Vol. 83 (1993) *ACTA PHYSICA POLONICA A* No. 2

ACOUSTIC WAVE PROPAGATION IN MERCURY IN CONSTANT EXTERNAL MAGNETIC FIELD

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(Received October 13, 1992; in final form November 27, 1992)

The paper gives a theoretical analysis of the effect of an external constant magnetic field on the propagation of ultrasonic waves in electrically conducting liquids as well as the results of measurements carried out in mercury. The theoretical part is based on Euler's equation, the equation of continuity, the thermodynamical equation, and Maxwell's equations. In the experimental part we propose and apply two methods for the measurement of the ultrasonic propagation velocity and its variations, as well as a pulse method perfected by the use of analog memory for the determination of the amplitude absorption coefficient. The correctness of the theoretical basis underlying the calculation of the small changes in propagation velocity induced by the magnetic field is confirmed by experiment. The amplitude absorption coefficient determined experimentally is considerably greater than that calculated theoretically for the medium studied.

PACS numbers: 43.35.+d

1. Introduction

A plane longitudinal acoustic wave propagating in an electrically conducting medium in an externally applied magnetic field generates an electric current which interacts with the field. The interactions in the medium give rise to mechanical force which opposes the particle motion.

If the magnetic induction vector $\boldsymbol{B}$ is at right angles to the propagation vector $\boldsymbol{k}$ of the wave, the current induced in the medium attains its maximum. This geometry of the experiment should ensure the best conditions of observation of the magnetohydrodynamical effect.

Our theoretical considerations show that the interactions between the two fields affect the complex propagation vector $\boldsymbol{k}$, leading to additional absorption as the result of eddy currents and velocity dispersion. Measurements in mercury, however, in spite of the very high sensitivity of the method (the accuracy reached about 4 mm/s), failed to disclose changes in the propagation velocity but permitted the observation of an increase in the absorption coefficient of the wave when the external magnetic field was switched on.

2. Theoretical analysis

The analysis of ultrasonic wave propagation in liquid conducting media in a magnetic field involves the use of Euler's equation of motion, the equation of continuity, Maxwell's equations, and the thermodynamical equation [1–3].

On neglecting laminar viscosity, temperature conductivity, and other dissipative processes, the equation of motion of the conducting medium in the magnetohydrodynamical approximation takes the following form:

$$\rho\frac{\partial \boldsymbol{v}}{\partial t} = -\nabla p + \boldsymbol{F}, \tag{1}$$

where $\boldsymbol{F} = \boldsymbol{j} \times \boldsymbol{B}$ is the Lorentz force per unit volume of the medium.

The other equations are these:

— the equation of continuity

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \cdot \boldsymbol{v}) = 0, \tag{2}$$

— the Maxwell equations

$$\nabla \times \boldsymbol{H} = \boldsymbol{j} + \frac{\partial \boldsymbol{D}}{\partial t}, \tag{3}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \tag{4}$$

— and the equation of state, restricted to its linear term

$$p' = \left(\frac{\partial p}{\partial \rho}\right)_s \rho' = c_\infty^2 \rho'. \tag{5}$$

Transforming Eqs. (1)–(4), taking into account the generalized Ohm law and a basic formula of vector calculus, we arrive at the following differential equation [1]:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) + \frac{1}{\sigma \mu_r \mu_0}\nabla^2 \boldsymbol{B}, \tag{6}$$

where $\boldsymbol{v}$ is the velocity of the elements of volume of the medium (acoustic velocity), $\boldsymbol{B}$ — the resultant vector of magnetic induction, σ — the specific electric conductivity of the medium, μ_0 — the magnetic permeability of vacuum, and μ_r — the coefficient of relative magnetic permeability of the medium. On the assumption of infinitely small amplitudes of the waves

$$\rho' \ll \rho_0, \quad \rho = \rho_0 + \rho',$$

$$p' \ll p_0, \quad p = p_0 + p',$$

Eqs. (1) and (2) reduce to

$$\rho_0\frac{\partial \boldsymbol{v}}{\partial t} = -\nabla p' + \boldsymbol{F}, \tag{7}$$

$$\frac{\partial \rho'}{\partial t} + \rho_0 \nabla \boldsymbol{v} = 0. \tag{8}$$

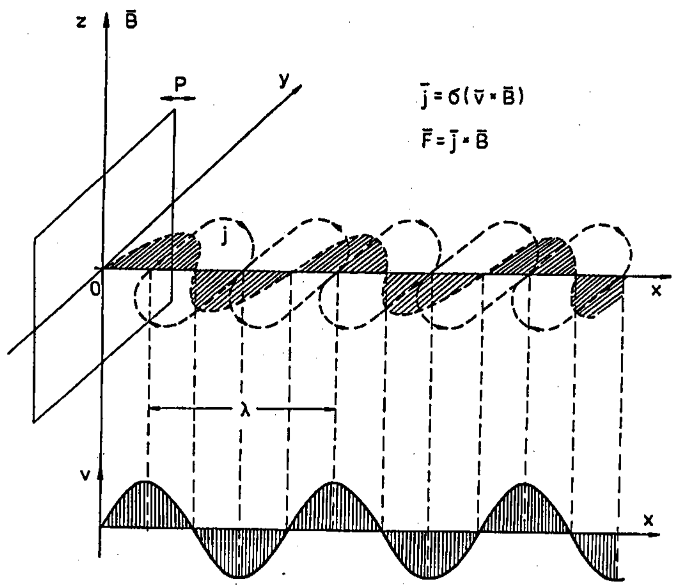

Fig. 1. The induction of eddy currents in an electrically conducting liquid medium in a strong external magnetic field at propagation of an ultrasonic wave in the medium.

Time derivation of the equation of state (5) and Euler's equation (7) with the expression (8) inserted leads to

$$\rho_0 \frac{\partial^2 \boldsymbol{v}}{\partial t^2} = c_\infty^2 \rho_0 \nabla^2 \boldsymbol{v} + \frac{\partial \boldsymbol{F}}{\partial t}. \tag{9}$$

Furthermore, if the magnetic field $\boldsymbol{B}_0$ is applied along the z-axis of coordinates (see Fig. 1) and if the plane ultrasonic wave propagates parallel to the x-axis, Eq. (9) can be rewritten in the form

$$\frac{\partial^2 v_x}{\partial t^2} = c_\infty^2 \frac{\partial^2 v_x}{\partial x^2} + \frac{1}{\rho_0} \frac{\partial F_x}{\partial t}, \tag{10}$$

where the force component F_x is given by the Lorentz formula and the first law of Maxwell as follows:

$$F_x = [\boldsymbol{j} \times \boldsymbol{B}]_x = \frac{1}{\mu_r \mu_0} [(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}]_x . \tag{11}$$

Consequently, the wave equation for the acoustic velocity v_x becomes

$$\frac{\partial^2 v_x}{\partial t^2} = c_\infty^2 \frac{\partial^2 v_x}{\partial x^2} + \frac{1}{\rho_0 \mu_r \mu_0} \frac{\partial}{\partial t} [(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}]_x . \tag{12}$$

Since the magnetic field existing within the acoustically excited medium is a superposition of the external field B_0 and the field b produced by the eddy currents, the resultant field is $\boldsymbol{B} = \boldsymbol{k}(B_0 + b)$, where $\boldsymbol{k}$ is the versor along the z-axis. Hence, inserting the respective field and velocity components into (6) we obtain

$$\frac{\partial}{\partial t} [\boldsymbol{k}(B_0 + b)] = \nabla \times [(v_x)\boldsymbol{i} \times \boldsymbol{k}(B_0 + b)] + \frac{1}{\sigma \mu_r \mu_0} \nabla^2 \boldsymbol{k}(B_0 + b). \tag{12a}$$

2.1. Ultrasonic wave propagation in perfectly conducting media

If the medium is assumed as perfectly conducting ($\sigma \approx \infty$), Eq. (12a) implies that

$$\frac{\partial b}{\partial t} = -B_0 \frac{\partial v_x}{\partial x}. \tag{13}$$

The vector product component occurring in (11) is now

$$[(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}]_x \approx -B_0 \frac{\partial b}{\partial x}. \tag{14}$$

Inserting Eqs. (13) and (14) into Eq. (12), we finally get the following wave equation for the velocity of an acoustic wave in a perfectly conducting medium upon action of an external magnetic field:

$$\frac{\partial^2 v_x}{\partial t^2} = \left(c_\infty^2 + \frac{B_0^2}{\rho_0 \mu_r \mu_0}\right) \frac{\partial^2 v_x}{\partial x^2}, \tag{15}$$

showing that, with increasing magnetic field strength, the propagation velocity of the wave increases and no dispersion intervenes

$$c_0^2 = c_\infty^2 + \frac{B_0^2}{\rho_0 \mu_r \mu_0}. \tag{15a}$$

2.2. Ultrasonic wave propagation in media with finite electric conductivity

The situation is radically different if the conductivity of the medium is finite. We now have

$$\frac{\partial b}{\partial t} = -B_0 \frac{\partial v_x}{\partial x} + \frac{1}{\sigma \mu_r \mu_0} \frac{\partial^2 b}{\partial x^2} \tag{16}$$

and Eq. (12) takes the form

$$\frac{\partial^2 v_x}{\partial t^2} = c_\infty^2 \frac{\partial^2 v_x}{\partial x^2} - \frac{B_0}{\rho_0 \mu_r \mu_0} \frac{\partial}{\partial x} \left[\frac{\partial b}{\partial t}\right]. \tag{17}$$

In order to derive the equation of dispersion the solution of which will permit the determination of the propagation velocity as well as the damping, one assumes the acoustic velocity v_x and the field b as harmonically variable

$$v_x = v_0 e^{j(\omega t - kx)}, \quad b = b_0 e^{j(\omega t - kx)}.$$

Inserting the appropriate partial derivatives of these quantities into (16) and (17), we arrive at the following set of equations:

$$\left(j\omega + \frac{k^2}{\sigma \mu_r \mu_0}\right) b - jkB_0 v_x = 0, \tag{18}$$

$$\frac{B_0}{\rho_0 \mu_r \mu_0} \omega k b + \left(c_\infty^2 k^2 - \omega^2\right) v_x = 0. \tag{19}$$

To solve it, we equate the determinant of its coefficients to zero and obtain the following algebraic dispersion equation, of the 4th degree in k:

$$k^4 - k^2 \left(\frac{\omega^2}{c_\infty^2} - j\frac{\omega \sigma \mu_0 \mu_r c_0^2}{c_\infty^2}\right) - \frac{j\sigma \mu_0 \mu_r \omega^3}{c_\infty^2} = 0. \tag{20}$$

This enables us to obtain a physically meaningful root of Eq. (20) in the form

$$k \approx \frac{\omega}{c_\infty}\sqrt{1-\frac{(c_0^2-c_\infty^2)c_0^2}{\beta^2\omega^2+c_0^4}}\left[1-\frac{\mathrm{j}(c_0^2-c_\infty^2)\beta\omega}{2(\beta^2\omega^2+c_0^4)}\right], \tag{21}$$

where

$$\beta = \frac{1}{\sigma\mu_\mathrm{r}\mu_0}.$$

The real part of (21) provides the following expressions for the propagation velocity of the wave and its variations:

$$c(\omega, B) = c_\infty \left[\frac{\left(\frac{\omega}{\sigma\mu_0\mu_\mathrm{r}}\right)^2 + \left(c_\infty^2 + \frac{B_0^2}{\rho\mu_\mathrm{r}\mu_0}\right)^2}{\left(\frac{\omega}{\sigma\mu_0\mu_\mathrm{r}}\right)^2 + c_\infty^2\left(c_\infty^2 + \frac{B_0^2}{\rho\mu_\mathrm{r}\mu_0}\right)}\right]^{0.5}, \tag{22}$$

$$\Delta c(B,\omega) = c_\infty\left[\left(\frac{(\sigma\mu_0\mu_\mathrm{r})^2\left(c_\infty^2+\frac{B_0^2}{\rho\mu_\mathrm{r}\mu_0}\right)^2+\omega^2}{\left(c_\infty^2+\frac{B^2}{\rho\mu_\mathrm{r}\mu_0}\right)c_\infty^2(\sigma\mu_0\mu_\mathrm{r})^2+\omega^2}\right)^{0.5}-1\right], \tag{23}$$

whereas the imaginary part gives its amplitude absorption coefficient

$$\alpha(\omega) = \frac{\omega^2\beta}{c_\infty}\sqrt{\frac{c_\infty^2c_0^2+\beta^2\omega^2}{\beta^2\omega^2+c_0^4}}\frac{(c_0^2-c_\infty^2)}{2(\beta^2\omega^2+c_0^4)}. \tag{24}$$

With the propagation velocity and amplitude absorption coefficient available, we get the absorption of the wave per wavelength

$$\mu(\omega) = \alpha(\omega)\lambda(\omega) = \frac{2\pi\alpha(\omega)c(\omega)}{\omega} = \frac{\pi\omega\beta(c_0^2-c_\infty^2)}{\beta^2\omega^2+c_0^4}. \tag{25}$$

It will be remembered that the maximum absorption per wavelength occurs for $\mathrm{d}\mu/\mathrm{d}\omega = 0$, that is for

$$\pi\beta(c_0^2-c_\infty^2)\frac{c_0^4-\omega^2\beta^2}{(\beta^2\omega^2+c_0^4)^2} = 0, \tag{26}$$

i.e.,

$$c_0^4 - \omega^2\beta^2 = 0. \tag{27}$$

This enables us to evaluate the frequency f_m for which absorption per wavelength becomes maximal

$$f_\mathrm{m} = \frac{c_0^2}{2\pi\beta} = \frac{\mu_0\mu_\mathrm{r}\sigma c_\infty^2}{2\pi} + \frac{\sigma B_0^2}{2\pi\rho}. \tag{28}$$

We give an evaluation of f_m for mercury — a liquid on which we carried out measurements applying a magnetic field — assuming the following values for the quantities occurring in experiment

$$B_0 = 1\ \mathrm{T},\quad \rho = 13551\ \mathrm{kg/m^3},\quad \sigma = 1.0438\times10^6\ \mathrm{S/m},$$

$$c_\infty = 1451\ \mathrm{m/s},\quad \mu_\mathrm{r}\approx 1,\quad \mu_0 = 4\pi\times10^{-7}\ \mathrm{H/m}.$$

We obtained $f_m = 440126.12 + 12.25$ Hz $= 440138.37$ Hz. Thus, a magnetic field applied to a liquid diamagnetic causes but an insignificant shift of the absorption maximum of the wave towards higher frequencies.

Inserting (28) into (25), we obtain an expression for the additional absorption per wavelength at f_m caused by the magnetic field

$$\mu_m = \frac{\pi}{2}\left(\frac{B_0^2}{\rho\mu_r\mu_0 c_\infty^2 + B_0^2}\right). \tag{29}$$

Using the same numerical values as above we get

$$\mu_m = \frac{\pi}{2(13551 \cdot 4\pi \cdot 10^{-7} \cdot 1451^2 + 1)} = 0.0000436.$$

Likewise, on inserting (28) into (24), we obtain for the amplitude coefficient of additional absorption of the wave at f_m in the presence of the constant magnetic field

$$\alpha(\omega_m) = \frac{\sigma B_0^2\sqrt{c_0^2 + c_\infty^2}}{4\sqrt{2}\rho c_0 c_\infty}. \tag{30}$$

With the above numerical values we find $\alpha(\omega_m) = 0.01327$ m^{-1}.

However, according to the expression (23), the evaluation of the change in propagation velocity of the ultrasonic wave on application of an external magnetic field to mercury under the same conditions should lead to $\Delta c(f_m) = 0.01$ m/s.

3. Phase methods for measurements of the ultrasonic wave velocity

Theoretical studies [4, 5] on the influence of a static magnetic field on the velocity of ultrasonic waves in a conducting fluid predict extremely small changes in velocity. For this reason we adopted and perfected two phase methods permitting measurements of the absolute value as well as the variations of the ultrasonic propagation velocity [7, 9, 10, 12, 16]. While measuring the absolute velocity, the change in phase (in a cell of constant length l) was obtained by varying the frequency of the wave. The velocity was determined from the recorded change in frequency and the corresponding change in phase. On the other hand, the variations in propagation velocity of the ultrasonic wave induced by the external magnetic field (at constant length l of the cell and constant frequency f of the wave) were determined on the basis of the changes in phase recorded.

Apparatus of this type is especially useful for measurements of acoustic quantities in a medium in a magnetic field acting in a volume restricted by the small distance between the poles of the electromagnet.

We attained a high degree of accuracy by appropriate adapting the component elements and applying a digital phase detector. However, in our measurements involving the presence of a magnetic field we applied the pulse method; here, we used a laboratory setup from "Matec Instruments" supplemented with an additional measuring block with analog memory admitting of high accuracy.

3.1. The experimental setup

By applying a phase detector at the output receiver, as shown in Fig. 2, very good results were obtained when measuring the absolute value of the propagation velocity in liquid medium. Two signals are fed to the phase detector, one of which comes from the reference branch. The propagation velocity hence determined (for any intermediate frequency) amounts to

$$c = 2\pi l \frac{\Delta f}{\Delta \phi}. \tag{31}$$

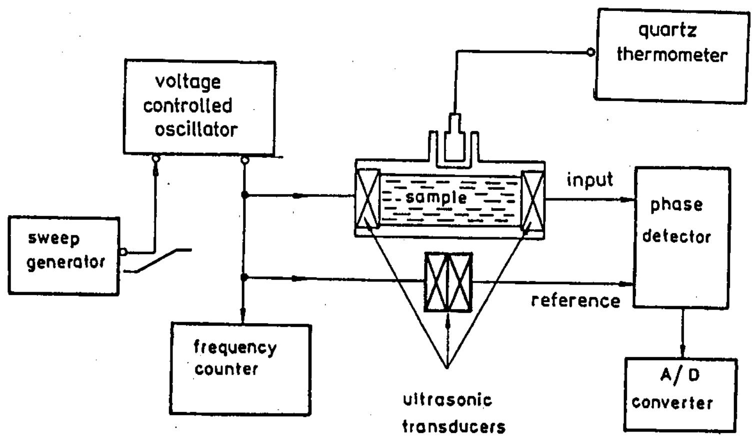

Fig. 2. The measuring setup used for the determination of the absolute value of ultrasonic wave propagation by the phase method, with tunable continuous wave.

Obviously, in order that c shall be measured with good accuracy, it is necessary that Δf and $\Delta \phi$ were related linearly. This condition is fulfilled only if the wave propagating in the cell is a running wave and if the liquid exhibits no dispersion. The phase detector used by us is a circuit constructed with digital technique, having an ideally linear characteristic.

In reality, however, certain undesirable physical processes not accounted for by Eq. (31) intervene in the measurements. They are due, among others, to multiple reflection at both ends of the cell [10]. To eliminate reflection (to ensure that only the running wave shall reach the receiver transducer) we inserted a system of diaphragms 4 mm thick in the measuring cell [7, 8].

3.2. Determinations of the changes in velocity

For the measurement of small changes in ultrasonic wave propagation velocity in liquid media we applied the setup shown in Fig. 3. The setup is based on phase detection for the wave propagating in the medium, the properties of which vary under the influence of external factors. To the two (the measuring and reference) inputs of the digital phase detector we fed the signal from the medium

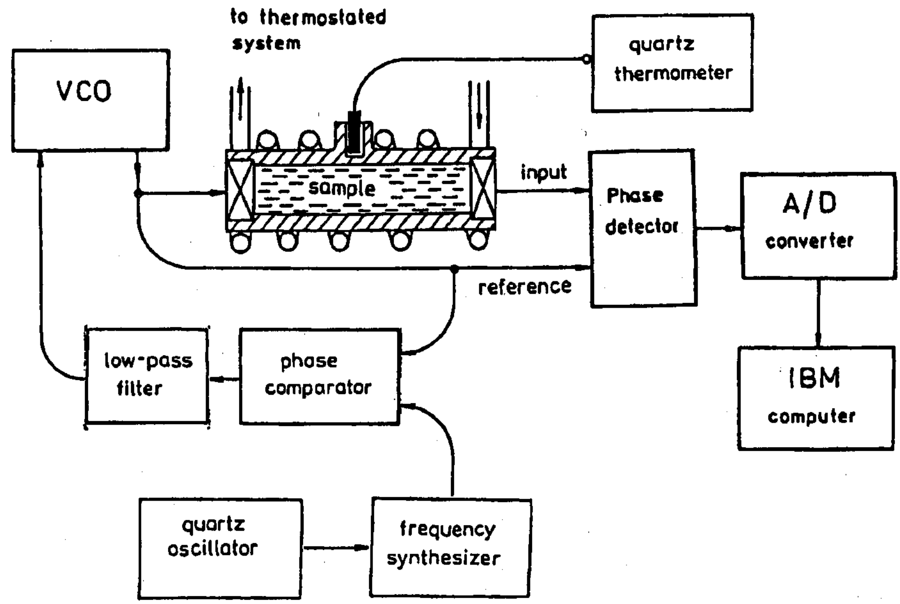

Fig. 3. Setup for the determination of changes in ultrasonic propagation velocity by the phase method.

and, respectively, that one coming directly from the voltage controlled oscillator high-stability generator.

The recording of the changes in velocity was carried out by comparing the phase of the measuring signal with that of the reference signal.

If, under conditions of thermodynamical equilibrium, the absolute value of the propagation velocity c_0 in the liquid is

$$c_0 = \frac{2\pi f l}{\phi_0}, \tag{32}$$

the corresponding phase ϕ_0 of the running wave is given by

$$\phi_0 = \frac{2\pi f l}{c_0}. \tag{33}$$

Under the action of the external factors the two quantities undergo variations and take the following values:

$$c_1 = c_0 + \Delta c = \frac{2\pi f l}{\phi_1}, \tag{34}$$

$$\phi_1 = \phi_0 + \Delta\phi_1 = \frac{2\pi f l}{c_1}. \tag{35}$$

With regard to Eqs. (33), (34) and (35), the change in velocity Δc is obtained in the form

$$\Delta c = -\frac{\Delta\phi_1 c_0}{\phi_0 + \Delta\phi_1}, \tag{36}$$

where ϕ_0 is the phase of the acoustic wave under stationary thermodynamical conditions in the measuring cell of length l, and $\Delta\phi_1$ is the change in phase caused by external factors.

3.3. Ultrasonic absorption coefficient measurements by the pulse method

In order to ascertain whether the external magnetic field affects the amplitude absorption coefficient of mercury, we proceeded to an experiment applying the pulse method. A block diagram of the setup, from "Matec Instrument", is shown in Fig. 4. Aiming at further enhancement of the accuracy we supplemented

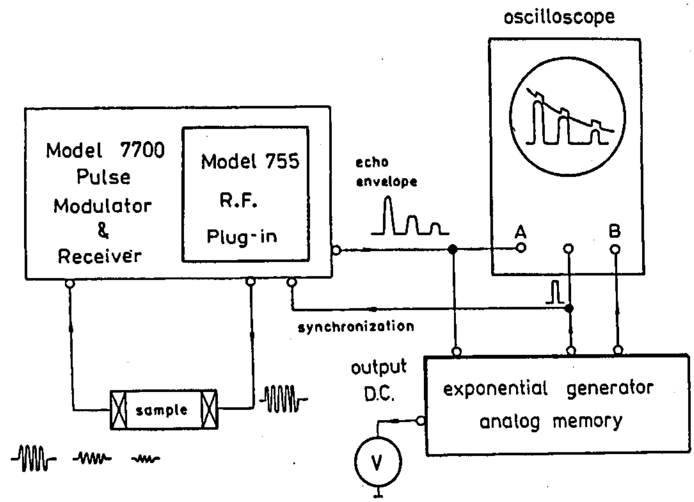

Fig. 4. Block diagram of the setup for measuring the absorption coefficient of ultrasonic waves in mercury.

the device with a circuit for the measurement of the amplitude of repeated pulses with analog memory [11, 14]. This enabled us to measure the pulses with usual digital voltmeters, permitting to achieve conveniently an accuracy of $1^\circ/_{00}$. The vanishing transient synchronized with the measured pulses was well adapted to visual comparison thus enabling us to dispose the two transducers strictly in parallel. This is obtained as soon as the peaks of successive pulses come into contact with the curve of exponential decay.

4. The results and analysis

We first determined the absolute value of the ultrasonic propagation velocity c_0 by the phase method with tunable frequency, at a fixed temperature in the absence of a magnetic field.

Figure 5 shows the graph of the velocity versus temperature, at the frequency $f = 2$ MHz maintained constant, for temperatures ranging from $-20°$C to $+25°$C by steps of 5°C.

We then measured the change in velocity Δc in the liquid under the action of an external magnetic field applied at right angles to the direction of propagation of the acoustic oscillations. The measurements were carried out for frequencies of 1, 2 and 4 MHz.

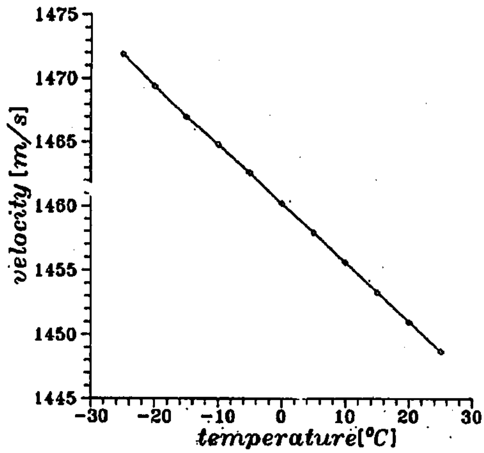

Fig. 5. Ultrasonic propagation velocity in mercury versus temperature, for $f = 2$ MHz.

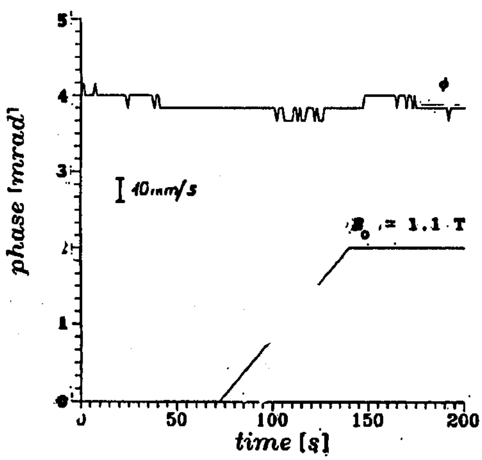

Fig. 6. The change in ultrasonic propagation velocity in mercury recorded on application of a magnetic field of $B = 1.1$ T, for $f = 4$ MHz and $T = 20°$C.

Figure 6 exemplifies the time evolution of the change in phase in mercury at $T = 20°$C, in a field of $B = 1.1$ T. The graphs obtained would appear to show that an external magnetic field causes no perceptible changes in the velocity of ultrasonic waves propagating in mercury.

Our measurements of the effect of the magnetic field on the absorption coefficient show that, in a diamagnetic medium, damping increases when the field is switched on (these measurements were performed without diaphragms with regard to the use of pulse technique).

Figure 7 shows an oscilloscopic picture of the ultrasonic pulses obtained in mercury at a frequency of $f = 4$ MHz and $T = 20°$C. On applying the magnetic field of strength 1.1 T the amplitude of the first pulse recorded decreased by 0.18 V. Prior to applying the field its amplitude amounted to $U_{10} = 3.62$ V, whereas immediately after application of the field it fell to $U_{1B} = 3.44$ V.

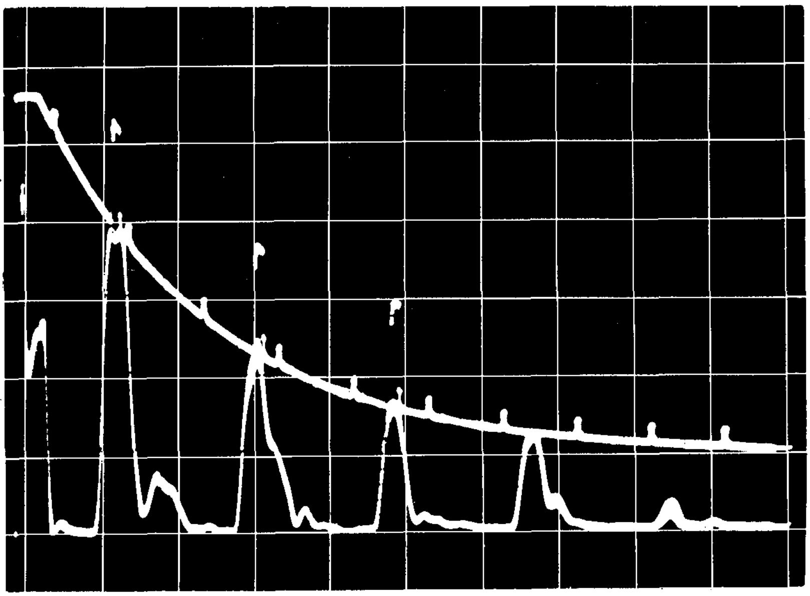

Fig. 7. Picture of the ultrasonic pulses, obtained on the oscilloscope screen, in the absence of magnetic field.

With regard to the distance $l = 2.7$ cm between the transducers, we thus obtained for the change in absorption coefficient $\Delta\alpha = 0.02$ cm^{-1} and for the change in absorption per wavelength $\Delta\alpha\lambda = 7.255 \times 10^{-4}$.

5. Conclusions

The absolute value of the ultrasonic propagation velocity c_0 in mercury obtained by the phase method with tunable frequency was found to be in agreement with the data reported in the literature [12, 13]. The slight differences do not exceed experimental error.

Theoretical considerations concerning anisotropy in the ultrasonic propagation velocity in electrically conducting liquids in an external static magnetic field point to an effect that is maximal for $\alpha = 90°$ between the wave and the field.

However, our measurements of the influence of the field on the propagation velocity in diamagnetic mercury performed at $\alpha = 90°$ failed to detect perceptible changes in velocity (theoretically of order 8 mm/s for 1 MHz). This result is in agreement with the theory, according to which the ultrasonic propagation velocity in electrically conducting *diamagnetic* liquids should not vary perceptibly on application of a magnetic field.

On application of the field we observed a decrease in amplitude of the pulses recorded, pointing to an increase in absorption of the ultrasonic wave. These additional losses can be due to the evolution of heat in mercury resulting from the eddy currents.

The comparison of the values of $\Delta\alpha$ and $\Delta\alpha\lambda$ obtained by us in the theoretical part of the present work and those reported in the experimental part leads to the conclusion that, in reality, the absorption of an ultrasonic wave propagating in an electrically conducting liquid in an external magnetic field is greater than that predicted by the theory.

It is to be presumed that the effect of a magnetic field on the propagation of ultrasonic waves will be much greater in the case of *magnetic liquids*, with regard to their specific properties. However, in this case, the theoretical approach to the problem will have to be quite different [6, 15].

Acknowledgment

The author wishes to express his indebtedness to Professor M. Łabowski for his helpful discussions throughout the present investigation.

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