Vol. 85 (1994) ACTA PHYSICA POLONICA A No. 2

Proceedings of the European Conference "Physics of Magnetism 93", Poznań 1993

UNIVERSAL SCALING OF BASIC PROPERTIES OF THE HEAVY-FERMION SUPERCONDUCTORS

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We show that the properties of the heavy-electron superconducting state induced by the interorbital kinetic exchange scale with the effective mass renormalization $m^*/m_0 \sim 1/T_K$. Explicitly, the pairing potential $\tilde{J} \sim J(m_0/m^*)\ln^2(m_0/m^*)$, where J is the magnitude of the bare Kondo coupling; the coherence length $\xi \sim T_K/T_c$, where T_c is the transition temperature, whereas the penetration depth $\lambda \sim (m^*/m_0)^{1/2}$ so that $\lambda/\xi \gg 1$. We also determine the scaling of magnetic critical fields.

PACS numbers: 74.70.Tx, 75.30.Mb, 71.28.+d

In this paper we predict a scaling of fundamental parameters characterizing a heavy-fermion superconductor, which extends the earlier analysis for the normal state [1]. This goal is achieved by considering the lattice Anderson model in which first order corrections in $1/U$, where U is the magnitude of the intraatomic f–f interaction, have been included [2] so as to generate an interorbital (hybrid) pairing. In this manner, both the Fermi liquid state of almost localized electrons, as well as their superconducting properties are obtained within a single framework. An earlier treatment [3] of superconductivity within the lattice Anderson model in the $U = \infty$ limit required higher-order $(1/N^2)$ correction to the mean-field slave-boson picture of the heavy electrons. Here, a stable superconducting phase appears already in the mean-field approximation for the pairing part and provides a universal scaling with the mass renormalization m^*/m_0, as discussed below.

We start from the effective Hamiltonian derived earlier [2] to the first nontrivial order in V/U, which was rederived in the slave boson representation of Zou and Anderson [4] and takes the form

$$\tilde{\mathcal{H}} = \sum_{mn\sigma} t_{mn} c^+_{m\sigma} c_{n\sigma} + \varepsilon_f \sum_{i\sigma} f^+_{i\sigma} f_{i\sigma} + \sum_{im\sigma} (V_{im} e_i f^+_{i\sigma} c_{m\sigma} + \text{H.c.}) - \sum_{im} (2V^*_{im} V_{in}/\bar{U}) b^+_{im} b_{in} + \sum_i \lambda_i (e^+_i e_i + \sum_\sigma f^+_{i\sigma} f_{i\sigma} - 1), \tag{1}$$

with

$$b^+_{im} \equiv (f^+_{i\uparrow} c^+_{m\downarrow} - f^+_{i\downarrow} c^+_{m\uparrow})/\sqrt{2}, \tag{2}$$

and $\bar{U} \equiv U - \tilde{\varepsilon}_f + \mu$.

The first three terms comprise the Anderson lattice model in $U = \infty$ limit. This formulation involves a single scalar boson and has been studied extensively in the last decade [1] in the limit $d_i^+ d_i \equiv 0$. The fourth term expresses the interorbital spin singlet pairing introduced before [2, 5], here reformulated in slave boson language. The last term contains a set of Lagrange multipliers $\{\lambda_i\}$ reflecting the local constraint which is imposed at every f-site due to the introduction of extra Bose field e.

One can easily diagonalize the single-particle part of (1) in the mean-field approximation and obtain the usual eigenenergies

$$E_{k\alpha} = (1/2)\left\{\varepsilon_{k} + \tilde{\varepsilon}_f - 2\mu + \alpha\left[(\varepsilon_{k} - \tilde{\varepsilon}_f)^2 + (2Ve)^2\right]^{\frac{1}{2}}\right\}, \tag{3}$$

where $\alpha = \pm 1$. The renormalized f-level position $\tilde{\varepsilon}_f$ is very close to the Fermi level and at $T = 0$ yields

$$\tilde{\varepsilon}_f - \mu \sim (W/2)\exp\left(-\frac{\mu - \varepsilon_f}{V^2\rho_0}\right), \tag{4}$$

where the bare band spans from $-W/2$ to $W/2$ and is assumed as featureless. The above energy difference is defined as $k_{\mathrm{B}}T_{\mathrm{K}}$, where T_{K} is customarily called the effective Kondo temperature [1]. The density of quasiparticle states at μ is then $\rho(\mu) = 1/2k_{\mathrm{B}}T_{\mathrm{K}}$ and therefore very large.

We now present the scaling of quantities with T_{K} in the superconducting phase. For that purpose we consider the case for which the number of particles is $n \leq 2$ per site so that only the lower hybridized band $E_{k-} \equiv E_{k}$ is occupied in the temperature range much smaller than the hybridization gap, $k_{\mathrm{B}}T \ll |V|e$. The effective Hamiltonian (1) transformed to the hybridized basis then has the form

$$\begin{aligned}\widetilde{\mathcal{H}}_{\mathrm{SB}} = {} & \sum_{k\sigma} \Psi_{k\sigma}^{+} E_{k} \Psi_{k\sigma} \\ & - \frac{V^2}{\bar{U}} \sum_{kk'} \frac{4V^2e^2}{[(\varepsilon_{k} - \varepsilon_f)^2 + 4V^2e^2]^{\frac{1}{2}}[(\varepsilon_{k'} - \varepsilon_f)^2 + 4V^2e^2]^{\frac{1}{2}}} \\ & \times \Psi_{k\uparrow}^{+}\Psi_{-k+q\downarrow}^{+}\Psi_{-k'\downarrow}\Psi_{k'+q\uparrow},\end{aligned} \tag{5}$$

where $\Psi_{k\sigma}^{+}$ is the creation operator of a hybridized a–c state. Generally, $\Delta_{k} \sim V_{k}$ has nodes for k points for which $V_{k} = 0$. In our model situation with $V_{k} = V$ the gap is never zero; therefore, we approximate the pairing potential by its average over occupied quasiparticle states. This leads to an effective k-independent potential

$$\tilde{J} \approx \frac{V^2}{\bar{U}}\left(\frac{k_{\mathrm{B}}T_{\mathrm{K}}}{Ve}\right)^2 \ln^2\left(\frac{k_{\mathrm{B}}T_{\mathrm{K}}}{|V|e}\right) = \frac{V^2\rho_0}{4\bar{U}} k_{\mathrm{B}}T_{\mathrm{K}} \ \ln^2(k_{\mathrm{B}}T_{\mathrm{K}}\rho_0). \tag{6}$$

In the limit of f electron localization $\tilde{J} \to 0$ (note that the pairing takes place only when $e \neq 0$, i.e. when the f holes exist and propagate). The disappearance of the pairing in the strict Kondo lattice limit ($e = 0, n_f = 1$) implies that our approach indeed describes pairing, not the singlet Kondo type of state.

The local nature of pairing in conjunction with the single-band nature of the problem allows us to derive explicitly the Ginzburg–Landau functional within the Lagrangian formalism for the Grassmann variables $\Psi_\sigma^+(r)$ and $\Psi_\sigma(r)$:

$$\mathcal{L}(\tau) = \int \mathrm{d}^3x \left\{\sum_{\sigma} \Psi_\sigma^+[p_0 + E(p)]\Psi_\sigma - \tilde{J}\Psi_\uparrow^+(x), \Psi_\downarrow^+(x)\Psi_\downarrow(x)\Psi_\uparrow(x)\right\}, \tag{7}$$

where $\Psi_\sigma = \Psi_\sigma(\boldsymbol{x}, \tau)$ $p_0 \equiv \partial_\tau$, and $E(\boldsymbol{p})$ is the eigenenergy $E_{\boldsymbol{k}}$ with $\boldsymbol{k}$ replaced by $(-\mathrm{i}\nabla)$. We also introduce the two-component Nambu notation $\Psi^+ \equiv (\Psi_\uparrow^+, \Psi_\downarrow)$ in (10) and apply the Hubbard–Stratonovich transformation to the quartic term. Such procedure reduces the partition function to the form

$$Z = \int \mathrm{D}\Psi \mathrm{D}\Psi^+ \mathrm{D}\Delta$$
$$\times \exp\left\{ -\int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3x \left[\Psi^+ \begin{pmatrix} p_0 + E(p), & -\tilde{\Delta} \\ -\tilde{\Delta}^+, & p_0 - E(p) \end{pmatrix} \Psi - \frac{|\tilde{\Delta}|^2}{\tilde{J}} \right] \right\}, \quad (8)$$

with $\tilde{\Delta} = \tilde{J}\Delta$. Integrating over the Grassmann variables and neglecting the part which does not depend explicitly on $\tilde{\Delta}$ we obtain

$$Z = \int \mathrm{D}\Delta$$
$$\times \exp\left\{ \mathrm{Tr}\ln\left[1 - \begin{pmatrix} 0, & \frac{1}{p_0 + E(p)}\tilde{\Delta} \\ \frac{1}{p_0 - E(p)}\tilde{\Delta}^+, & 0 \end{pmatrix} \right] - \beta \int \mathrm{d}^3x |\tilde{\Delta}|^2 \tilde{J} \right\}, \quad (9)$$

where the part $\{\ldots\}$ is called the effective action S_{eff}. Expanding $\exp(-S_{\mathrm{eff}})$ into the Taylor series, carrying out a Fourier expansion, and evaluating corresponding sums [6] one arrives at the Ginzburg–Landau functional F_{GL} in the form

$$F_{\mathrm{GL}} = -\frac{1}{2}\rho_0(m^*/m_0)$$
$$\times \int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3x \left\{ \left[\left(1 - \frac{T}{T_{\mathrm{c}}}\right) + \xi^2\nabla^2 \right] |\tilde{\Delta}|^2 - \frac{7}{16}\frac{\zeta(3)}{\pi^2 T_{\mathrm{c}}^2}|\tilde{\Delta}|^4 \right\}, \quad (10)$$

with $T_{\mathrm{c}} = 1.13D\exp(-1/\tilde{J}\rho(\mu)) \sim T_{\mathrm{K}}^{\frac{1}{2}}\exp(-k_{\mathrm{B}}T_{\mathrm{K}}/\tilde{J})$. The coherence length at $T = 0$ is

$$\xi_0^2 = \frac{7}{48}\frac{\zeta(3)}{\pi^2 T_{\mathrm{c}}^2}\left(\frac{k_{\mathrm{B}}T_{\mathrm{K}}}{Ve}\right)^4 \frac{k_{\mathrm{F}}^2}{m_0^2} \sim (T_{\mathrm{K}}/T_{\mathrm{c}})^2, \quad (11)$$

with k_{F} being the Fermi wave vector and $\zeta(x)$ is the Riemann zeta function.

To determine the London penetration depth we start with the substitution $\nabla \to \nabla - 2\mathrm{i}e_0\boldsymbol{A}/c$, where $\boldsymbol{A}$ is the vector potential. This produces the term $(1/2)m_A^2\boldsymbol{A}^2$ in S_{eff}, where

$$m_A^2 = \frac{2}{3}e_0^2\left(\frac{v_{\mathrm{F}}}{c^2}\right)^2\left(1 - \frac{T}{T_{\mathrm{c}}}\right)\left(\frac{k_{\mathrm{B}}T_{\mathrm{K}}}{Ve}\right)^4 \rho(\mu) \sim k_{\mathrm{B}}T_{\mathrm{K}}(1 - T/T_{\mathrm{c}})m_{\mathrm{BCS}}^2 \quad (12)$$

is the photon mass in the superconducting phase, and $m_{\mathrm{BCS}}^2 = 2e_0^2(v_{\mathrm{F}}/c)^2\rho_0/3$ is the mass if there were no enhancement due to the presence of the f level. The London penetration depth at $T = 0$ is $\lambda_0 = (\hbar/m_A) \sim T_{\mathrm{K}}^{-\frac{1}{2}}$. The last quantity enters the ratio $\kappa = \lambda/\xi$ which takes the form

$$\kappa = \{\sqrt{2/3}\xi_0 e_0 v_{\mathrm{F}}(\tilde{\varepsilon}_f/|V|e)\sqrt{\rho_0}\}^{-1} \sim T_{\mathrm{K}}^{-3/2}T_{\mathrm{c}}. \quad (13)$$

Note that we have used the relation $\xi \equiv \xi(T) = \xi_0/(1 - T/T_{\mathrm{c}})^{\frac{1}{2}}$. Close to f electron localization $T_{\mathrm{K}} \to 0$ and then $\kappa \gg 1$.

The expression (13) can be used to determine the thermodynamic critical magnetic field B_c via the relation $B_c^2/2 = -F_{GL}/V_0$, where V_0 is the volume of the system. Explicitly,

$$B_c = \left[\frac{2\pi^2}{7\zeta(3)}\rho(\mu)\right]^{\frac{1}{2}} (T_c - T) \sim (T_c - T)T_K^{-\frac{1}{2}}, \tag{14}$$

and therefore the first and second critical magnetic fields are

$$B_{c1} = \frac{\Phi_0}{2\pi\lambda^2}\ln(\lambda/\xi) \sim T_K \ln(T_K^{-3/2}T_c a), \tag{15}$$

where a is a constant, and

$$B_{c2} = \sqrt{2}\kappa B_c = 2\left[\frac{3\pi^2}{7\zeta(3)}\right]^{\frac{1}{2}} \frac{1}{\xi_0 e v_F}\left(\frac{Ve}{\tilde{\varepsilon}_f}\right)^2 \sim (T_c/T_K^2)(T_c - T). \tag{16}$$

We have calculated the derivative $B'_{c2} = -(dB_{c2}/dT)$ at $T = T_c$ and have plotted it as a function of $T_c\gamma^2$, as well as B'_{c1} at T_c as displayed for various systems in Fig. 1a and b. This figures indicate that heavy-fermion superconductors come out

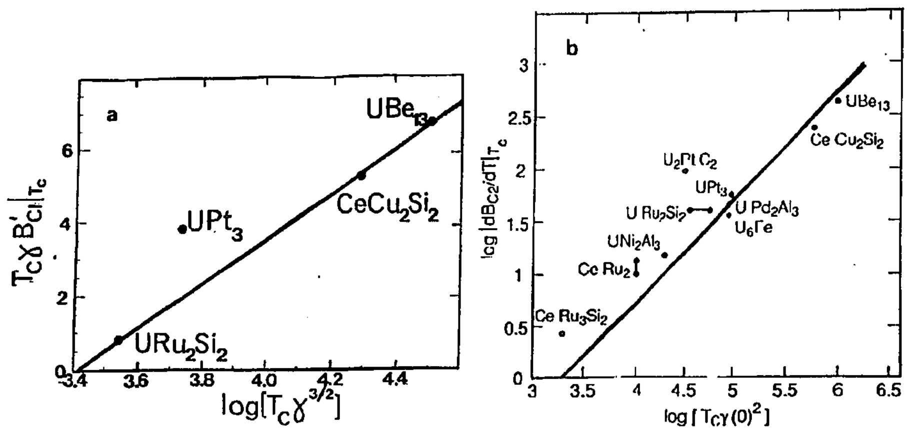

Fig. 1. Predicted linear scaling of first (a) and second (b) critical field derivatives at $T = T_c$ for various heavy-fermion superconductors (solid line). The solid line represents the result coming from the Ginzburg–Landau–Gorkov theory in the clean limit. For explanation see main text.

in clean limit rather than dirty. The data were extracted from the works listed in Ref. [7]. However, it is important to note that the proposed scaling is independent of pairing mechanism, as the coupling constant appears only via T_c.

In summary, we have presented a universal scaling picture for superconducting phase of heavy fermions, taking into account processes of the order V^2/U which produce the pairing. Even though F_{GL} is of standard form, the coefficients acquire unusually high values because of the factor m^*/m_0 Eq. (10).

The authors acknowledge the support of the Committee for Scientific Research, grant No. 2 0429 91 01.

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