ALFVÉN–MAGNETOSONIC WAVES INTERACTION

K. MURAWSKI

University of St Andrews, Department of Mathematical Sciences
Mathematical Institute, University of St Andrews, St Andrews KY16 9SS, Fife, Scotland

(Received September 11, 1991)

The nonlinear propagation of the Alfvén and magnetosonic waves in the solar corona is investigated in terms of model equations. Due to viscous effects taken into account the propagation of the Alfvén wave itself is governed by a Burgers-type equation. The Alfvén waves exhibit a tendency to drive both the slow and fast magnetosonic waves. For this process model equations are a generalization of the Zakharov equations. The propagation of the magnetosonic waves is described by linearized Boussinesq-type equations with ponderomotive terms due to the Alfvén wave. Both long and short Alfvén waves are considered. Also the limits of the slow and fast modes are investigated. An approximate shock wave solution has been found for a vertically propagating slow mode. Numerical results for the fast mode propagating perpendicular to the magnetic field show the effect of inhomogeneity and pumping on a shock as the solution of the homogeneous Burgers equation.

PACS numbers: 03.40.Kf, 52.35.Mw

1. Introduction

Although the problem of the propagation of linear magnetohydrodynamic waves in an inhomogeneous medium is of great interest in solar physics (e.g. [1]), it has not been yet investigated in sufficient detail. Moreover, the propagation of MHD waves has been studied mostly in the case where the Alfvén waves decouple from magnetosonic ones (see e.g. [2, 3]).

Few analytical calculations relating both the Alfvén and the magnetosonic waves have been attempted. There are some numerical simulations made by Hollweg et al. [4]. They conclude that shear Alfvén wave in a solar magnetic flux tube can drive sound waves which eventually dissipate into shocks. It is qualitatively suggested that Alfvén waves may heat the corona undirectly by driving the slow mode, with some of the properties of spicules. Sakai and Sonnerup [5] have derived model equations which describe the long dispersive Alfvén wave driving sound wave. On the other hand the equation governing the evolution of the fast wave envelope modulated by a slow wave driven by the ponderomotive force has been derived for a sausage wave travelling along magnetic photospheric slab with rigid walls by Sahyouni et al. [6]. Model equations describing dispersive Alfvén–magnetosonic waves interaction have been derived by Shukla et al. [7]. Coupling between magnetosonic waves and tearing mode has been described in terms of model equations by Sakai and Washimi [8]. Coupled nonlinear Schrödinger equations governing the interaction of sausage and kink surface waves in a plasma slab have been derived by Vladimirov et al. [9].

In all above mentioned cases the derived equations are some generalizations of the Zakharov equations originally derived for the Langmuir wave and slow density plasma response. A recent source of references on these equations can be found in Murawski et al. [10]. The Zakharov equations do not take, however, viscosity into account.

The purpose of this paper is to derive model equations describing the coupling between Alfvén and magnetosonic waves which are driven by the former. We are not going to treat resonant interaction between Alfvén waves. This process can lead to creation of new waves (see Wentzel [11]). Actually, the Alfvén wave can drive both the slow and fast mode. Due to the small value of the sound speed in comparison to the Alfvén one, we should expect weak coupling in the Alfvén–slow mode interaction and a strong one for the Alfvén–fast wave interaction. This process of driving of the fast wave can be even enhanced by the phase mixing effect caused by inhomogeneities in the embient magnetic field.

The paper is organized as follows. The next Section presents fundamental set of equations for the Alfvén and magnetosonic waves which are shortly described in Section 3. A dispersion relation for the viscous non-dispersive Alfvén wave is derived in Section 4. Burgers-type equations describing the Alfvén waves propagation are presented in Section 5. A case of linear polarization is also discussed. Equations which govern the Alfvén–magnetosonic waves interaction are derived in the next Section. Numerical results for the fast mode propagating perpendicular to the ambient magnetic field are shown in the subsection 6.1.2. The final part of the paper contains a short summary and conclusions.

2. Fundamental equations

Let us consider a viscous compressible plasma with infinite conductivity described by the equations of magnetohydrodynamics (e.g. [12]):

$$\rho_t + \nabla \cdot (\rho V) = 0, \tag{2.1}$$

$$\rho[V_t + (V \cdot \nabla) V] + \nabla p = \frac{1}{\mu}(\nabla \times B) \times B$$

$$+\eta_0 \nabla^2 V + (\eta_1 + \frac{1}{3}\eta_0)\nabla(\nabla \cdot V), \tag{2.2}$$

$$B_t = \nabla \times (V \times B), \tag{2.3}$$

$$\nabla \cdot B = 0, \tag{2.4}$$

$$\frac{\rho^\gamma}{\gamma-1}\left[\left(\frac{p}{\rho^\gamma}\right)_t+(\boldsymbol{V}\cdot\nabla)\frac{p}{\rho^\gamma}\right]=\eta_0\sum_{i,k=1}^{3}(V_{i x_k}+V_{k x_i}-\frac{2}{3}\delta_{ik}\nabla\cdot\boldsymbol{V})V_{i x_k}, \quad (2.5)$$

where ρ is the plasma density, $\boldsymbol{V}\equiv[u,v,w]$ the velocity ($V_1=u$, $V_2=v$, $V_3=w$), p the pressure, μ the magnetic permitivity, $\boldsymbol{B}\equiv[a,h,b]$ the magnetic induction, γ the ratio of specific heats, and $\eta_0\gg\eta_1$ the dynamic and bulk viscosity coefficients, respectively. The indice t denotes the partial differentiation with respect to time. In the above equations two important effects have been neglected: electron thermal conduction [13] and Braginskii viscosity vector simplified (e.g. [14, 15]).

We introduce a Cartesian coordinate system with z-axis parallel to the undisturbed inhomogeneous. (x dependent) magnetic field $B_0(x)$. In what follows we assume that all variables depend on x and z only.

The undisturbed state is characterized by $\boldsymbol{V}=0$, $\rho=\rho_0=\text{const.}$, $p=p_0(x)$, $\boldsymbol{B}_0=[0,0,B_0(x)]$.

3. Basic modes in a homogeneous medium

3.1. Alfvén wave

The driving force for the Alfvén wave is the magnetic tension alone. The dispersion relation is (for the inviscid plasma)

$$\omega=kV_A\cos\Theta.$$

So, it can not propagate in a direction perpendicular ($\Theta=\pi/2$) to the magnetic field. The Alfvén wave is transverse. The velocity perturbation is normal both to the applied magnetic field and the propagation direction. The magnetic field perturbation is perpendicular to $\boldsymbol{B}_0$. There are no pressure or density changes associated with the wave. The energy flows along the field at the Alfvén speed.

3.2. Fast wave

For the dissipation free medium we have got the following dispersion relation:

$$\frac{\omega^2}{k^2}=\frac{1}{2}\{c_s^2+V_A^2\pm[(c_s^2+V_A^2)^2-(2c_sV_A\cos\Theta)^2]^{\frac{1}{2}}\}. \quad (3.2)$$

The sign + corresponds to the fast wave. Its velocity becomes the faster of either V_A or c_s for $\Theta=0$: for $c_s\gg V_A$, $\omega/k\approx c_s$ — acoustical in character (longitudinal fluid motion) and for $V_A\gg c_s$, $\omega/k\approx V_A$ — magnetic in character (transverse to B fluid motion). It is roughly isotropic wave propagating fastest across the field. The fast wave is driven by tension and pressure forces. The gas and magnetic pressure variations are in phase. Thus, the fast mode is essentially a sound wave in the convection zone, photosphere, and lower chromosphere, but it becomes more like an Alfvén wave in the upper chromosphere and corona although it still has small compressions.

3.3. Slow wave

The sign $-$ in Eq. (3.2) corresponds to a slow wave. It assumes the velocity and type of wave structure of the slower of c_s and V_A for $\Theta = 0$ and in the two limiting cases: when $c_s \gg V_A$, $\omega/k \approx V_A \cos\Theta$ and when $V_A \gg c_s$, $\omega/k \approx c_s \cos\Theta$. The energy flow is confined to near the magnetic field direction. The wave is driven by tension and pressure forces. The gas and magnetic pressure variation are out of phase. In the corona the slow mode is more acoustic than magnetic.

4. Dispersion relation for the Alfvén wave

To derive model equations which govern the Alfvén wave propagation in the viscous plasma we need to know the dispersion relation in the homogeneous medium. In this way linearizing Eqs. (2.1–2.5) around the homogeneous undisturbed state we see that the equations for the Alfvén wave decouple and we get the following dispersion relation for parallel propagation ($e^{i(kz+\omega t)}$):

$$\omega^2 = V_A^2 k^2 + i\frac{\eta_0}{\rho_0}k^2\omega. \tag{4.1}$$

In the limit of long wavelength ($k \to 0$) waves we obtain

$$\frac{\omega}{k} = \pm V_A + \frac{i}{2}\frac{\eta_0}{\rho_0}k. \tag{4.2}$$

Note that essentially this expression is similar to the one for the inviscid and dispersive Alfvén wave (with the Hall term in the induction equation (2.3) included), see e.g. [16]. The important difference is that here stands imaginary unit i corresponding to a dissipation.

5. The fast wave propagation in the long waves limit

Let the dimensionless wave amplitude be equal to $\varepsilon \ll 1$. In order to compete with the nonlinearity for the dispersion, it is necessary that the correction in the dispersion relation for the long wavelength waves be of the order of ε^2. Thus, for the problem in question ($k \to 0$) it is necessary to introduce the following stretched variables:

$$\xi = \varepsilon^2(z - V_A t), \quad \tau = \varepsilon^4 t, \quad \zeta = \varepsilon^3 x. \tag{5.1}$$

The dissipation coefficient ε^2 can be defined as the ratio of a transverse length scale to a charactersitic wavelength. This scaling is essentially the same as for the homegeneous medium [17]. We assume here that V_A slowly depends on x or in other words it is locally constant. The corona is in fact highly structured across the field and this assumption can be only partially justified. This way we can solve analytically the problem and provide the insight into more complex phenomena which can be modelled by solving the full set of MHD Eqs. (2.1–2.5) numerically. For this moment, this problem is too advanced and we limit ourself to assumption (5.1). Note also that ε^2 describes the weakness of dispersion (e.g. [18]).

We use the following expansion:

$$f = f_0 + \varepsilon f_1 + \varepsilon^2 f_2 + \ldots, \tag{5.2}$$

where now ε describes the weakness of nonlinearity [18] and allow the density to be varied in time. The nonlinear coefficient ε can be defined as the ratio of a characteristic wave amplitude to a transverse length scale. Collection of terms at ε^3 (e.g. [19]) as a compatibility condition gives us coupled two-dimensional Burgers equations:

$$(V_A^2 - c_s^2)u_{1\tau} + \frac{V_A}{4}(V_\perp^2 u_1)_\xi - \frac{V_A^3}{2B_0}(\Psi u_1)_\xi + \frac{V_A^2}{4}V_{\perp\zeta}^2 - \frac{V_A^4}{2B_0}\Psi_\zeta - \frac{\eta_0}{2\rho_0}(V_A^2 - c_s^2)u_{1\xi\xi} = 0, \tag{5.3}$$

$$(V_A^2 - c_s^2)v_{1\tau} + \frac{V_A}{4}(V_\perp^2 v_1)_\xi - \frac{V_A^3}{2B_0}(\Psi v_1)_\xi - \frac{\eta_0}{2\rho_0}(V_A^2 - c_s^2)v_{1\xi\xi} = 0, \tag{5.4}$$

$$\Psi_\xi + \frac{B_0}{V_A}u_{1\zeta} = 0, \tag{5.5}$$

where

$$\Psi \equiv -\frac{B_0}{V_A}\int u_{1\zeta}\,\mathrm{d}\xi,$$

$$V_\perp^2 \equiv u_1^2 + v_1^2.$$

Note that B_0/V_A is a constant. Similar equations have been derived by Ruderman [16] and Mjølhus and Wyller [17]. They, however, have taken resistivity instead of viscosity into account.

Equations (5.3–5.5) are difficult to solve analytically. Because there is a lack of corresponding discussion in the literature, we will simply consider some one-dimensional cases.

5.1. One-dimensional case

In the one-dimensional case $\partial\zeta = 0$ which implies $\Psi = 0$ and we get coupled Burgers equations (the subscript 1 is dropped):

$$f_\tau + \beta(V_\perp^2 f)_\xi - \alpha f_{\xi\xi} = 0, \tag{5.1.1}$$

where $f \equiv u$ or v, and the nonlinear β and dissipative α coefficients are defined as:

$$\alpha \equiv \frac{\eta_0}{2\rho_0}, \quad \beta \equiv \frac{V_A}{4(V_A^2 - c_s^2)}.$$

Note that α does not depend on x but β does.

5.2. Energy equation for the coupled Burgers equations

Equation (5.1.1) can be written in the differential "conservation" form:

$$V_{\perp\tau}^2 + \frac{3}{2}\beta V_{\perp\xi}^4 - 2\alpha(uu_{\xi\xi} + vv_{\xi\xi}) = 0 \tag{5.2.1}$$

or in the equivalent integral form:

$$\Big(\int_{-\infty}^{\infty} V_\perp^2\,\mathrm{d}\xi\Big)_\tau - 2\alpha\int_{-\infty}^{\infty}(uu_{\xi\xi} + vv_{\xi\xi})\,\mathrm{d}\xi = \text{const.}. \tag{5.2.2}$$

The first term in this equation describes energy. So, we see that energy decreased in time.

Equation (5.1.1) can also be rewritten in the form:

$$f_\tau + (\beta V_\perp^2 f - \alpha f_\xi)_\xi = 0, \tag{5.2.3}$$

which says that a momentum along the axis is conserved.

5.3. Linear polarization

Now we restrict ourself to a linear polarization. So,

$$u = V_\perp \cos\phi, \quad v = V_\perp \sin\phi, \tag{5.3.1}$$

where ϕ is a constant polarization angle. In this case we obtain a modified Burgers equation (with a cubic nonlinear term):

$$V_{\perp\tau} + 3\beta V_\perp^2 V_{\perp\xi} - \alpha V_{\perp\xi\xi} = 0. \tag{5.3.2}$$

Looking for stationary solutions

$$V_\perp \equiv V(\zeta \equiv \xi - s\tau), \quad s > 0, \tag{5.3.3}$$

and integrating (5.3.2) over ζ we get the following ordinary differential equation:

$$\alpha V_\zeta = \beta V^3 - sV + l, \quad l = \text{const.}, \tag{5.3.4}$$

which is very convenient for the phase analysis (e.g. [20]). For $\alpha > 0$ and $\beta < 0$ there are no physical solutions. In the corona, however, both coefficients are greater than zero. In this case the phase analysis leads to the conclusions that we should expect finite solutions for $-l_m < l \le l_m$, where $l_m \equiv (2s/3)\sqrt{s/3\beta}$. Under this constraint for each value of the free parameter l there are two shocks characterized by:

1) expansion shock: $a < V < b$ and increases with ζ,
2) compression shock: $b < V < c$ and decreases with ζ,

where $a < b < c$ are the roots of the equation made from the right hand side (r. h. s.) of Eq. (5.3.4). For $l = l_m$, $b = c$ and the expansion shock solution is given by

$$\log \left| \frac{V-a}{V-b} \right| + \frac{a-b}{V-b} = (b-a)^2 \frac{\beta}{\alpha}\zeta + \text{const.} \tag{5.3.5}$$

Otherwise the shock solutions are described by

$$\frac{\log|V-c|}{c^2-(b+a)c+ab} + \frac{\log|V-b|}{(b-a)(b-c)} + \frac{\log|V-a|}{(a-b)(a-c)} = \frac{\beta}{\alpha}\zeta + \text{const.} \tag{5.3.6}$$

Let us now dimensionalize Eq. (5.3.2) in the following way

$$U \to V_A U, \quad \tau \to T\tau, \quad \xi \to X\xi,$$

with $T = X/V_A$. For typical coronal conditions [12]: $V_A = 2 \times 10^6$ m/s, $c_s = 2 \times 10^5$ m/s, $\eta_0 = \frac{1}{2}$ g/(cm s), $\rho_0 = 1.5 \times 10^{-15}$ g/cm^3, $X = 10^8$ m, and $B = 10$ Gauss, we get that the nonlinear β and dissipative α coefficients of the dimensionless equation are approximately equal to 0.25 and -10^{-3}, respectively. Hence, we deduce that the fast waves in the corona are weakly damped. Thus we must look for other mechanisms which can explain wave damping. One of such ways is described in the next Section.

6. Alfvén–magnetosonic waves interaction

The Alfvén wave drives both the slow and fast waves. So, let us consider a mix of these waves. Because in the corona the sound speed is very small the Alfvén–fast mode interaction is very strong. In other words, because in the corona

$v_f \approx V_A$, the process of driving of the fast waves is more efficient than the corresponding one for the slow mode. Due to a phase mixing we can expect that in the upper corona the fast waves can propagate obliquely to the magnetic field. Other directions of the propagation are also allowed but due to the inhomogeneity in the x-direction waves are mostly damped. In the following sections we consider both short and long Alfvén waves limits. The short Alfvén wave interacts itself (selfmodulation) and also is modulated by the magnetosonic wave response.

6.1. Magnetosonic waves driven by long Alfvén wave

To study the long Alfvén-magnetosonic waves interaction the physical quantities are divided into the following parts:

$$f(x,z,t) = f_0(x) + \varepsilon f_1 + \varepsilon^2 f_2 + ..., \tag{6.1.1}$$

where $f_0(x)$ is the undisturbed state, f_1 and f_2 describe the long shear Alfvén and magnetosonic waves, respectively. Because the magnetosonic waves are driven by the Alfvén wave we should expect that the former ones are in lower magnitude. Thus f_1 describes the quantities

$$f_1 : u_1 \neq 0, \quad v_1 \neq 0, \quad a_1 \neq 0, \quad h_1 \neq 0. \tag{6.1.2}$$

Other first order quantities are taken to be zero. Due to the dispersion relation for the Alfvén wave we use the same variables stretching as in Sec. 5, (see Eq. (5.1)). From Eqs. (2.1–2.5) collecting terms at ε, we get

$$a_1 = -\frac{B_0}{V_A} u_1, \quad h_1 = -\frac{B_0}{V_A} v_1.$$

Note that B_0/V_A does not depend on x.

Collecting terms at ε^3 we get equations describing the Alfvén wave propagation and written here in the laboratory reference frame (∂_τ, ∂_ζ, and ∂_ξ have been replaced by $\partial_t + V_A\partial_z$, ∂_x, and ∂_z, respectively):

$$\begin{aligned} &u_{1t} + V_A u_{1z} - \frac{\eta_0}{2\rho_0} u_{1zz} + \frac{1}{4}(u_1^2 + v_1^2)_x + \frac{V_A}{2B_0}[(b_2 u_1)_z + b_2 u_{1z}] \\ &+\frac{1}{2}[(w_2 u_1)_z + w_2 u_{1z}] - \frac{1}{2\rho_0}(V_A \rho_2 u_{1z} - p_{2x}) = 0, \end{aligned} \tag{6.1.3}$$

$$\begin{aligned} &v_{1t} + V_A v_{1z} - \frac{\eta_0}{2\rho_0} v_{1zz} + \frac{V_A}{2B_0}[(b_2 v_1)_z + b_2 v_{1z}] \\ &+\frac{1}{2}[(w_2 v_1)_z + w_2 v_{1z}] - \frac{V_A}{2\rho_0}\rho_2 v_{1z} = 0. \end{aligned} \tag{6.1.4}$$

Note that the Alfvén wave variables are coupled to the vertical magnetic field, velocity, density and pressure. The process of interaction is thus much more complicated than in the case of the Zakharov equations.

The magnetosonic waves are described by equations which are obtained at ε^2:

$$\begin{aligned} &u_{2tt} - c_A^2 u_{2xx} - c_{Ax}^2 u_{2x} - \delta_1 u_{2xxt} - V_A^2 u_{2zz} - \frac{\eta_0}{\rho_0} u_{2zzt} - c_s^2 w_{2xz} - \delta_2 w_{2xzt} = \\ &-\frac{1}{2}(u_1^2 + v_1^2)_{xt} - (\gamma - 1)\frac{\eta_0}{\rho_0}(\frac{4}{3}{u_{1x}}^2 + {u_{1z}}^2 + {v_{1x}}^2 + {v_{1z}}^2)_x, \end{aligned} \tag{6.1.5}$$

$$v_{2tt} - V_A^2 v_{2zz} - \frac{\eta_0}{\rho_0}(v_{2xx} + v_{2zz})_t = 0, \tag{6.1.6}$$

$$w_{2tt} - c_s^2 w_{2zz} - \delta_1 w_{2zzt} - \frac{\eta_0}{\rho_0} w_{2xxt} - c_s^2 u_{2xz} - \delta_2 u_{2xzt} =$$
$$-\frac{1}{2}(u_1^2 + v_1^2)_{zt} - (\gamma - 1)\frac{\eta_0}{\rho_0}(\frac{4}{3}{u_{1x}}^2 + {u_{1z}}^2 + {v_{1x}}^2 + {v_{1z}}^2)_z, \tag{6.1.7}$$

where

$$c_A^2 \equiv c_s^2 + V_A^2, \ \delta_1 \equiv \frac{3\eta_1 + 4\eta_0}{3\rho_0}, \ \delta_2 \equiv \frac{3\eta_1 + \eta_0}{3\rho_0}.$$

Additionally, we must include equations describing an evolution of b_2, ρ_2, and p_2 with which the Alfvén wave is coupled

$$b_{2t} + (B_0 u_2)_x = 0, \tag{6.1.8}$$

$$\rho_{2t} + \rho_0(u_{2x} + w_{2z}) = 0, \tag{6.1.9}$$

$$p_{2t} - c_s^2 \rho_{2t} + p_{0x} u_2 = (\gamma - 1)\eta_0(\frac{4}{3}{u_{1x}}^2 + {u_{1z}}^2 + {v_{1x}}^2 + {v_{1z}}^2). \tag{6.1.10}$$

Note the ponderomotive terms at the r. h. s. of (6.1.5) and (6.1.7) and lack of any one in (6.1.6). Thus we see that the flow is driven due to the existence of gradients in this direction described by the first terms of the r. h. s. These equations contain also the pseudo-damping term $c_{Ax}^2 u_{2x}$ connected with a phase mixing. We can prove it by considering the equation:

$$u_{tt} - c_A^2 u_{xx} - c_{Ax}^2 u_x = 0$$

and assuming that locally a wave may be represented as:

$$u \sim e^{i(kx - \omega t)}$$

to get

$$\omega^2 = c_A^2 k^2 - i c_{Ax}^2 k.$$

So, the $c_{Ax}^2 u_x$ term is a (pseudo-)damping one.

6.1.1. Slow mode limit

In the slow mode limit b_2, u_2, $v_2 \ll w_2$ and time changes are negligible with respect to the Alfvén time $\partial_t \ll V_A \partial_x$. Then the Alfvén wave is described by

$$u_{1t} + V_A u_{1z} - \frac{\eta_0}{2\rho_0} u_{1zz} + \frac{1}{4}(u_1^2 + v_1^2)_x$$
$$+\frac{1}{2}[(w_2 u_1)_z + w_2 u_{1z}] - \frac{V_A}{2\rho_0}\rho_2 u_{1z} + \frac{1}{2\rho_0} p_{2x} = 0, \tag{6.1.11}$$

$$v_{1t} + V_A v_{1z} - \frac{\eta_0}{2\rho_0} v_{1zz} + \frac{1}{2}(w_2 v_1)_z + \frac{1}{2} w_2 v_{1z} - \frac{V_A}{2\rho_0}\rho_2 v_{1z} = 0. \tag{6.1.12}$$

The slow mode propagation is governed by

$$w_{2tt} - c_s^2 w_{2zz} - \delta_1 w_{2zzt} - \frac{\eta_0}{\rho_0} w_{2xxt} = -\frac{1}{2}(u_1^2 + v_1^2)_{zt}$$
$$-(\gamma - 1)\frac{\eta_0}{\rho_0}(\frac{4}{3}{u_{1x}}^2 + {u_{1z}}^2 + {v_{1x}}^2 + {v_{1z}}^2)_z. \tag{6.1.13}$$

Additionally, the equations for p_2 and ρ_2 take the form:

$$p_{2t} - c_s^2 \rho_{2t} = (\gamma - 1)\eta_0 (\frac{4}{3} {u_{1x}}^2 + {u_{1z}}^2 + {v_{1x}}^2 + {v_{1z}}^2), \tag{6.1.14}$$

$$\rho_{2t} + \rho_0 w_{2z} = 0. \tag{6.1.15}$$

6.1.2. Slow wave propagating vertically in a homogeneous medium

Let us consider a case in which the long Alfvén wave drives only propagating vertically slow wave and look for approximate solutions. Now, Eqs. (6.1.11–6.1.15) will be rewritten in the case of the homogeneous field. Neglecting in them some terms proportional to η_0 due to a weak influence of the viscosity on the long Alfvén wave we can write equations which describe the Alfvén wave:

$$u_{1t} + V_A u_{1z} + \frac{1}{2}[(w_2 u_1)_z + w_2 u_{1z}] - \frac{V_A}{2\rho_0}\rho_2 u_{1z} = 0, \tag{6.1.16}$$

$$v_{1t} + V_A v_{1z} + \frac{1}{2}[(w_2 v_1)_z + w_2 v_{1z}] - \frac{V_A}{2\rho_0}\rho_2 v_{1z} = 0, \tag{6.1.17}$$

the slow mode

$$w_{2tt} - c_s^2 w_{2zz} - \delta_1 w_{2zzt} = -\frac{1}{2}(u_1^2 + v_1^2)_{zt}, \tag{6.1.18}$$

and corresponding equations for ρ_2, p_2, and b_2:

$$\rho_{2t} + \rho_0 w_{2z} = 0, \tag{6.1.19}$$

$$b_{2t} - \frac{B_0}{\rho_0}\rho_{2t} = B_0 w_{2z}, \tag{6.1.20}$$

$$p_2 = c_s^2 \rho_2. \tag{6.1.21}$$

Note that this equation is adiabatic. So at the level of these calculations there is no coronal heating. It is usefull, however, to provide approximate solution for the propagating vertically slow wave even for this case. The full problem does not seem to be solvable in an analytical way.

Let us assume that the Alfvén wave is linearly polarized in the x-direction ($v_1 = 0$) and look for the stationary solutions ($\xi = z - ct$). Then, we get

$$\rho_2 = \frac{\rho_0}{c} w_2, \tag{6.1.22}$$

$$w_{2\xi} = \frac{c_s^2 - c^2}{c\delta_1} w_2 + \frac{1}{2\delta_1} u_1^2 + l, \qquad l = \text{const.}, \tag{6.1.23}$$

$$u_1 = l_2 \left[2(V_A - c) + (2 - \frac{V_A}{c}) w_2\right]^{-1/(2-\frac{V_A}{c})}, \qquad l_2 = \text{const.} \tag{6.1.24}$$

Substituting (6.1.24) into (6.1.23), we obtain

$$w_{2\xi} = a w_2 + \frac{l_3^2}{(b + d w_2)^{2/d}} + l, \tag{6.1.25}$$

where we have used the notation

$$a \equiv \frac{c_s^2 - c^2}{c\delta_1}, \qquad l_3^2 \equiv \frac{l_2^2}{2\delta_1}, \qquad b \equiv 2(V_A - c), \qquad d \equiv 2 - \frac{V_A}{c}.$$

We consider now the following case

$$d<0 \quad (c<\frac{V_A}{2}) \quad \text{and} \quad \frac{2}{d}=-n \ (c=\frac{n}{2(n+1)}V_A).$$

Equation (6.1.25) takes then the form:

$$w_{2\xi}=l_3^2(\frac{n+2}{n+1}V_A-\frac{2}{n}w_2)^n+aw_2+l, \tag{6.1.26}$$

where n is an arbitrary number. In the case of $n=2$ ($c=V_A/3$) we obtain a shock wave with an amplitude l_3^{-2} and moving on the "background" $4V_A/3-a/2l_3^2$

$$w_2=\frac{4}{3}V_A-\frac{a}{2l_3^2}+l_3^{-2}\tan(\frac{1}{2}\xi+l_4). \tag{6.1.27}$$

Choosing other values of n we can find other solutions. This problem, however, is not discussed in this paper.

6.1.3. Fast mode limit

Now, $w_2 \ll u_2$, v_2 and $c_s\partial_z \ll \partial_t$. The Alfvén wave equations are:

$$u_{1t}+V_Au_{1z}-\frac{\eta_0}{2\rho_0}u_{1zz}+\frac{1}{4}(u_1^2+v_1^2)_x+\frac{V_A}{2B_0}[(b_2u_1)_z+b_2u_{1z}]$$
$$-\frac{V_A}{2\rho_0}\rho_2u_{1z}+\frac{1}{2\rho_0}p_{2x}=0, \tag{6.1.28}$$

$$v_{1t}+V_Av_{1z}-\frac{\eta_0}{2\rho_0}v_{1zz}+\frac{V_A}{2B_0}[(b_2v_1)_z+b_2v_{1z}]-\frac{V_A}{2\rho_0}\rho_2v_{1z}=0. \tag{6.1.29}$$

The fast magnetosonic wave propagation is governed by the following equations:

$$u_{2tt}-c_A^2u_{2xx}-c_{A\,x}^2u_{2x}-\delta_1u_{2xxt}-V_A^2u_{2zz}-\frac{\eta_0}{\rho_0}u_{2zzt}=$$
$$-\frac{1}{2}(u_1^2+v_1^2)_{xt}-(\gamma-1)\frac{\eta_0}{\rho_0}(\frac{4}{3}{u_{1x}}^2+{u_{1z}}^2+{v_{1x}}^2+{v_{1z}}^2)_x, \tag{6.1.30}$$

$$v_{2tt}-v_A^2v_{2zz}-\frac{\eta_0}{\rho_0}(v_{2xx}+v_{2zz})_t=0. \tag{6.1.31}$$

Equations for b_2, ρ_2, and p_2 take the following form:

$$\rho_{2t}+\rho_0u_{2x}=0, \tag{6.1.32}$$

$$p_{2t}-c_s^2\rho_{2t}+p_{0x}u_2=(\gamma-1)\eta_0\left(\frac{4}{3}{u_{1x}}^2+{u_{1z}}^2+{v_{1x}}^2+{v_{1z}}^2\right), \tag{6.1.33}$$

$$b_{2t}+(B_0u_2)_x=0. \tag{6.1.34}$$

6.1.4. Fast mode propagating in the x-direction. Numerical results

In the previous section, we have applied the expansion method to derive model equations for the Alfvén–magnetosonic waves interaction. Here, turning our interest to the fast wave (there is only the fast wave in a medium) propagating perpendicular to the magnetic field, we develop the reductive Taniuti–Wei's method ([21], see also [18] for a review of methods) to obtain the inhomogeneous

Burgers equation in the limit of long wavelength waves. For this aim we expand the quantities into the series:

$$f = f_0(\xi) + \sum_{n=1}^{\infty} \varepsilon^n f_n \tag{6.1.35}$$

and due to the spatial inhomogeneity introduce the following coordinate stretching:

$$\tau = \varepsilon \int \left(\frac{\mathrm{d}x}{c_{\mathrm{A}}} - \mathrm{d}t \right), \quad \xi = \varepsilon^2 x. \tag{6.1.36}$$

Substitution of the expansion (6.1.35) and (6.1.36) into Eqs. (2.1–2.5) leads at ε^2 to the inhomogeneous Burgers equation:

$$u_{1\xi} + \beta u_1 u_{1\tau} - \alpha u_{1\tau\tau} + \frac{1}{2}(\ln c_A)_\xi u_1 = 0, \tag{6.1.37}$$

where the nonlinear β and dissipative α coefficients are defined as

$$\beta = \frac{(\gamma+1)c_s^2 + 3V_{\mathrm{A}}^2}{2c_{\mathrm{A}}^4}, \quad \alpha = \frac{3\eta_1 + 4\eta_0}{6\rho_0 c_{\mathrm{A}}^3}. \tag{6.1.38}$$

Let us now use dimensionless variables as follows:

$$u_1 = V_{\mathrm{A}}^* u, \quad \tau = T^* t, \quad \xi = X^* x, \tag{6.1.39}$$

where f^* denotes the constants typical coronal values described in Sec. 5.

This Section describes a perpendicularly propagating fast wave in a sea of Alfvén waves which produce a ponderomotive force. Due to this process the fast wave gains some energy and the Alfvén wave amplitude is reduced. It seems to be acceptable to assume that the Alfvén wave reduces slowly its amplitude and a good choice is to take an exponential dependence. In order to represent this pumping effect, in the new variables the equation takes the following form:

$$u_x + \beta u u_t - \alpha u_{tt} + \frac{1}{2}(\ln c_{\mathrm{A}})_x u = b\mathrm{e}^{-\Gamma x} \sin^2 t. \tag{6.1.40}$$

Now, the dimensionless coefficients are described by

$$\alpha = \frac{3\eta_1 + 4\eta_0}{6\rho_0 c_{\mathrm{A}}^3} \frac{V_{\mathrm{A}}^{*2}}{X^*}, \quad \beta = V_{\mathrm{A}}^{*2}[(\gamma+1)c_s^2 + 3V_{\mathrm{A}}^2]/2c_{\mathrm{A}}^4 \tag{6.1.41}$$

and the ponderomotive term due to the Alfvén wave has been added phenomenologically to the right hand side: $b\mathrm{e}^{-\Gamma x}\sin^2 t$. Note that the pseudo-damping term $\frac{1}{2}(\ln c_{\mathrm{A}})_x u$ is a consequence of the spatial inhomogeneity taken into consideration.

Equation (6.1.40) shows that if viscosity is dropped, the linear terms on the left hand side give $u^2 c_{\mathrm{A}} =$ const. implying that wave energy flux density is conserved. The term proportional to u is however a pseudo-damping one because the energy $\int_{-\infty}^{\infty} u^2 \mathrm{d}x$ is not conserved if c_{A} is a function of the coordinate x, $c_{\mathrm{A}} = c_{\mathrm{A}}(x)$. The other way of proving it is to get a linear dispersion relation from (6.1.40). After Fourier analysis of the left hand side of it, we get

$$k = \mathrm{i}\alpha\omega^2 + \frac{\mathrm{i}}{2}(\ln c_{\mathrm{A}})_x.$$

So, we see that both u_{tt} and u terms are responsible for damping of the wave. The second derivative term is responsible for viscous damping because α is dependent on the viscosity coefficients η_0 and η_1. The second damping term is connected with the inhomogeneity and thus is called pseudo-damping term.

For the homogeneous Burgers equation, both the viscous α and the nonlinear β coefficients are positive. So, there are only shock solutions for which $u_\xi < 0$, where the phasor $\xi = t - x$.

6.1.5. Numerical results

The homogeneous Burgers equation possesses an expansion (in ξ) shock wave solution written as

$$u(\xi) = \frac{1}{\beta}\left[1 + \tanh\left(-\frac{\xi}{2\alpha}\right)\right], \tag{6.1.42}$$

where $\xi = t - x$. This shock has been taken as an initial condition (for $x = 0$) for the inhomogeneous Burgers equation with the ponderomotive term due to the Alfvén wave. The equation has been numerically solved in the time interval $-5 < t < 15$ using the flux centred transport (FCT) technique developed by Boris and Book [22]. The method has also been described by Schnack and Killeen [23]. The algorithm has been tested by checking conservation laws. The steep coronal shock is presented in Fig. 1a. This is an exact solution for this equation. Solving this equation numerically we have to apply boundary condition which modify the solution. The exact solution is defined on the infinite interval, whereas that one got numerically can be only simulated on a finite interval.

In order to study the effect of the inhomogeneity on the shock wave we have considered the case of a decreasing magnetic field. The magnetic field has been described by

$$B(x) = B_0^* - \frac{B_0^*}{x_{\max}}x. \tag{6.1.43}$$

So, at $x = x_{\max} = 15$ the magnetic field is zero. The equilibrium condition

$$p_0 + \frac{B_0^2}{2\mu} = \text{const.} \tag{6.1.44}$$

has been also taken into account.

The numerical results are presented in Fig. 1. The effect of the inhomogeneity is twofold. First, the wave is attenuated. It is a consequence of the appearence of the pseudo-damping term in (6.1.40). For example, at $x = 15$ the amplitude of the homogeneous shock was less than 0.8 whereas in the case of the decreasing magnetic field the amplitude was less than 0.04 (Fig. 1d). Second, the velocity of the wave is increased. The homogeneous shock moves approximately twice more slowly than the one under the decreasing magnetic field (Fig. 1d).

Finally, the effect of the ponderomotive forces has been taken into consideration. This way the force, exerted by the Alfvén wave, has been phenomenologically represented as $b\exp(-\Gamma x)\sin t$. For $b = u\ (x = 0, t = 0)/15$ and $\Gamma = 2/15$, numerical results are shown in Fig. 2. The Alfvén wave transfers its energy into the shock thus slowing down the attenuation. However, at $x = 15$ it is clearly seen that the pseudo-damping effects are more competitive than the ponderomotive ones and the shock is much attenuated. Furthermore, due to the periodic nature of the ponderomotive force the sinusoidal oscillations decreases more slowly than the shock. So, the shock progressively loses its identity, see Fig. 2c.

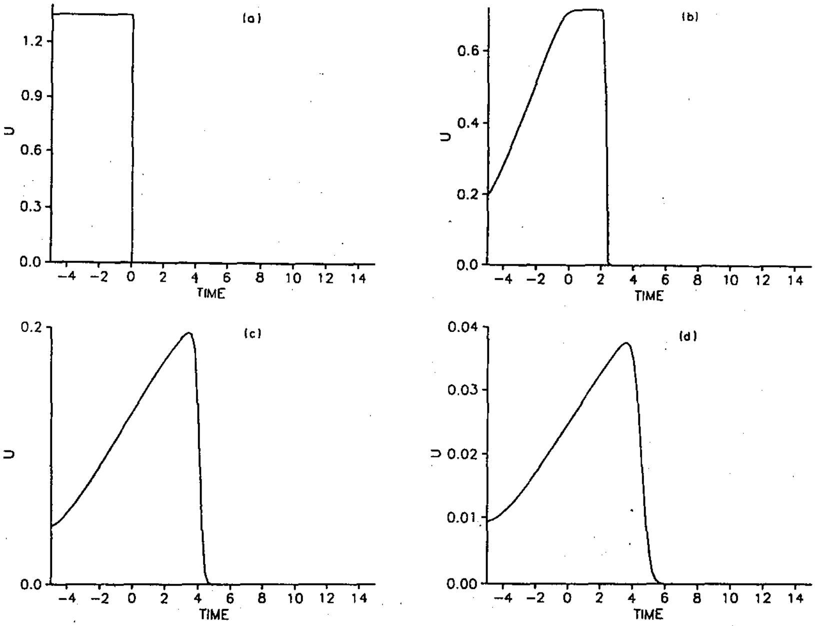

Fig. 1. A shock as the solution of the homogeneous Burgers equation under the effect of decreasing in x magnetic field at a) $x = 0$, b) $x = 3$, c) $x = 9$, and d) $x = 15$.

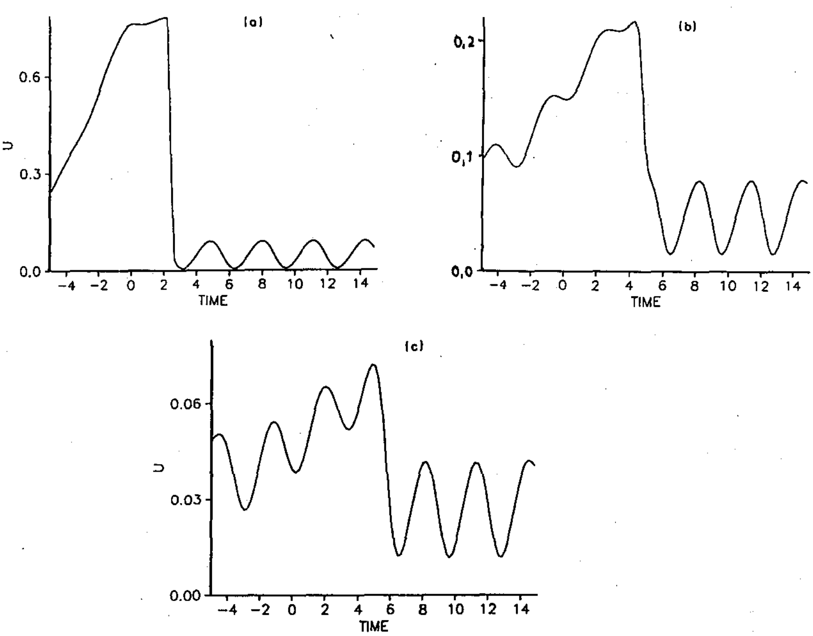

Fig. 2. As in Fig. 1, but here the ponderomotive force is included.

6.2. Magnetosonic waves driven by short Alfvén waves

For a packet of Alfvén waves modulated by the magnetosonic waves (which are driven by the former) we use the following expansion:

$$f(x,z,t)=\sum_{n=1}^{\infty}\varepsilon^n f_n,\qquad f_n=\sum_{m=-\infty}^{\infty} f_n^{(m)}(\xi,\zeta,\tau)\mathrm{e}^{\mathrm{i}m(kz-\omega t)},$$

$$f_n^{(-l)}=f_n^{(l)*},\quad \text{and}\quad f_1^{(l)}=0,\qquad l\neq \pm 1,$$

$$\xi=\varepsilon(z-\lambda t),\ \ \tau=\varepsilon^2 t,\ \ \zeta=\varepsilon x. \tag{6.2.1}$$

Whereas the magnetosonic waves variables are expanded as

$$f=f_0(x)+\varepsilon^2 f_2+\cdots. \tag{6.2.2}$$

Collection of terms at ε leads to

$$a_1^{(1)}=-B_0\frac{k}{\omega}u_1^{(1)}, \tag{6.2.3}$$

$$h_1^{(1)}=-B_0\frac{k}{\omega}v_1^{(1)}. \tag{6.2.4}$$

Note a similarity of these expressions to the corresponding ones for the long Alfvén waves. Now, however, we have got a phase velocity which is usually different from V_{A}. An expression for the group velocity λ is found from the equations at ε^2

$$\lambda=\omega_k. \tag{6.2.5}$$

Finally, from ε^3, we obtain (for the Alfvén wave) complex coefficients nonlinear Schrödinger equations:

$$\mathrm{i}(u_t+\lambda u_z)+\alpha_1 u_{zz}+\alpha_2 u_{xx}+\delta u+\beta_1 b_2 u-kw_2 u+\beta_2\rho_2 u=0, \tag{6.2.6}$$

$$\mathrm{i}(v_t+\lambda v_z)+\alpha_1 v_{zz}+\alpha_3 v_{xx}+\delta v+\beta_1 b_2 v-kw_2 v+\beta_2\rho_2 v=0, \tag{6.2.7}$$

where

$$u\equiv u_1^{(1)},\ \ v\equiv v_1^{(1)},$$

and the coefficients are defined as follows:

$$\alpha_1=\frac{\rho_0 V_{\mathrm{A}}^2\omega(1-\lambda k/\omega)^2-\mathrm{i}\eta_0\omega^2}{\rho_0(V_{\mathrm{A}}^2k^2+\omega^2)},$$

$$\alpha_2=-\mathrm{i}\frac{3\eta_1+4\eta_0}{3\rho_0}\frac{\omega^2}{V_{\mathrm{A}}^2k^2+\omega^2},$$

$$\alpha_3=-\mathrm{i}\frac{\eta_0}{\rho_0}\frac{\omega^2}{V_{\mathrm{A}}^2k^2+\omega^2},$$

$$\beta_1=-2\frac{k^2V_{\mathrm{A}}^2\omega}{B_0(V_{\mathrm{A}}^2k^2+\omega^2)},$$

$$\beta_2=\frac{\omega^3}{\rho_0(V_{\mathrm{A}}^2k^2+\omega^2)},$$

$$\delta=-\frac{\mathrm{i}}{2}\frac{\eta_0}{\rho_0}k^2.$$

The magnetosonic waves' propagation is governed by linearized Boussinesq-type equations:

$$\begin{aligned}
&u_{2tt} - c_{\mathrm{A}}^2 u_{2xx} - \delta_1 u_{2xxt} - c_{\mathrm{A}x}^2 u_{2x} - V_{\mathrm{A}}^2 u_{2zz} - \frac{\eta_0}{\rho_0} u_{2zzt} \\
&-c_s^2 w_{2xz} - \delta_2 w_{2xzt} = -\frac{1}{2}|u|^2{}_{xt} - \frac{1}{2}\left|\frac{V_{\mathrm{A}}^2 k^2}{\omega^2} v\right|^2_{xt} \\
&-(\gamma-1)\frac{\eta_0}{\rho_0}\left[\frac{8}{3}|u_x|^2 + 2|v_x|^2 + 2k^2(|u|^2 + |v|^2)\right]_x
\end{aligned} \tag{6.2.8}$$

$$\begin{aligned}
&v_{2tt} - V_{\mathrm{A}}^2 v_{2zz} - \frac{\eta_0}{\rho_0}(v_{2xx} + v_{2zz})_t = -(uv^*{}_x + u^* v_x)_t \\
&+\frac{V_{\mathrm{A}}^2 k}{B_0 \omega}\left[u(B_0 \frac{k}{\omega} v^*)_x + u^*(B_0 \frac{k}{\omega} v)_x\right]_t ,
\end{aligned} \tag{6.2.9}$$

$$\begin{aligned}
&w_{2tt} - c_s^2 w_{2zz} - \frac{\eta_0}{\rho_0} w_{2xxt} - \delta_1 w_{2zzt} - c_s^2 u_{2xz} - \delta_2 u_{2xzt} = -\frac{V_{\mathrm{A}}^2 k^2}{2\omega^2} \\
&\times(|u|^2 + |v|^2)_{zt} - (\gamma-1)\frac{\eta_0}{\rho_0}\left[\frac{8}{3}|u_x|^2 + 2|v_x|^2 + 2k^2(|u|^2 + |v|^2)\right]_z .
\end{aligned} \tag{6.2.10}$$

The ponderomotive terms are derived by a method of averaging over the fast variables (x, z, t) in the Alfvén wave. Note that the r. h. s. of Eq. (6.2.9) disappears for the homogeneous field. Thus, the y components of the magnetosonic waves are driven only in the case of the inhomogeneous field. Additionally, we must include equations for b_2 and ρ_2:

$$b_{2t} + (B_0 u_2)_x , \tag{6.2.11}$$

$$\rho_{2t} + \rho_0 u_{2x} + \rho_0 w_{2z} = 0. \tag{6.2.12}$$

7. Conclusions

Short and long Alfvén waves propagate according to Burgers-type and complex-coefficients-nonlinear Schrödinger equations, respectively. For the derivation of the complex coefficient nonlinear Schrödinger equation see also [24]. Its solutions have been discussed recently by Stenflo [25] and Stenflo et al. [26]. The long Alfvén wave is more weakly damped due to the phase mixing effect (which is described by the $-\alpha(x)u_{xx}$ term) than the short wave. The Alfvén wave drives the slow and fast magnetoacoustic waves because of gradients in the z and x direction. These waves are damped both by the viscosity and the phase mixing effect. Thus, they cascade their energies into lower scales. The x component of the fast wave is additionally damped due to inhomogeneity $(-c_{\mathrm{A}x}^2 u_{2x})$ cascading its energy to lower scales. The y component of the fast mode is driven only by the short Alfvén wave because of inhomogeneity. The short Alfvén wave is not coupled with the pressure p_2 as the long Alfvén wave is.

The numerical calculations performed for the fast wave propagating perpendicular to the ambient inhomogeneous magnetic field have shown that due to the inhomogeneity the shock wave is attenuated and increases its velocity. The periodic ponderomotive force has also been taken into account to show that waves go into lower and lower scales. This mechanism gives us a little more insight into the process of damping of the fast mode due to the inhomogeneous magnetic field and probably into a process of corona heating.

One would also have thought that an interesting problem would be to show how far the waves must propagate to create shocks. This would be a calculation relevant to coronal heating. Work in this direction is in a progress and will be published elsewhere.

Acknowledgements

The author wishes to express his sincere thanks to the Solar Theory Group at St Andrews, especially Dr Bernard Roberts and Prof. Eric Priest, for their encouragement and help, to Dr Luigi Nocera for correspondence, and to the SERC for financial support.

References

[1] B. Roberts, in: *Proc 4th European Meeting on Solar Physics. The Hydromagnetics of the Sun*, Noordwijkerhout, ESA SP-220, The Netherlands, 1984, p. 137.
[2] G. Mann, *J. Plasma Phys.* **40**, 281 (1988).
[3] B.K. Som, B. Dasgupta, V.L. Patel, M.R. Gupta, *Phys. Fluids B* **1**, 2340 (1989).
[4] J.V. Hollweg, S. Jackson, D. Galloway, *Solar Phys.* **75**, 35 (1982).
[5] J.-I. Sakai, B.U. Ö Sonnerup, *J. Geophys. Res.* **88**, 9069 (1983).
[6] W. Sahyouni, I. Zhelyazkov, P. Nenovski, *Sol. Phys.* **115**, 17 (1988).
[7] P.K. Shukla, G. Feix, L. Stenflo, *Astrophys. Space Sci.* **147**, 383 (1988).
[8] J.-I. Sakai, H. Washimi, *Astrophys. J.* **258**, 823 (1982).
[9] S.V. Vladimirov, L. Stenflo, M.Y. Yu, *Phys. Lett.* **153**, 144 (1991).
[10] K. Murawski, E. Infeld, J. Ziemkiewicz, *Wave Motion* **13**, 337 (1991).
[11] D.G. Wentzel, *Sol. Phys.* **39**, 129 (1974).
[12] E. Priest, *Solar Magnetohydrodynamics*, Reidel, Dordrecht, 1982.
[13] M. Ruderman, *Sol. Phys.* **131** , 11 (1991).
[14] L. Nocera, E.R. Priest, *Geophys. Astrophys. Fluid Dyn.* **37**, 193 (1986).
[15] L. Nocera, E.R. Priest, J.V. Hollweg, *Geophys. Astrophys. Fluid Dyn.* **35**, 111 (1986).
[16] M. Ruderman, *Fluid Dyn.* **22**, 299 (1987).
[17] E. Mjølhus, J. Wyller, *Phys. Scr.* **33**, 442 (1986).
[18] K. Murawski, *Z. Natforsch. A* **41**, 1186 (1986).
[19] A. Jeffrey, T. Kawahara, *Asymptotic Methods in Nonlinear Wave Theory*, Pitman, London 1982.
[20] K. Murawski, *Aust. J. Phys.* **40**, 593 (1987).
[21] T. Taniuti, C.C. Wei, *J. Phys. Soc. Jpn.* **24**, 941 (1968).

[22] J.P. Boris, D.L. Book, *J. Compt. Phys.* **20**, 397 (1976).
[23] D. Schnack, J. Killeen, *J. Comput. Phys.* **35**, 110 (1980).
[24] N. Asano, *J. Phys. Soc. Jpn* **36**, 861 (1974).
[25] L. Stenflo, *J. Phys. A, Math. Gen.* **21**, L499 (1988).
[26] L. Stenflo, M.Y. Yu, P.K. Shukla, *Phys. Scr.* **40**, 257 (1989).