Vol. 81 (1992) *ACTA PHYSICA POLONICA A* No 4–5

CARRIER-INFLUENCED BISTABLE BEHAVIOUR IN RING CAVITY CONFIGURATION

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(Received October 2, 1991)

Bistable output–input light characteristics in the exciton spectral region have been examined in a ring cavity configuration taking ito account the influence of the presence of free carriers in the cavity as well as of the density-dependence of the exciton damping. The resulting modifications could serve as a means to optimize the optical bistable device operation.

PACS numbers: 42.65.Po; 78.20.Dj

1. Introduction

The interest in the phenomenon of optical bistability (hereafter called simply OB) has grown very rapidly over the last years. OB is defined as a presence of two stable states of an optical system under one and the same stationary irradiation. The simplest example of such a device is a Fabry–Perot resonator filled with a material whose refractive index is intensity-dependent. The possibility of OB was first theoretically predicted by Szoke et al. [1]. In semiconductors OB was observed experimentally for the first time by Miller et al. [2]. It is now well-known that OB results from the interplay between nonlinearity and feedback. Up to date, various kinds of optical nonlinearity and feedback, both with and without resonator, have been proposed and investigated in detail in a great deal of works

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[3–8]. In all these works, however, the authors have not yet considered the influence of the free carriers that usually exist in real experimental conditions on the phenomenon of OB. In fact, free carriers play an important role and may cause serious effects in many physical processes in highly excited materials. The authors of [9] and [10] accounted for the contribution of free electron–hole pairs to the one-photon absorption and luminescence in the spectral region of P-zone, respectively. In [11] the exciton–electron and exciton–hole interactions were taken into account in the phase transitions for the excitonic excitations in semiconductors. And, quite recently the authors of e.g. [12] and one of us [13] have paid attention to the influence of free carriers on the turbulence and chaotic behaviour of dynamical systems. Our group itself has just dealt with the intrinsic excitonic OB in an exciton–electron–hole system [14, 15] and shown that both the holding intensity and the hysteresis loop size of the OB are decreased for increasing electron–hole pair concentration. Suggestions based on the theory of the OB are also made in [15] for possible experimentally measuring the exciton-carrier coupling constant.

In this paper we would like to present a theoretical treatment of the OB in a ring cavity filled with an excited semiconductor containing sample a finite concentration of free electron–hole pairs. As we shall show, the changes of the OB picture induced by the presence of free carriers and by the dependence of the exciton damping on the quasi-particle density might be used for better operating bistable devices.

For convenience we shall utilize throughout the paper the unit system with $\hbar = c = V = 1$, where $\hbar$, c, and V are the Planck constant, the velocity of light and the sample volume, respectively.

2. Basic equations

Let us consider a ring cavity (see Fig. 1 and its description later) which is anticipated to contain a number of free electron–hole pairs whose origin of appearance is not specified. Supposing now that the cavity is irradiated by an

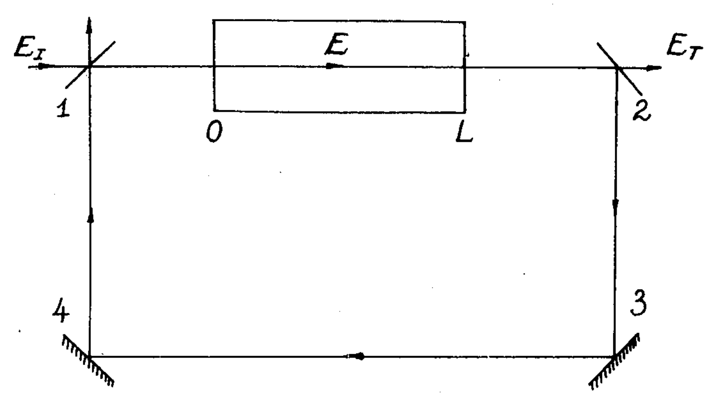

Fig. 1. Ring cavity configuration.

externally driving laser field with given wave vector $\boldsymbol{k}$, complex amplitudes $E_{\boldsymbol{k}}^{(\pm)}$ and frequency Ω_k. The laser frequency is assumed to be in resonance with the exciton energy so that the exciton will be generated via the photoabsorption. These excitons also interact between themselves and with the carriers. The carriers, however, are coupled only one to another and to the excitons but not to the laser field because the latter is resonant with the excitonic transition but not with the band-to-band one. The Hamiltonian describing the system under consideration then can be written as

$$H = H_1 + H_2, \tag{1}$$

where

$$H_1 = \sum_{p} \omega_{\mathrm{x}}(p) a_{p}^{+} a_{p} - g_k (E_{\boldsymbol{k}}^{(-)} \mathrm{e}^{-\mathrm{i}\Omega_k t} a_{\boldsymbol{k}}^{+} + \mathrm{h.c.}$$
$$+ \sum_{p,q,l} \left[\frac{1}{2} \nu_{\mathrm{x-x}}(p,q,l) a_{p+l}^{+} a_{q-l}^{+} a_p a_q \right.$$
$$\left. + \nu_{\mathrm{x-e}}(p,q,l) a_{p+l}^{+} e_{q-l}^{+} e_p a_q + \nu_{\mathrm{x-h}}(p,q,l) a_{p+l}^{+} h_{q-l}^{+} h_p a_q \right] \tag{2}$$

and

$$H_2 = \sum_{p} \omega_{\mathrm{e}}(p) e_{p}^{+} e_{p} + \sum_{p} \omega_{\mathrm{h}}(p) h_{p}^{+} h_{p}$$
$$+ \frac{1}{2} \sum_{p,q,l} V_l \left(e_{p+l}^{+} e_{q-l}^{+} e_p e_q + h_{p+l}^{+} h_{q-l}^{+} h_p h_q - 2 e_{p+l}^{+} h_{q-l}^{+} h_p e_q \right). \tag{3}$$

In Eqs. (2) and (3) $a_{p}^{+}(a_p)$, $e_{p}^{+}(e_p)$ and $h_{p}^{+}(h_p)$ are boson operators for excitons and fermion operators for electrons and holes with wave vector (or momentum) p and frequency (or energies) $\omega_{\mathrm{x}}(p)$, $\omega_{\mathrm{e}}(p)$ and $\omega_{\mathrm{h}}(p)$, respectively. g_k is the exciton-light coupling factor. $\nu_{\mathrm{x-x}}(p,q,l)$, $\nu_{\mathrm{x-e}}(p,q,l)$ and $\nu_{\mathrm{x-h}}(p,q,l)$ are the exciton–exciton, exciton–electron and exciton–hole interaction potentials, whose analytic expressions can be found e.g. in [11]. V_l is the Coulomb potential. The light field can be represented in terms of its positive and negative frequency parts

$$E_{\boldsymbol{k}} = E_{\boldsymbol{k}}^{(+)} \mathrm{e}^{+\mathrm{i}\Omega_k t} + E_{\boldsymbol{k}}^{(-)} \mathrm{e}^{-\mathrm{i}\Omega_k t}. \tag{4}$$

The problem of OB in the system can be solved semi-classically by utilizing the equation of motion of the $\boldsymbol{k}$-mode exciton operator and the wave equation for the light field. These equations have the forms

$$\mathrm{i}\frac{\partial a_{\boldsymbol{k}}}{\partial t} = (\omega_{\mathrm{x}}(k) - \mathrm{i}\gamma_k) a_{\boldsymbol{k}} - g_k E_{\boldsymbol{k}}^{(-)} \mathrm{e}^{-\mathrm{i}\Omega_k t}$$
$$+ \frac{1}{2} \sum_{p,q} \nu_{\mathrm{x-x}}(p,q,k-p) a_{p+q-k}^{+} a_q a_p$$
$$+ \sum_{p,q} \left[\nu_{\mathrm{x-e}}(p,q,k-p) e_{p+q-k}^{+} e_q a_p + \nu_{\mathrm{x-h}}(p,q,k-p) h_{p+q-k}^{+} h_q a_p \right] \tag{5}$$

and

$$\frac{\partial^2 E_{\boldsymbol{k}}^{(-)}}{\partial x^2} - \frac{\partial^2 E_{\boldsymbol{k}}^{(-)}}{\partial t^2} = 4\pi g_k \frac{\partial^2 a_{\boldsymbol{k}}}{\partial t^2}, \tag{6}$$

where γ_k is phenomenologically introduced damping of the $\boldsymbol{k}$-mode exciton. The complex amplitudes of the macroscopic field can be represented as follows:

$$E_{\boldsymbol{k}}^{(\pm)} = E\mathrm{e}^{\mp(\mathrm{i}kx+\mathrm{i}\varphi(x,t))}, \tag{7}$$

in which φ and E are real functions of the space x and time t. Moreover, in the slowly varying approximation the field amplitude E is a function which alters in time and space much more slowly as compared to the phase of the wave $-\mathrm{i}\Omega_k t + \mathrm{i}kx + \mathrm{i}\varphi(x,t)$. This approximation which will be applied in this paper can be formulated by the following inequalities:

$$\left|\frac{\partial E}{\partial x}\right| \ll k|E|; \qquad \left|\frac{\partial E}{\partial t}\right| \ll \Omega|E|. \tag{8}$$

Equation (5) after averaging over the states of the system and invoking to the Hatree–Fock approximation becomes (see more details in [14])

$$\mathrm{i}\frac{\partial\langle a_{\boldsymbol{k}}\rangle}{\partial t} = [\omega_{\mathrm{x}}(k) + \nu N^{\mathrm{ex}} + \mu N^{\mathrm{e}} - \mathrm{i}\gamma_k]\langle a_{\boldsymbol{k}}\rangle - g_k E_{\boldsymbol{k}}^{(-)}\mathrm{e}^{-\mathrm{i}\Omega_k t}. \tag{9}$$

For the stationary regime we look for a particular solutions of Eq. (9) in the form

$$\langle a_{\boldsymbol{k}}\rangle = A_{\boldsymbol{k}}\mathrm{e}^{-\mathrm{i}\Omega_k t+\mathrm{i}kx+\mathrm{i}\varphi(x,t)} \tag{10}$$

with $A_{\boldsymbol{k}}$ being the quantity to be determined. Substituting Eq. (10) into Eq. (9) we get

$$-g_k E = [(\nu N^{\mathrm{ex}} + \mu N^{\mathrm{e}} - \Delta_k) - \mathrm{i}\gamma_k]A_{\boldsymbol{k}}, \tag{11}$$

where $N^{\mathrm{ex}} = \sum_{\boldsymbol{q}}\langle a_{\boldsymbol{q}}^{+}a_{\boldsymbol{q}}\rangle = \sum_{\boldsymbol{q}} N_{\boldsymbol{q}}^{\mathrm{ex}}$ is the exciton density of all possible modes in the system and $\Delta_k = \Omega_k - \omega_{\mathrm{x}}(k)$ is frequency detuning. In all calculations above, following [11] we have used the approximative values of the various coupling constants as $\nu_{\mathrm{x-x}}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{l}) \approx \nu_{\mathrm{x-x}}(0,0,0) \equiv \nu = 52\pi I_{\mathrm{x}} a_{\mathrm{x}}^3/3$ (I_{x} and a_{x} are the exciton binding energy and Bohr radius) and $\nu_{\mathrm{x-e}}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{l}) \approx \nu_{\mathrm{x-h}}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{l}) \approx \nu_{\mathrm{x-e}}(0,0,0) = \nu_{\mathrm{x-h}}(0,0,0) \equiv \mu/2 = 24\pi I_{\mathrm{x}} a_{\mathrm{x}}^3$ and have assumed that the electron density $N^{\mathrm{e}} = \sum_{\boldsymbol{q}}\langle e_{\boldsymbol{q}}^{+}e_{\boldsymbol{q}}\rangle = \sum_{\boldsymbol{q}} N_{\boldsymbol{q}}^{\mathrm{e}}$ and the hole density $N^{\mathrm{h}} = \sum_{\boldsymbol{q}}\langle h_{\boldsymbol{q}}^{+}h_{\boldsymbol{q}}\rangle = \sum_{\boldsymbol{q}} N_{\boldsymbol{q}}^{\mathrm{h}}$ are equal. Noticing further that $N_{\boldsymbol{k}}^{\mathrm{ex}} = |A_{\boldsymbol{k}}|^2$ is the density of the $\boldsymbol{k}$-mode excitons which are governed directly by the monochromatic classical light field and thus behave as a coherent and macroscopically occupied mode leading approximately to $N_{\boldsymbol{k}}^{\mathrm{ex}} \approx N^{\mathrm{ex}}$ we may cast Eq. (11) into

$$g^2E^2 = [(\nu N^{\mathrm{ex}} + \mu N^{\mathrm{e}} - \Delta)^2 + \gamma^2]N^{\mathrm{ex}}, \tag{12}$$

where and onwards the index $\boldsymbol{k}$ is not written anymore for brevity.

At this moment it is worth mentioning that the exciton damping γ introduced "by hand" when deriving the equation of motion (5) is so far a constant, i.e. independent of any parameters. In fact, due to the many-body nature of the problem γ, as a rule, must depend on the system parameters. Most naturally it should be a function of, say, exciton density as well as electron and hole ones because these quasi-particles are coupled to each other and their interactions of course determine their dephasing lifetimes. A microscopic approach using Feynman diagrams beyond the Hartree–Fock approximation should yield explicitly analytic expressions for the density-dependent exciton damping via the imaginary part of the exciton self-energy Σ_X. Here, however, for simplicity we just touch the question in a phenomenological manner by considering the various couplings between the quasi-particles as complex quantities instead of real ones. That means we shall add imaginary parts to ν and μ to have $\nu \to \nu - i\chi_1$ and $\mu \to \mu - i\chi_2$ with χ_1 and χ_2 being responsible for the dependence of the exciton damping on exciton- and electron–hole pair densities, respectively. After doing so, Eq. (12) gets a more general form

$$[(\nu N^{ex} + \mu N^e - \Delta)^2 + (\gamma + \chi_1 N^{ex} + \chi_2 N^e)^2] N^{ex} = g^2 E^2. \tag{13}$$

Since Eq. (13) is a cubic (with respect to N^{ex}) equation, it might exhibit optical bistability under certain conditions. Obviously, if we put $\nu = 0$ and $\chi_1 = 0$ in Eq. (13), i.e. if we neglect the exciton–exciton interaction, Eq. (13) becomes a linear one and no multistability can appear. This reveals that it is namely the exciton–exciton interaction what plays the decisive role in generating OB in our problem. Physically, such an interaction induces an energy shift (blue or red depending on the sign of ν) and/or a spectral broadening of the exciton energy level that cause discontinuous exciton density jumps when the light intensity is sweeping back and forth forming thus a density gap which corresponds to the unstable solution of Eq. (13). For convenience some normalized dimensionless quantities are introduced which are proportional to exciton density ($\tilde{N}$), carrier density (ρ) and frequency detuning (δ)

$$\tilde{N} = \frac{\nu N^{ex}}{\gamma}; \qquad \rho = \frac{\mu N^e}{\gamma}; \qquad \delta = \frac{\Delta}{\gamma}. \tag{14}$$

Inserting (14) into (13) gives

$$[(\tilde{N} + \rho - \delta)^2 + (1 + \xi_1 \tilde{N} + \xi_2 \rho)^2] \tilde{N} = g^2 E^2, \tag{15}$$

where $\xi_1 = \chi_1/\nu$ and $\xi_2 = \chi_2/\mu$. Then, from Eq. (5) and Eq. (6) combined with Eq. (15) we can easily obtain the following equation for the field amplitude E:

$$\frac{\partial E}{\partial x} = -\frac{2\pi \Omega_k^2 \gamma^2}{k} \frac{\tilde{N}(1 + \xi_1 \tilde{N} + \xi_2 \rho)}{E}. \tag{16}$$

Equations (15) and (16) are basic equations for further consideration of the phenomenon of OB in a ring cavity taking into account the existence of free carriers and the nonlinearity originated from the density-dependent exciton damping.

3. Ring cavity configuration

In this section we shall study the carrier-influenced optical bistability in a ring cavity configuration within the framework of the mean field approximation [17]. Such a framework allows us to obtain the light-light kind of the bistable behaviour (for light-density and light-light kinds of OB see e.g. [18]).

Let us denote by L the length of a resonator confined between two identical mirrors 1 and 2 whose reflection and transmission coefficients are respectively labelled by R and T. These mirrors are assumed lossless, so that

$$R + T = 1. \tag{17}$$

Together with two other 100% reflection mirrors 3 and 4 the configuration drawn in Fig. 1 forms the so-called ring cavity which we shall concern in this paper. It is worth to remind the well-known boundary conditions [17] for the field amplitudes at points $x = 0$ and $x = L$ of the resonator which look as

$$E(L) = \frac{E_{\rm T}}{\sqrt{T}}; \qquad E(0) = \sqrt{T}E_{\rm I} + RE(L), \tag{18}$$

where $E(L) = E(x = L)$, $E(0) = E(x = 0)$ and $E_{\rm I}$, $E_{\rm T}$ are amplitudes of the incident and transmited fields. We again introduce the normalized dimensionless input $E_{\rm int}$ and output $E_{\rm out}$ field amplitudes as follows:

$$E_{\rm out} = \frac{E_{\rm T}}{E_{\rm S}\sqrt{T}}; \quad E_{\rm int} = \frac{E_{\rm I}}{E_{\rm S}\sqrt{T}}; \quad \tilde{N}_L = \tilde{N}(x = L); \quad E_{\rm S}^2 = \frac{\gamma^3}{\nu g^2}. \tag{19}$$

Now integrating Eq. (16) from 0 to L within the mean field approximation, we have two state equations for the system which read

$$E_{\rm int} = E_{\rm out} + 2C\frac{\tilde{N}_L}{E_{\rm out}}(1 + \xi_1\tilde{N}_L + \xi_2\rho), \tag{20}$$

$$E_{\rm out}^2 = [(\tilde{N}_L + \rho - \delta)^2 + (1 + \xi_1\tilde{N}_L + \xi_2\rho)^2]\tilde{N}_L, \tag{21}$$

where C is a parameter of the theory and defined as

$$C = \frac{\alpha L}{4T} \tag{22}$$

with

$$\alpha = \frac{4\pi\Omega^2 g^2}{k\gamma}. \tag{23}$$

Transparently, if we ignore the existence of the carriers as well as the density-dependence of the exciton damping, Eqs. (20) and (21) are exactly reduced to Eqs. (4.3) and (4.4) in [8]. On the other hand, the case of $\nu = \mu = \chi_2 = 0$ recovers that in [16]. Thus our consideration is more general than the others. In general, the conditions for OB to occur, i.e. for $E_{\rm out}$ to be three-valued function of $E_{\rm int}$ will look very complicated. Nevertheless, in the simplest situation when $\delta = \rho$ and $\xi_1 = 0$ the above-mentioned condition is found to be very simple: OB will take place when the parameter C is greater than a certain critical value $C_{\rm CR}$, namely

$$C > C_{\rm CR} = (5 + 2\sqrt{6})(1 + \xi_2\rho). \tag{24}$$

Since ρ (the presence of the cariers) and ξ_2 (the dependence of the exciton damping on the carrier density) enter the condition for the occurrence of OB (see Eq. (24)), it is expected that the operation of a bistable device could be controlled by beforehand preparing the sample with an appropriate carrier concentration. For controlling one should keep in mind that the condition (24) is valid only for a particular case of $\delta = \rho$ and $\xi_1 = 0$. The most general conditions for arbitrary parameters entering Eqs. (20) and (21) being very complicated are not written down here. For plotting, however, we can directly handle the system of Eqs. (20) and (21) numerically.

In order to see how the presence of the carriers modifies the OB characteristics we have drown in Figs. 2 and 3 the output-input light dependence for several choices of the parameters. Fig. 2 corresponds to $C = 30, \xi_1 = \xi_2 = \delta = 0$ and $\rho = 0$

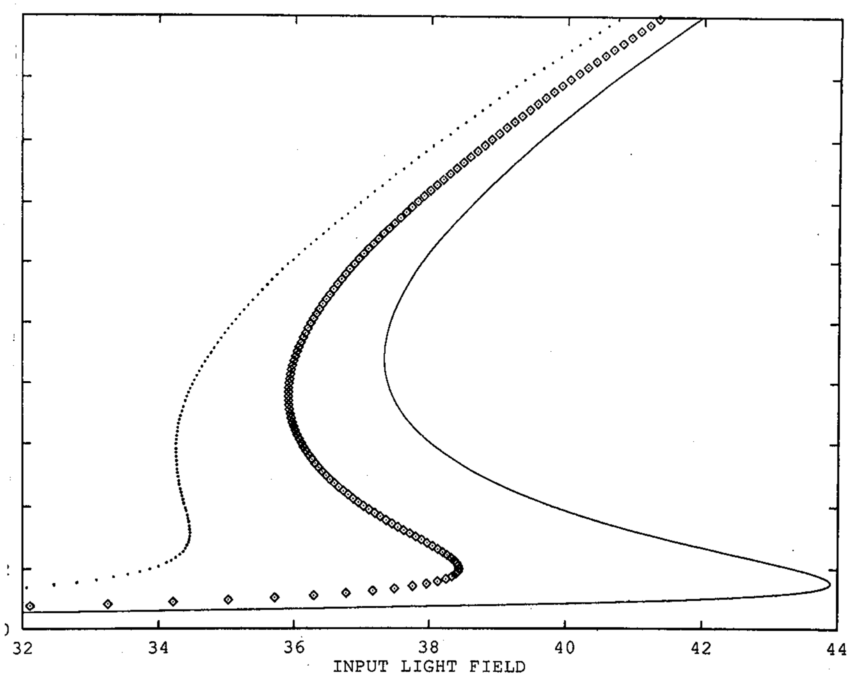

Fig. 2. The output light field E_{out} versus the input light field E_{int} with $C = 30$, $\xi_1 = \xi_2 = 0$, $\delta = 0$ and $\rho = 0$ (solid curve); $\rho = 0.3$ (curve with squares) and $\rho = 0.6$ (curve with dots).

(solid curve), $\rho = 0.3$ (curve with squares) and $\rho = 0.6$ (curve with dots), while in Fig. 3 we choose $C = 30$, $\rho = 0.2$, $\delta = 0$ and different ξ_i such as $\xi_1 = \xi_2 = 0$ (solid curve), $\xi_1 = \xi_2 = 0.025$ (curve with squares) and $\xi_1 = \xi_2 = 0.05$ (curve with dots). As it is clear from the figures, the increase in the carrier concentration removes the domain within which the optical bistability occurs (hereafter referred shortly to as OB domain) to the low field side (see Fig. 2). On the other hand, the increase in the dependence of the exciton damping on the carrier/exciton density shifts the OB domain to the high field side (see Fig. 3). These two opposite influences might be properly combined by adjusting carrier concentration and by selecting materials with necessary density-dependent exciton damping towards possible optimizations of the operation of bistable devices.

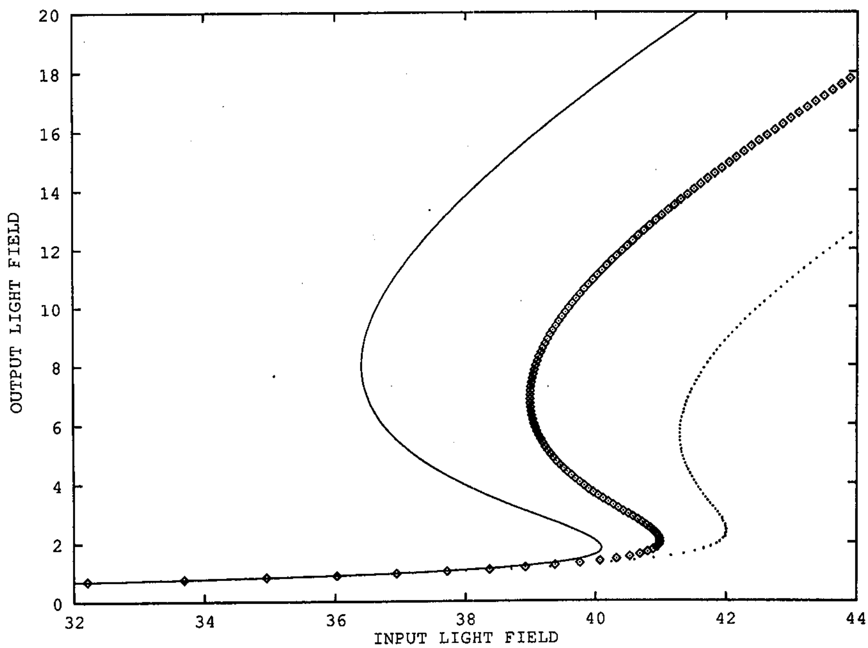

Fig. 3. The same as in Figure 2 but $\rho = 0.2$ and $\xi_1 = \xi_2 = 0$ (solid curve), $\xi_1 = \xi_2 = 0.025$ (curve with squares) and $\xi_1 = \xi_2 = 0.05$ (curve with dots).

4. Conclusion

We have investigated the bistable behaviour of the output light versus input one in a ring cavity configuration in the vicinity of the exciton resonance taking simultaneously into account the existence of free electron–hole pairs in the cavity and the nonlinearity due to the density-dependent exciton damping. We do hope the results obtained, though only qualitative, could be more or less helpful also for technologists on the way to construct the optical computer which is based on the principle of optical bistable devices.

Acknowledgments

One of the authors (NBA) would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for the hospitality at the International Centre for Theoretical Physics (ICTP), Trieste, Italy. NBA considers his pleasant duty to recognize with gratitudes the Associateship of the ICTP and the kind helps of Professor P.N. Butcher, Professor E. Tosatti, Professor Yu Lu and others from the ICTP Condensed Matter Group. He is also indepted to Prof. M. Tomak for a critical reading of the manuscript.

A part of this work was initiated in Vietnam where the constant support and encouragement from Professor Nguyen Van Hieu, Professor Dao Vong Duc and Professor Nguyen Thanh are very gratefully acknowledged by all the authors.

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