spinless proton. Rather loosely, we call this the hydrogen atom, which we study assuming that the electron is in the $1s$ configuration. The second situation consists of a free electron in the absence of any external potential. We study this second case in the assumption that the momentum of the electron and of the vacuum photons scattering on it are both smaller than mc.

2. Spatial structure of the electromagnetic energy density distribution around the hydrogen atom

Let us consider a nonrelativistic, neutral source, modelled as a spinless hydrogen atom with the proton fixed at $r = 0$, and study the energy density distribution at distances larger than the Bohr radius a_0.

Let the atom be coupled to the electromagnetic field through the minimal coupling interaction. In the Coulomb gauge the Hamiltonian of the whole system has the following form [5]:

$$H = \frac{p^2}{2m} - \frac{e^2}{x} + \int \mathrm{d}^3r' \left[\frac{E^2_{\perp}(r') + B^2(r')}{8\pi}\right] + \frac{e}{mc}\boldsymbol{A}_{\perp}(\boldsymbol{x})\cdot\boldsymbol{p} + \frac{e^2}{2mc^2}A_{\perp}(\boldsymbol{x})^2, \tag{2.1}$$

where $-e$ and m are respectively the electron charge and mass, and $\boldsymbol{A}_{\perp}(\boldsymbol{x})$ is the transverse vector potential operator. In the dipole approximation, the vector potential is taken at $\boldsymbol{x} = 0$.

The ground state $|\psi\rangle$ of the interacting system is evaluated by standard perturbation theory, up to order e^2, starting from the bare ground state $|0\rangle = |\phi_{100}; \{0_{k,\sigma}\}\rangle$ in which the atom is in the state $n = 1$, $l = 0$, $m = 0$ and the field is in the vacuum state [5]. Evidently $|\psi\rangle$ contains also one- and two-photon states.

An analytic evaluation of the electric and magnetic energy density can be performed in the so-called "near zone", within a radius smaller than the characteristic atomic transition wavelength, and in the "far zone", or "radiation zone", beyond this characteristic distance. Two qualitatively different behaviours of the field dominate in each zone.

In the near zone ($a_0 \ll r \ll \lambda_{n1}$), only virtual photons of frequency higher than the lower atomic transition frequency can be taken into account. This leads to the following approximated expressions for the square of the electric and magnetic fields:

$$\langle\psi|E^2(r)|\psi\rangle \approx \text{Z.P.} + \frac{2e^2}{r^6}\langle\phi_{100}|x^2|\phi_{100}\rangle, \tag{2.2}$$

$$\langle\psi|B^2(r)|\psi\rangle = \text{Z.P.} - \frac{5e^2\hbar}{2\pi mc}\frac{1}{r^5} + \frac{\alpha\hbar c2}{m^2c^23}p_a^2\frac{1}{4}, \tag{2.3}$$

where $p_a^2 = \hbar^2/a_0^2$. The dominant term in the energy density distribution has a r^{-6}-dependence, due to the dipole term (2.2):

$$\mathcal{H}(r) \approx \frac{1}{4\pi}\frac{e^2}{r^6}\langle\phi_{100}|x^2|\phi_{100}\rangle + \frac{1}{4\pi}\text{Z.P.} \tag{2.4}$$

In Eq. (2.2), r^{-5}-dependent terms, contributed by the square of the transverse field and by the interference between the longitudinal and the transverse field [5], have not been written explicitly; they are in fact overcome by the r^{-6}-dependent term in this zone. Among them the average value of the square of the transverse field is particularly interesting

$$\langle\psi|E_\perp^2(r)|\psi\rangle = \text{Z.P.} + \frac{5e^2\hbar}{2\pi mc}\frac{1}{r^5}. \tag{2.5}$$

In the far zone ($r \gg \lambda_{n1}$) one cannot neglect retardation effects because they cancel all the r^{-6}-dependent instantaneous contributions. In this zone the relevant virtual photons have frequencies smaller than the atomic frequency; this gives the following approximate quantum averages for the square of the electric and magnetic fields [5]:

$$\langle\psi|E^2(r)|\psi\rangle = \text{Z.P.} + \frac{23}{2\pi}\hbar c\alpha\frac{1}{r^7} + \text{O}(r^{-8}), \tag{2.6a}$$

$$\langle\psi|B^2(r)|\psi\rangle = \text{Z.P.} - \frac{7}{2\pi}\hbar c\alpha\frac{1}{r^7} + \text{O}(r^{-8}), \tag{2.6b}$$

The far zone fields depend on the static polarizability of the atom in the ground state

$$\alpha = \frac{2}{3}\sum_{N,M}\frac{\langle|\phi_{N1M}|e\boldsymbol{x}|\phi_{100}\rangle|^2}{\hbar\omega_{N1}}. \tag{2.7}$$

As a consequence, the energy density has the following dependence [5]:

$$\mathcal{H}(r) = \frac{1}{\pi^2}\hbar c\alpha\frac{1}{r^7} + \frac{1}{4\pi}\text{Z.P.} \tag{2.8}$$

The results above can be related to the magnetic and electric interaction energies with a test body. The interaction energy between the source and a test body having static electric polarizability α_E can be derived from Eq. (2.6a) and coincides with the Casimir–Polder potential [12]:

$$\Delta\mathcal{E}_E = -\frac{1}{2}\alpha_E\langle\psi|E^2(r)|\psi\rangle = -\frac{23}{4\pi}\hbar c\alpha\alpha_E\frac{1}{r^7}. \tag{2.9a}$$

The interaction energy between the source and a test body with static magnetic polarizability α_M can be derived from Eq. (2.6b) and it is [12]:

$$\Delta\mathcal{E}_M = -\frac{1}{2}\alpha_M\langle\psi|B^2(r)|\psi\rangle = +\frac{7}{4\pi}\hbar c\alpha\alpha_M\frac{1}{r^7}. \tag{2.9b}$$

3. Electromagnetic energy density distribution around a free electron

In the previous paragraph we have seen that the internal dynamics of a neutral source has effects on the local observables of the field in the surrounding space. If one considers a charge point source, one expects that the field will be influenced only by emission and reabsorption of photons, accompanied by recoil of the charged source. This effect on the local observables of the field can be analysed modelling the source as a charged free particle. In particular, one can consider a spinless electron, having mass m and charge $-e$.

A free quantum particle having a defined momentum is described by a wave function which is not localized; as a consequence, in the dressed state of the system field+source, the field observables have an average value which is uniform in space, since the field interacts with a source which is in fact delocalized. This feature of the model makes it necessary to introduce an appropriate formalism, which allows to identify a "cloud", as an object having a defined space structure. For example, it is possible to relate the position of the source to the field observables using quantum averages of correlation operators. An appropriate correlation operator which can be introduced has the following form [9]:

$$G_F(\rho) = \int d^3r F(r+\rho/2)|r-\rho/2\rangle\langle r-\rho/2|, \tag{3.1}$$

where $F(r+\rho/2)$ is any operator which describes local observables of the field. The correlation operator defined above describes the field at distance ρ from the position of the electron and involves an integration over all possible positions of the electron. Moreover, this operator can be related to integral observables. In fact, by integrating over ρ, and by changing variable $\{r+\rho/2 = r_1;\ r-\rho/2 = r_2\}$ one obtains the integral of the field observable F over all space

$$\int d^3\rho G_F(\rho) = \int d^3r_1 F(r_1)\int d^3r_2|r_2\rangle\langle r_2| = \int d^3r_1 F(r_1). \tag{3.2}$$

Using this correlation operator, one can calculate the quantum average of the correlated fields, their mean square values and the correlated energy density, at distance ρ from the electron position [9].

As discussed above, in order to calculate local field observables at distances ρ larger than the Compton wavelength, a nonrelativistic approach is justified, provided the electron is nonrelativistic. The photon cloud of the free electron can then be studied starting from the minimal coupling form Hamiltonian, in the Coulomb gauge [9]:

$$H = \frac{p^2}{2m} + \int \mathrm{d}^3 r' \left[\frac{E_\perp^2(r') + B^2(r')}{8\pi}\right] + \int \mathrm{d}^3 r' \frac{E_\parallel^2(r')}{8\pi} + \frac{e}{mc} \boldsymbol{A}_\perp(\boldsymbol{x}) \cdot \boldsymbol{p} + \frac{e^2}{2mc^2} A_\perp(\boldsymbol{x})^2, \tag{3.3}$$

where the vector potential $\boldsymbol{A}_\perp(\boldsymbol{x})$ is taken at the position $\boldsymbol{x}$ of the electron. The correlation operators are averaged on state $|\psi\rangle$, which is evaluated by standard perturbation methods [5] from the bare state $|\boldsymbol{p}'; \{0_{\boldsymbol{k},\sigma}\}\rangle$, in which the electron has a bare momentum $\boldsymbol{p}'$ and the field is in the vacuum state [9].

The quantum values of correlation operators are defined as the average value of operators of the form (3.1), with F appropriately chosen.

On the state defined above, the electric field ($F = \boldsymbol{E}_\parallel(\boldsymbol{\rho}) + \boldsymbol{E}_\perp(\boldsymbol{\rho})$) and the magnetic field ($F = \boldsymbol{B}(\boldsymbol{\rho})$), correlated with the electron position at distance ρ, have quantum average different from zero.

The quantum average of the correlated electric field turns out to be equal, for $p' \ll mc^2$, to the Coulomb field of a charge e [9]:

$$\langle\psi|G_{\boldsymbol{E}}(\rho)|\psi\rangle = -e\frac{\boldsymbol{\rho}}{\rho^3}. \tag{3.4}$$

Only the longitudinal part of the field contributes to (3.4), while the transverse part is zero on the average [5].

The average value of the magnetic field is different from zero and coincides with the classical magnetic field of a free charge in motion with velocity $\boldsymbol{v} = \boldsymbol{p}'/m$ [9]:

$$\langle\psi|G_{\boldsymbol{B}}(\rho)|\psi\rangle = -\frac{e}{mc}\boldsymbol{p}' \times \frac{\boldsymbol{\rho}}{\rho^3}. \tag{3.5}$$

The zero point field plays no role in the quantities above. In the correlated mean square values of the fields, however, new terms appear, connected with the vacuum fluctuations [9]. Let us start by considering the square of the electric field. From operator (3.1), with $F = E^2 = (\boldsymbol{E}_\perp + \boldsymbol{E}_\parallel)^2$, we have

$$\langle\psi|G_{\boldsymbol{E}^2}(\rho)|\psi\rangle - \text{Z.P.} = \frac{e^2}{\rho^4} + \frac{5e^2\hbar}{2\pi mc}\frac{1}{\rho^5}. \tag{3.6}$$

The first term on the right hand side of Eq. (3.6) is entirely given by the longitudinal contribution and indeed coincides with the square of the classical field; hence it varies as ρ^{-4}. The second term has a ρ^{-5} dependence; it is entirely transverse and it has no counterpart in Eq. (3.4). In the same way as for the zero point field, one can say that there are fluctuating corrections to the field, which do not appear in the average of the linear terms but give corrections to their mean

square values. Such contributions are quantum effects, arising from the interaction between the electron and the fluctuating zero point field.

The term of interference between transverse and longitudinal field vanishes.

The average value of the correlated square magnetic field, derived from Eq. (3.1) with $F = B^2$, is [9]:

$$\langle\psi|G_{\boldsymbol{B}^2}(\rho)|\psi\rangle - \text{Z.P.} = \frac{e^2}{m^2c^2}\frac{p'^2\sin^2\theta}{\rho^4} - \frac{5e^2\hbar}{2\pi mc}\frac{1}{\rho^5}, \tag{3.7}$$

where θ is the angle between $\boldsymbol{p}'$ and $\boldsymbol{\rho}$. It consists of two terms; the first on the right hand side coincides with the square of the classical magnetic field, Eq. (3.5). The second term is a quantum contribution, which, in analogy with the mean square value of the transverse electric field, has a fluctuating nature. Like its electric counterpart, it decreases with distance as ρ^{-5}.

An important aspect of these results is that the magnetic and the electric ρ^{-5}-dependent terms are equal and opposite. Therefore, there is no contribution to the total energy density distribution coming from these quantum corrections. However, they can have separately observable effects, because they can be related to the interaction between a free charged particle and a test body having magnetic or electric polarizability.

In fact, analogously to the case of the hydrogen atom, one can evaluate the interaction energy between the electron and a test body.

The quantum corrections, due to the transverse term alone, to the interaction energy between the free charge and a test body having static electric polarizability α_{E} can be derived from Eq. (3.6) and it turns out to be

$$\Delta\mathcal{E}_{\mathrm{E}}^{\mathrm{f}} = -\frac{1}{2}\alpha_{\mathrm{E}\perp}\langle\psi|G_{\boldsymbol{E}_\perp{}^2}(\rho)|\psi\rangle = -\frac{1}{2}\alpha_{\mathrm{E}}\left[\frac{5e^2\hbar}{2\pi mc}\frac{1}{\rho^5}\right], \tag{3.8}$$

The interaction energy between the electron with $\boldsymbol{p}' = 0$ and a test body with static magnetic polarizability α_{M} can be derived from Eq. (3.7) and it is

$$\Delta\mathcal{E}_{\mathrm{M}}^{\mathrm{f}} = -\frac{1}{2}\alpha_{\mathrm{M}}\langle\psi|G_{\boldsymbol{B}}(\rho)|\psi\rangle = \frac{1}{2}\alpha_{\mathrm{E}}\left[\frac{5e^2\hbar}{2\pi mc}\frac{1}{\rho^5}\right]. \tag{3.9}$$

4. Conclusions

As we have seen, QED interaction between a slow electron and the zero point field modifies field local observables in the neighbouring of the source. These effects can be discussed in terms of a virtual photon cloud extended in the surrounding space.

If the electron is bound to a proton in the way described in Sec. 2, the field is influenced by the source's internal structure. It is possible to distinguish two zones, where it behaves in a qualitatively different way: a near zone and a far zone, separated by the characteristic wavelength of the hydrogen atom. The fundamental difference between these two zones is due to the effect of retardation [12], which must be taken into account in the far region. In this zone, the field properties are connected to the static magnetic and electric polarizabilities of the

source [13], as can be seen from Eqs. (2.6) and (2.7); the electromagnetic energy density distribution in this zone vary with the distance r from the atom as r^{-7}.

In the near zone, the dominant term is the longitudinal electric dipole field, which determines an r^{-6} dependence for the energy density distribution. The longitudinal term can be explained as due to the quantum fluctuations of the atomic dipole moment. Moreover, the interaction between the source and the zero point field gives fluctuating corrections to the transverse electric field and to the magnetic field, which manifest themselves through r^{-5}-dependent contributions to the mean square values of both the transverse electric field, Eq. (2.5) and the magnetic field, Eq. (2.3), and also through a term of interference between the transverse and the longitudinal electric fields in the electric energy density.

In the case of a free electron, it is possible to speak of a cloud of virtual photons having a well defined spatial structure, if one correlates the field observables to the position of the particle at given distances.

It is instructive to compare the cloud of a free electron with the cloud surrounding the electron bound within the hydrogen atom.

In the fields correlated to the position of the free electron classical and quantum contributions can be distinguished. The correlated longitudinal electric field coincides with the classical field of a point charge. In addition to this, a fluctuating transverse contribution appears due to interaction with the quantum vacuum. Comparison of Eqs. (2.5) and (3.6) shows that the quantum average of the square transverse electric field has the same form both for the hydrogen atom in the near zone and for the electron. In the case of the free electron, the interference between the longitudinal and the transverse field is zero [9].

The correction to the square magnetic field both in the near zone of the hydrogen atom, Eq. (2.3), and in the space surrounding of the electron, Eq. (3.7), is contributed by two terms, whose spatial structure is quite similar. The r^{-4}-dependent term present in both expressions is a classical contribution, due to the motion of the electron. The difference in the coefficients comes from the fact that one must average the momentum of the electron bound in the hydrogen atom, while the free electron has a momentum with fixed direction and modulus [9]. Both in Eq. (2.3) and (3.7), the r^{-5}-dependent terms are identical, and have the same nature, being determined by the interaction with the vacuum fluctuations of the field.

On the other hand, the electromagnetic field energy density in the far zone of the hydrogen atom, as given by (2.6) and (2.8), is completely different from anything we find for the free electron. This far-zone field is heavily influenced by the structure of the eigenvalue spectrum through the appearance of the ground-state electric polarizability of the atom (2.7).

We are led to conclude that the transverse part of the electromagnetic energy density, which in the minimal coupling scheme must be regarded as the cloud of virtual photons surrounding the slow electron is very much the same whether the electron is bound in the hydrogen atom or it is free, provided we are in a zone not too distant from the electron itself. As we move away from the electron, however, the differences between a bound and a free electron become important, and, at large distances from the electron, the form of the virtual cloud is essentially

determined by the dynamical properties of the eigenstates of the electron in the potential which binds it to the centre of motion.

This seems reasonable, since the high-frequency photons, which constitute most of the virtual cloud at small distance from the electron, are not much influenced by the form of the eigenvalue spectrum of the bound electron, whereas the low-frequency photons obviously are, their energy being of the same order as the characteristic energy differences in the eigenvalue spectrum of the bound electron.

Acknowledgments

The authors acknowledge partial financial support by Comitato Regionale Ricerche Nucleari e Struttura della Materia, by Regione Siciliana Assassorato Beni Culturali ed Ambiente and by Ministero della Ricerca Scientifica e Tecnologica.

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