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CORNER DISSIPATION IN QUANTUM HALL SYSTEMS

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A description of the onset of dissipation in the integer quantum Hall effect is given, where the electric field across the sample is expressed through a time dependent vector potential. This brings the essentially time dependent, non-stationary nature of the problem into focus. The electric field induces transitions between the levels of the disorder broadened Landau band. Above a critical electric field the particles are driven upwards in energy space beyond the Fermi level, which leads to dissipation since the accumulated energy is lost to the heat bath after τ_{in}, the time between two inelastic events. Thus the dissipated power is obtained without the use of the traditional (linear response) transport formulae. As an application we investigate the dissipation in the corner region of a Hall bar. The results are in reasonable accordance with recent experiments exploiting the fountain pressure effect.
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1. Introduction

One of the striking features of the quantum Hall effect (QHE) [1] is that the samples have vanishingly small diagonal resistivity ρ_{xx}, which means that the current flow is dissipationless. However, there is ample evidence that a well-chosen set of several parameters like the magnetic field $\boldsymbol{B}$, the mobility μ, and the current density j (electric field strength) is necessary to realize the integer QHE (IQHE). In a finite sample it is entirely possible that there are domains in which one or more of the above parameters do not fall into the right regime. In an earlier work [2] (see also [6]) we have pointed out how the dissipationless state is destroyed when the current density or, equivalently, the electric field exceeds a certain limit. Recently a new experimental technique was used to demonstrate that in a Hall bar there is always dissipation in the two opposite corners, where the current enters and leaves the sample. The observation was based on the fountain pressure effect of superfluid helium [3], where local changes in the temperature induce the formation of tiny droplets on the heated surface covered with a thin helium film. In this paper we attempt to explain the observed features.

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2. General theoretical approach

A general theoretical approach to the IQHE has been developed [4], which showed that the full time evolution (beyond the linear response approximation) is crucial for the microscopic explanation of the IQHE. Illustrations of these general results have been given by explicit calculations in semi-realistic model systems [4]. We consider the Hamiltonian

$$H = (1/2m)\,[(h/2\pi\mathrm{i})\nabla - (q/c)\boldsymbol{A}(r)]^2 + V(r) - q\boldsymbol{E}\cdot r. \tag{1}$$

Here $\boldsymbol{E}$ is a homogeneous electric field (the macroscopic field) and $V(r)$ is a disorder potential. $\boldsymbol{B} = (0,0,B) = \mathrm{curl}\boldsymbol{A}(r)$, $\boldsymbol{A}(r) = (0,Bx,0)$; $r = (x,y,0)$. For simplicity we take $\boldsymbol{E} = (0,E_y,0)$. After a time dependent gauge transformation $\boldsymbol{A}'(r) = \boldsymbol{A}(r) - c\boldsymbol{E}t$ we first consider the time dependent Schrödinger equation in the adiabatic limit ($E_y \to 0$). Here the energy levels $\varepsilon(t)$ become rather similar to conventional Bloch dynamics; all degeneracies are lifted [5] and the (adiabatic) spectrum $\{\varepsilon_s(t)\}$ consists of a series of quasi-horizontal lines, fluctuating with the time period $\Delta t = h/|qE_yL_y|$ (with boundary conditions as in [4]). In view of Sec. 3 we remark that L_y can be considered as the width of a subdomain (of a macroscopic sample) over which $\boldsymbol{E}(r)$ is constant.

A sufficiently high electric field may induce non-adiabatic transitions between the adiabatic energy levels. The probability P for such transitions depends on the energy ε of the state and on the strength of the electric field E_y. In a disorder broadened Landau band $P = P(\varepsilon)$ is highest for states at the middle of the band, and decreases rapidly towards the edges, where $P(\varepsilon)$ is many orders of magnitude smaller [6]. Further, if $P(\varepsilon) \approx 0$, the state is insulating, and if $P(\varepsilon) \approx 1$, the velocity of the state tends to the unperturbed classical value cE_y/B [2]. Each adiabatic state in the broadened Landau band is characterized by a threshold value $E_y = E^{\mathrm{th}}(\varepsilon)$, which marks the transition between the two regimes, i.e. $P(\varepsilon) \approx 0$ (≈ 1) if $E_y \ll (\gg)\ E^{\mathrm{th}}(\varepsilon)$. Therefore, in each broadened Landau band there are two energies (mobility edges), where $P(\varepsilon)$ starts to be appreciably different from zero. These mobility edges depend on E_y.

These general results have been illustrated in a model system, where the values of $P(\varepsilon)$ could be calculated explicitly [4]. However, in the general case, i.e. for arbitrary disorder potentials, no satisfactory theory exists for the calculation of the $P(\varepsilon)$ at present. Nevertheless, it has been possible to estimate the highest threshold field in a band (i.e. for states at the band edges) for a typical disorder potential in a quantum Hall system. For this highest threshold field the order of magnitude of 100 V cm^{-1} was obtained [2, 6]. This is consistent with experimental breakdown fields E_{b} of the IQHE (E_{b} is the field strength $|\boldsymbol{E}| = E_{\mathrm{b}}$, where dissipation starts to set in in a quantum Hall system).

Consider the case, where the Fermi level E_{F} lies at a band edge. If $E_y < E_{\mathrm{b}}$, we have $P(\varepsilon_{\mathrm{F}}) \approx 0$ (see preceding section). Therefore, no net energy increase above ε_{F} can occur (i.e. the dissipation is zero). If now E_y is increased, the mobility edges are pushed towards the band edges, which are reached if $E_y = E_{\mathrm{b}}$. Then transitions to levels above ε_{F} are induced. This energy increase above ε_{F} lasts until the electron suffers an inelastic collision and loses its excess energy to the heat bath.

3. Dissipation in the corners of a Hall bar

In the interior of a Hall bar $\boldsymbol{E}$ is not constant. Far away from the contacts the equipotential lines are parallel to the direction of the bar, but they converge (in a radially symmetric way) towards two diagonally opposite corners. (This behaviour can be explained from Maxwell's equations and the appropriate boundary conditions [7].) Hence on a circle with radius r from such a corner point the field $\boldsymbol{E}(r)$ points in the azimuthal direction and its value varies as $1/r$. Consequently, there always exists a critical radius r_b, ,where $|\boldsymbol{E}(r)|$ equals E_b (≈ 100 V cm^{-1}), i.e. there is dissipation in the two domains ($r < r_b$) at the incoming and outgoing corners (where the electrons enter and leave the Hall bar) and (noticeable) dissipation is absent elsewhere in the bar.

At the perimeter of each of these (quarter) circles with radius r_b the IQHE begins to hold, i.e.

$$I = (\pi/2) r_b E_b (q^2/h) \times \text{integer}, \tag{2}$$

where I is the total current between source and drain. Relation (2) must hold equally well in the incoming and in the outgoing corner, and it shows that the radius r_b is linear in I. Such a linear relation has recently been observed [3]. However, the experiments [3] have also shown that (2) holds only for sufficiently low currents I; when I attains a critical value I_c, the symmetry between the two corners is suddenly lost. Here the helium drop (which is a measure for the total dissipation) at the incoming corner continues to grow linearly with I, but the drop at the outgoing corner rapidly shrinks to almost half of its size, and then it remains practically constant in size when I is further increased.

This asymmetry may have the following origin. Before an electron loses its accumulated energy it has travelled a distance l_{in} (the velocity $v[r(t)]$ in the corner integrated over the time τ_{in} between two inelastic events). Therefore, at the outgoing corner, a part of the excess energy is dissipated only in the three-dimensional contact outside the Hall bar. For this to be a possible mechanism l_{in} must be an appreciable fraction of r_b. In the corner, where $E < E_b$ one has $v \geq 10^5$ cm s^{-1}. Concerning τ_{in} we see that in a similar system the inelastic diffusion length L_{in}^d was measured to be 4.4×10^{-7} m at 1 K, while the diffusion constant D measured with a pulsed technique [8] was slightly less than 5×10^{-5} cm^2 s^{-2}, whence $\tau_{in} = {L_{in}^d}^2/D \approx 4 \times 10^{-9}$ s. This leads to l_{in} ($\approx v\tau_{in}$) ≥ 4 μm. This value fits into the proposed scenario. Note also that all the excess energy is transported out of the system by those electrons starting at a radius $r_{in} = l_{in}$. The radius r_{in} grows more rapidly with I than r_b. In addition, the highest dissipated power per unit surface is created within this radius r_{in}. All this leads to the decrease in dissipation in the total area inside r_b, but which becomes sensible only beyond a sufficiently high value of I, in consistency with the experiments.

In conclusion, we have explained the experimentally observed symmetric dissipation in the corners of a Hall bar, and proposed a tentative explanation of the asymmetric dissipation occurring above a critical current $I > I_c$.

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