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TEMPERATURE DEPENDENCE OF THE ELECTRON RAMAN LIGHT SCATTERING IN NORMAL METALS AND SUPERCONDUCTORS

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Within the anisotropic BCS model the temperature dependence of the electron Raman light scattering in normal metals and superconductors is investigated. In either case analytical expressions describing the intensity of scattered light as a function of the Raman frequency and temperature are derived.

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1. Introduction

The discovery of high-temperature superconductors has led to an increase of interest in electron light scattering. In this class of substances a thresholdless of electronic Raman light scattering is observed. In the range $\omega \ll 2\Delta$ ($\omega = \omega_i - \omega_s$, where ω_i (ω_s) is the frequency of incident (scattered) light) the linear dependence of scattering cross-section on ω is observed. This can be caused, e.g., by spatial inhomogeneities in the energy gap (2Δ) in the sample and/or the type of electron pairing leading to the vanishing of the gap on the lines [1] or points on the Fermi surface. By now, the role of local heating by laser spot in a system with small heat conduction remains unclear, although there exists a relatively simple way of measuring the temperature basing on the peak-to-peak Stokes $I(\omega)$ and anti-Stokes $I(-\omega)$ intensities distance. It holds $I(-\omega) = I(\omega)\exp -\hbar\omega/kT)$.

This paper is devoted to investigations of the effect of temperature on electronic light scattering in an anisotropic metal in normal and superconducting state.

2. The model, cross-section and intensity of scattered light

The interaction part of the Hamiltonian of an anisotropic superconductor in an electromagnetic field has the following form:

$$\mathcal{H} = \int \mathrm{d}^3 r M(r)\psi_\alpha^+(r)\psi_\alpha(r) - \frac{g}{2}\int \mathrm{d}^3 r \mathrm{d}^3 r' V(r,r')\psi_\alpha^+(r)\psi_\gamma^+(r')\psi_\gamma(r')\psi_\alpha(r), \tag{1}$$

$M(r) = \frac{1}{m}\left(\frac{e}{c}\right)^2 \boldsymbol{A}^{(\mathrm{i})}(r)\boldsymbol{A}^{(\mathrm{s})}(r)$; $\boldsymbol{A}^{(\mathrm{i})}$ ($\boldsymbol{A}^{(\mathrm{s})}$) is the vector potential of the incident scattered electromagnetic wave. In Eq. (1) we neglect a term linear in $\boldsymbol{A}$, which if taken into account would give a small contribution (of order ω/ω_i) to the cross-section.

The cross-section is described by the expression

$$\mathrm{d}\sigma \propto \left\langle \mathcal{S}^+\mathcal{S}\right\rangle, \tag{2}$$

where the angular brackets have the meaning of a quantum-mechanical and statistical average over the Gibbs ensemble at finite temperature. The transition matrix can be written in the form

$$\mathcal{S} = -\left\{ \int \mathrm{d}x M(x)\psi_\alpha^+(x)\psi_\alpha(x) - \frac{g}{2}\int \mathrm{d}x\mathrm{d}x' V(x,x') \times \left[C_{\gamma\alpha}(x',x)\psi_\alpha^+(x)\psi_\gamma^+(x') + A_{\alpha\gamma}(x,x')\psi_\gamma(x)\psi_\alpha(x')\right] \right\}, \tag{3}$$

$$C_{\alpha\beta}(x,x') = \left\langle T_\tau(\overline{\psi}_\alpha(x)\overline{\psi}_\beta(x'))\right\rangle, \qquad A_{\alpha\beta}(x,x') = \left\langle T_\tau(\overline{\psi}_\alpha^+(x)\overline{\psi}_\beta^+(x'))\right\rangle,$$

$$x = r,\tau, \qquad V(x,x') = \delta(\tau-\tau')V(r,r'), \qquad M(x) = M(r)\delta(\tau). \tag{4}$$

The bars over the field operators in (4) mean that they are written in the interaction representation taking into account the Hamiltonian term quadratic in the electromagnetic field.

In the case of the singlet pairing of electrons and $V(r,r') = \delta(r-r')$ we have $\hat{A}(x,x') = \hat{C}(x,x')$, and the form of the expressions can be simplified.

After substituting (3) into (2) we obtain

$$\mathrm{d}\sigma \propto \int \mathrm{d}^4 q \int \mathrm{d}^4(y-x)\mathrm{e}^{\mathrm{i}q(x-y)}\mathcal{K}(x-y), \tag{5}$$

where

$$\mathcal{K}(x-y) = 2\alpha^2\Big\{ \left[\mathcal{G}(x-y)\mathcal{G}(y-x) - \mathcal{F}^+(x-y)\mathcal{F}^+(y-x)\right] + 2\left[R(q)+R^*(q)\right]\left[\mathcal{G}(x-y)\mathcal{F}^+(y-x) - \mathcal{F}^+(x-y)\mathcal{G}(y-x)\right] - 2|R(q)|^2\left[\mathcal{G}^2(x-y) + \mathcal{G}^2(y-x) + \mathcal{F}^{+2}(x-y) + \mathcal{F}^{+2}(y-x)\right] \Big\}, \tag{6}$$

$\alpha = \boldsymbol{e}^{(\mathrm{i})}\,\boldsymbol{e}^{(\mathrm{s})}/m$, where $\boldsymbol{e}^{(\mathrm{i})}$ ($\boldsymbol{e}^{(\mathrm{s})}$) is the unit vector of polarization of the incident (scattered) light.

$$R(q) = gT\sum_\omega \int \frac{\mathrm{d}^3 p}{(2\pi)^3}\mathcal{F}^+(p)\mathcal{G}(p-q)$$

$$\times \left\{ 1 - gT \sum_{\omega} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \left[\mathcal{G}(-p)\mathcal{G}(p-q) + \mathcal{F}^+(-p)\mathcal{F}^+(p-q) \right] \right\}^{-1}, \quad (7)$$

and Fourier transforms of the Green functions are of the following form (see, e.g., [5]):

$$\mathcal{G}(p,\omega) = -\frac{\mathrm{i}\omega + \xi_p}{\omega^2 + \xi_p^2 + \Delta^2(p)}, \qquad \mathcal{F}^+(p,\omega) = \frac{\Delta(p)}{\omega^2 + \xi_p^2 + \Delta^2(p)}, \quad (8)$$

$$\frac{p^2}{2m} - \varepsilon_{\mathrm{F}} = \frac{p_0}{m}(|p| - p_0) = \xi_p, \qquad p_0 \approx (2m\varepsilon_{\mathrm{F}})^{1/2}.$$

The expression (5) can be written in the form [2–4]:

$$\mathrm{d}\sigma = \frac{4}{\pi}\left(\frac{e^2}{\hbar c^2}\right)^2 \frac{I(\omega)}{[(n+1)^2 + \kappa^2]^2} \mathrm{d}\Omega \mathrm{d}\omega, \quad (9)$$

$$I(\omega) = \int_0^\infty \mathrm{d}q f(q) \rho(q,\omega), \quad (10)$$

$$f(q) = \frac{16(n^2+\kappa^2)(\omega_{\mathrm{i}}/c)^2}{[q^2 - (2\omega_{\mathrm{i}}/c)^2(n^2-\kappa^2)]^2 + 64(\omega_{\mathrm{i}}/c^4)n^2\kappa^2}, \quad (11)$$

where n and κ are the refractive index and absorption coefficient, respectively, at the frequency of the incident light

$$\rho(q,\omega) = -\frac{1}{\pi} \frac{\mathrm{Im}\, K^{\mathrm{R}}(q,\omega)}{1 - \exp(-\hbar\omega/kT)}. \quad (12)$$

In order to obtain the function $K^{\mathrm{R}}(q,\omega)$ one has to find the discrete Fourier transform of the function $\mathcal{K}(x-y)$ with respect to $\omega_0 = 2\pi\nu T$ and subsequently to perform analytical continuation on the imaginary axis $\omega_0 = -\mathrm{i}\omega$, so that the obtained function $K^{\mathrm{R}}(q,\omega)$ has no singularities in the upper complex half-plane of the variable ω.

The anisotropy of interaction of the system with electromagnetic field will be taken into account by introducing an inverse mass tensor, i.e., we shall substitute in (6), $\alpha \longrightarrow e_j^{(\mathrm{i})} m_{jk}^{-1} e_k^{(\mathrm{s})}$.

Due to anisotropy of interaction among electrons there appears a kernel $V(n,n')$ depending on the directions of the impulses of electrons. We assume that the kernel factorizes, i.e., $V(n,n') = \varphi(n)\varphi(n')$ and $n \ll \kappa$.

If $vq \ll \max(T_{\mathrm{c}},\omega)$, we get

$$\mathrm{Im}\, K^{\mathrm{R}}(q,\omega) = -\frac{2\pi}{\omega}\left\langle \frac{\theta(\omega - 2\Delta(n'))}{[\omega^2 - 4\Delta^2(n')]^{1/2}} \times \left| \alpha\Delta(n') - \varphi(n') \frac{\langle \alpha\Delta(n)\varphi(n)I(n,\omega,T)\rangle}{\langle \varphi(n)I(n,\omega,T)\rangle} \right|^2 \right\rangle \tanh\frac{\omega}{4T}, \quad (13)$$

$$I(n,\omega,T) = \frac{\mathrm{i}\pi\theta(\omega - 2\Delta)}{(\omega^2 - 4\Delta^2)^{1/2}} \tanh\frac{\omega}{4T} - \omega \int_\Delta^\infty \frac{\mathrm{d}\omega'}{(\omega^2 - 4\omega'^2)(\omega'^2 - \Delta^2)^{1/2}} \tanh\frac{\omega'}{2T}. \quad (14)$$

In (13) and below $\langle\ldots\rangle = (2\pi)^{-3}\int \frac{dS_F}{v}$, and v is the maximum value of the normal (to the Fermi surface) component of the Fermi velocity.

In the opposite case $vq \gg \max(T_c, \omega)$, $(vq = vq\mu)$ we obtain

$$\operatorname{Im} K^R(q,\omega) = -\frac{2\pi}{q}\left\langle \frac{\alpha^2}{\pi}\delta(\mu)\left[\int_\Delta^\infty d\omega' \frac{\omega'(\omega'+\omega) - \Delta^2}{\{(\omega'^2-\Delta^2)[(\omega'+\omega)^2-\Delta^2]\}^{1/2}} \times\left(\tanh\frac{\omega'+\omega}{2T} - \tanh\frac{\omega'}{2T}\right) + \theta(\omega - 2\Delta)\int_\Delta^{\omega-\Delta} d\omega' \frac{\omega'(\omega-\omega')+\Delta^2}{\{(\omega'^2-\Delta^2)[(\omega-\omega')^2-\Delta^2]\}^{1/2}}\right]\right\rangle. \tag{15}$$

2.1. A normal metal

In the normal metal $\Delta = 0$ there exists exclusively the region $vq > \omega$ and from (13) we get

$$\operatorname{Im} K^R(q,\omega) = -\frac{2\pi\omega}{q}\left\langle \frac{\alpha^2}{v}\delta(\mu)\right\rangle. \tag{16}$$

On substituting (16) and (12) into (10) we get

$$I(\omega) \propto \frac{\omega}{1-\exp(-\omega/T)}\begin{cases} \delta^2\ln(2v/\omega\delta) & \text{for } \omega \ll v/\delta, \\ 4\delta^{-2}(v/\omega)^4 & \text{for } \omega \gg v/\delta. \end{cases} \tag{17}$$

The dependence described by (17) is shown in Fig. 1, $\delta = c/\kappa\omega_i$.

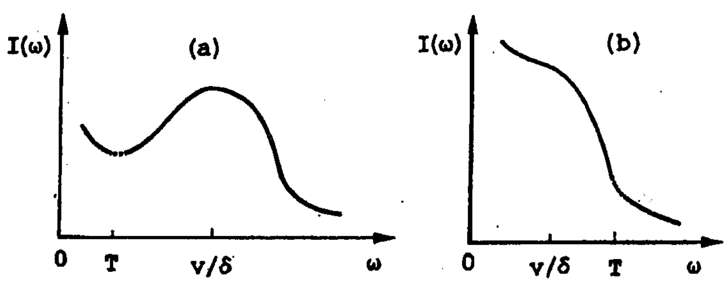

Fig. 1. Intensity of light scattered by a normal metal for typical values of the Fermi velocity — $v \approx 10^8$ cm/s and $\delta \approx 10^{-5}$ cm ($v/\delta \approx 50$ cm^{-1}, 1 cm^{-1} $\longrightarrow$ 1.44 K), schematically (a) at low temperature, $T \ll v/\delta$, (b) at high temperature, $T \gg v/\delta$.

2.2. Superconductor

The spectrum of the light scattered by the superconductor is composed of a background and a component which has its maximum at $\omega > 2\Delta_{\min}$. The factor determining the temperature dependence of the background at $T \ll \Delta_{\min}$ is of the form

$$I(\omega) \approx \delta^2[T(\omega+T)]^{1/2}\ln(2/q_{\min}\delta)\exp(-\Delta_{\min}/T) \quad \text{for} \quad q_{\min}\delta \ll 1 \tag{18}$$

and

$$I(\omega) \approx (2/\delta q_{\min}^2)^2 [T(\omega + T)]^{1/2} \exp(-\Delta_{\min}/T) \quad \text{for} \quad q_{\min}\delta \gg 1, \tag{19}$$

where $vq_{\min} = \max(T_c, \omega)$.

The contribution coming from the creation of pairs near the threshold has the form

$$I(\omega) \approx (\omega - 2\Delta_{\min})^{1/2} [1 - \exp(-2\Delta_{\min}/T)]^{-1} \tanh(\Delta_{\min}/2T). \tag{20}$$

3. Conclusions

For a normal metal, at small transfer frequencies, $\omega < T$, the cross-section instead of decreasing with frequency as $\omega \ln(2v/\omega\delta)$ (as would be the case at absolute zero) increases as $T \ln(2v/\omega\delta)$ (see (17)). In a superconductor the light scattering appears below the frequency threshold $\omega < 2\Delta_{\min}$ (if $\Delta_{\min} \neq 0$). The cross-section depends exponentially on temperature (see, (18)–(20)) and the frequency dependence goes as $[T(\omega + T)]^{1/2}$. Above the threshold the value of the cross-section slightly increases at the transition to the superconducting state.

The electron, Raman light scattering in a normal metal has not been observed yet.

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