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THE INFLUENCE OF SURFACE ROUGHNESS ON EQUILIBRIUM STATES OF SUPERFLUID ^{3}He IN CONFINED GEOMETRY

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The superfluid ^{3}He is considered in confined geometry for arbitrary thickness when surface roughness is taken into account. The equilibrium states, the critical size when the two-dimensional state is realized exclusively, and the critical thickness below which superfluidity is destroyed are defined, found and discussed with reference to the essential parameters of a system. Some particular cases are examined analytically. The examples of exact numerical solutions are presented.

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1. Introduction

The progress in experimental research of superfluid He^3 in confined geometry disposes us to give some theoretical explanations and predictions. Although recently much attention has been paid to this problem [1–9] the presented results have been obtained under some restrictions imposed on thickness of a system and concerning the equilibrium state symmetry (pure planar state) [1–4] or on wall boundary conditions (specular reflecting surface) [5–8]. In the present paper we generalize the problem and take into account the surface irregularities. Additionally we treat the thickness of the flat system as a continuous external parameter, which manifests in dependence of the equilibrium state on the system size [8]. Nevertheless we still omit the quantum interference effects, since for the system thickness much longer than the interparticle separation (i.e. the Fermi wavelength $1/p_F$) they can be easily averaged out [5].

2. Formalism

We apply the self-consistent Green function formalism in the framework of weak-coupling approximation where the flat geometry of the system is induced in accordance with Ref. [1]. We consider the slab geometry with one diffusely reflecting surface and a facing specular surface parallel to xy-plane and located at L distance. The separation length L can vary from the distance d such that $1/p_F \ll d \leq \xi_0$, where $p_F = (3\pi^2\rho)^{1/3}$ is the Fermi momentum of a bulk system, ρ — density of a bulk system and $\xi_0 \sim 40$ nm is the superfluid coherence length at zero temperature (i.e. the de Broglie length of the Cooper pair), to infinity which corresponds to a standard bulk system. The geometry of the problem causes that the perpendicular momentum component p_z is quantized, i.e. $p_z = p_F n/N_0$ where $n = 0, 1, \ldots, N-1$, $N_0 = p_F L/\pi$ and $N = [N_0]+1$ (here $[N_0]$ denotes the nearest integer less than N_0). Note that in analogy to Refs. [4, 5] we can write $N_0 = N - \beta$ $(0 < \beta \leq 1)$ and since the β parameter expresses local fluctuations of the distance L, all macroscopic magnitudes should be averaged over β. This statement stays in opposition to results achieved in Ref. [2] where the discrete spectrum of single-quasiparticle excitations led to effects resembling the de Haas–van Alphen effect. We claim that because of the local fluctuation we are not able to control the thickness of the system as a whole, hence merely microscopic magnitudes oscillate locally together with fluctuation of L, which cannot determine the realistic physical results [4–6]. Moreover in Ref. [6] it was proved that the averaging over β parameter modifies the physical results for very thin films only (i.e. if L separation length is comparable with interparticle distance $L \sim 1/p_F$), and therefore this effect can be neglected for the considered thickness scale.

For the diffusely scattering boundary surface we employ a versatile "Randomly Rippled Wall" model and a one-parameter (Gaussian) model of the surface bump distribution proposed by Chaplik and Entin [10] and adopted by Tešanović and Valls [1]. Such model constitutes the simplified problem of Dirac's delta type, where only the average height of surface irregularities is included. The extended two parameter model with surface *bump height* and *width* can be introduced in accordance with Ref. [11]. Some results reached for such approximation can be found in Ref. [3], where it was pointed out that all gap components are suppressed and more so as *bump height* increases relative to *width*. Since this effect seems to be small, however, we restrict ourselves to the one-parameter model.

In order to consider the specified problem we use the Matsubara–Green function formalism exact for systems with confined geometry and porous substrate surface roughness. Since such formalism was thoroughly elaborated in [1] and supplemented in [2] we do not report it here in detail, specifying only modifications which appear in relation to standard three-dimensional Green function formalism. Namely, due to the confined slab geometry the ϑ-angle integration is replaced by appropriate summation over n. Thus, the averaging over the Fermi surface reduces to the form [6]:

$$\langle \ldots \rangle_{\text{bulk}} = \int \frac{d\Omega}{4\pi} \ldots \longrightarrow \langle \ldots \rangle = \int_0^{2\pi} \frac{d\varphi}{2\pi} \frac{1}{N_0} \sum_{n=0}^{N-1} \ldots \tag{1}$$

On the other hand inclusion of the surface roughness causes that the Matsubara frequency ω_k is modified by the normal part of the self-energy $\Sigma_n(i\omega_k)$, and therefore in all basic equations considered below the renormalized form of the frequency $\tilde{\omega}_k(n) = \omega_k + i\Sigma_n(i\omega_k)$ should be used.

Moreover, in contradiction to the papers [1, 2] we admit more general form of the unitary energy gap. It causes that the equilibrium state is determined by such factors as the system size and the surface roughness.

Finally, in order to complete the review of the applied formalism we give some remarks concerning the assumed accuracy. We neglect the effects connected with the integration over the Fermi surface neighbourhood (particle–hole asymmetry) [6]. The chemical potential μ is found self-consistently [1]. We assume additionally that the Cooper pair creates the coherent state oriented in xy-plane (for details see below). Note that although the porous substrate is certainly causing some disorder in the system, this effect, however, can be neglected because it affects the equilibrium state in a very thin film only (in comparison with the considered system size). Such treatment of the problem is fully justified since the omitted effects are of the same order as others standardly ignored, as e.g. strong coupling effects.

Let us now formulate the basic assumptions and equations which allow to solve the specified problem.

2.1. The order parameter

For the assumed symmetry conditions we can neglect the non-unitary states and hence the order parameter can be taken in the following form:

$$\hat{\Delta} = \Delta(\varepsilon) d \cdot \sigma i\sigma_y, \tag{2}$$

where

$$d = \left[(1-\varepsilon)^{1/2}\hat{p}_x,\ (1-\varepsilon)^{1/2}\hat{p}_y,\ (1+2\varepsilon)^{1/2}\hat{p}_z\right] \tag{3}$$

is oriented in xy-plane [12]. For such defined states the ε parameter varies from $\varepsilon = -1/2$ (the pure bulk planar state) to $\varepsilon = 1$ (the pure bulk polar state). For $\varepsilon = 0$ we deal with isotropic BW state. Note that the established parameterization allows us to consider all possible unitary p-wave states simultaneously, using sole parameter ε. The only exception constitutes the ABM (axial) state for which:

$$d = \sqrt{\frac{3}{2}}\,(\hat{p}_x + i\hat{p}_y)\,\hat{z}. \tag{4}$$

Nevertheless, since in the framework of weak-coupling approximation the ABM and planar states are degenerated, and their free energies are equal to each other, we can study the planar state instead.

For the state given by Eq. (3) we can write:

$$\hat{\Delta}\hat{\Delta}^{+} = \Delta^2(\varepsilon)|d|d|^2,\ \ |d|^2 = 1 - \varepsilon + 3\varepsilon p_z^2, \tag{5}$$

which can be also rewritten in the following equivalent form:

$$\hat{\Delta}\hat{\Delta}^{+} = \Delta_2^2(\varepsilon)\,(1-\hat{p}_z^2) + \Delta_1^2(\varepsilon)\hat{p}_z^2 \tag{6}$$

and hence (cf. [3–5]):

$$\Delta_2(\varepsilon) = \Delta(\varepsilon)\,(1-\varepsilon)^{1/2}, \qquad \Delta_1(\varepsilon) = \Delta(\varepsilon)\,(1+2\varepsilon)^{\frac{1}{2}} \tag{7}$$

are the planar and polar (perpendicular) gap components which vanish for $\varepsilon = 1$ and $\varepsilon = -1/2$, respectively. Note that for $\varepsilon = -1/2$ the form of Eq. (5) is identical as for ABM state.

2.2. The inclusion of the surface roughness

The effects of surface roughness on the properties of superfluid ^{3}He in confined geometry are overwhelmingly determined by the pair-breaking nature of random scattering. However, it is assumed that particles experience random impurity scattering only in vicinity of the boundary surface, i.e. within a layer comparatively thin with respect to considered distance L. Therefore these effects can be considered by means of standard perturbation expansion method, and their intensity strongly depend on the system size. Since the more thorough elucidation of the problem is given in Ref. [1], here we discuss only some features of the regarded effects.

The influence of the substrate roughness on the superfluid ^{3}He system within the framework of the considered approach is stated by means of the following parameter [1, 10]:

$$\Gamma_0 = \frac{\pi^4 w^2}{2mL^6} N_0^5, \tag{8}$$

which modifies the self-energy (and subsequently the Matsubara frequency) in the self-consistent manner (see below).

The assumed accuracy allows us to transform Eq. (8) to the form:

$$\Gamma_0 = \frac{\mu y}{\pi^2 l}, \tag{9}$$

where $y = w^2 p_F^4$ parameter is the dimensionless measure of the surface roughness and $l = p_F L/\pi$ is the dimensionless distance. Rewriting Γ_0 parameter in more convenient form we arrive at:

$$\frac{\Gamma_0}{45\pi T_c^{\text{bulk}}} = \frac{\alpha y}{2l}, \qquad \alpha = \frac{2T_F}{45\pi^3 T_c^{\text{bulk}}}. \tag{10}$$

From the experimental data on superfluid ^{3}He [13] we have: $T_c^{\text{bulk}} = 0.8$ mK, $T_F = 1.64$ K at $p = 0$ bar and $T_c^{\text{bulk}} = 2.6$ mK, $T_F = 1.02$ K at $p = 34$ bar (melting pressure), and finally for α we obtain following inequality: $2.939 \geq \alpha \geq 0.562$, that is to say $\alpha \sim 1$. Renormalizing the y parameter we can replace it by $\tilde{y} = \alpha y$. Moreover, we state that the pressure weakens the roughness effects.

3. The basic equations

Since the presented problem constitutes the extension of the considerations contained in Refs. [1, 2] we omit all formal derivations inserted there and present the basic equations suitable for this problem, only.

3.1. The gap equation

In order to determine the state realized in the system with flat geometry and surface roughness we have to solve the gap equation. In the most general case for the p-pairing the gap equation attains the following vectorial form:

$$\Delta d_j = 3f\Delta\hat{p}_k\langle\hat{p}_k P(\varepsilon)d_j\rangle, \tag{11}$$

where $d_j = d_{kj}\,\hat{p}_k$, $f = N(0)g_1$ is dimensionless pairing interaction constant and the kernel $P(\varepsilon)$ will be defined and discussed in detail in the next subsection.

Since in the considered problem the order parameter (2) is assumed to be a function of only two independent parameters ε and Δ, by virtue of Eq. (3) the gap equation (11) resolves itself to the system of two linearly independent scalar forms (cf. [5] and Appendix A):

$$1 = \frac{3}{2}f\langle(1-x^2)P(\varepsilon)\rangle, \tag{12}$$

$$1 = 3f\langle x^2 P(\varepsilon)\rangle. \tag{13}$$

Note that the above equations correspond to the bulk pure planar (axial) and pure polar state gap equations, respectively. In our case, however, for an arbitrary ε the suitable state is realized provided that both equations are satisfied coincidently (cf. [6, 7]).

In order to employ the above equations for the specified problem we can rewrite Eqs. (12) and (13) in the form of the following set of equations:

$$2f^{-1} = (3-a)\langle P(\varepsilon)\rangle - \langle(3x^2-a)P(\varepsilon)\rangle, \tag{14}$$

$$f^{-1} = a\langle P(\varepsilon)\rangle + \langle(3x^2-a)P(\varepsilon)\rangle \tag{15}$$

and solving them we obtain

$$(1-a)f^{-1} = \langle(3x^2-a)P(\varepsilon)\rangle, \tag{16}$$

$$f^{-1} = \langle P(\varepsilon)\rangle. \tag{17}$$

The former equation enables us to determine the equilibrium state for flat geometry (i.e. to determine the ε parameter). Note that for the bulk system $\varepsilon = 0$. Instead, the latest equation allows us to find the energy gap amplitude for the equilibrium state. In this manner we have recovered two independent scalar equations adequate for the description of the problem.

On the other hand multiplying Eq. (12) by $2(1-\varepsilon)/3$ and adding it to Eq. (13) multiplied by $(1+2\varepsilon)/3$ we get an extra equation:

$$f^{-1} = \langle |d|^2 P(\varepsilon) \rangle, \tag{18}$$

which defines the relation between ε and $\Delta(\varepsilon)$ for arbitrary states. We emphasize that the macroscopic forms of the above equations are reached after averaging over the β parameter (cf. Appendix A). Then, Eqs. (16) and (17) reduce to the forms

$$\frac{f^{-1}}{2l} = \langle (3x^2-1) P(\varepsilon) \rangle_{\text{bulk}}, \tag{19}$$

$$f^{-1} = \frac{f^{-1}}{2l} + \langle P(\varepsilon) \rangle_{\text{bulk}}, \tag{20}$$

whereas Eq. (18) receives the form

$$f^{-1} = \frac{(1-\varepsilon) f^{-1}}{2l} + \langle |d|^2 P(\varepsilon) \rangle_{\text{bulk}}. \tag{21}$$

Note that the set of Eqs. (19)–(20) can be reduced to Eq. (21) for all ε. Thus, if $\varepsilon = -1/2$ ($l = l_{c1}$, see below), joint Eqs. (19) and (20) are equivalent to Eq. (21) and for $l < l_{c1} (\varepsilon = -1/2)$ we can consider only the latest equation.

3.2. The kernel $P(\varepsilon)$

Since the kernel $P(\varepsilon)$ derived in the framework of Green function approach possesses a few equivalent representations, we discuss it in detail.

The most general form of $P(\varepsilon)$ taking into account the substrate roughness is as follows:

$$P(\varepsilon) = \frac{2\pi T}{\Delta} \sum_{k=1}^{k_D} \left(\tilde{\omega}_k^2 + |d|^2 \right)^{-1/2}, \tag{22}$$

where $\tilde{\omega}_k = \omega_k + \Gamma_0 U x^2$, $\omega_k = (2k-1)\pi T$ and

$$U = \left\langle \frac{(\omega/\Delta) + U(\Gamma_0/\Delta) x^2}{\left[\left((\omega/\Delta) + U(\Gamma_0/\Delta) x^2 \right)^2 + |d|^2 \right]^{1/2}} x^2 \right\rangle \tag{23}$$

is a function of (ω/Δ), (Γ_0/Δ) and ε; $k_D = [(\omega_D + \pi T)/2\pi T]$, where ω_D denotes the cut-off frequency, for which $\Delta \ll \omega_D \ll \mu$. Note that the k_D parameter is a function of temperature, it tends to infinity if T tends to zero. It causes that the above representation becomes inconvenient for very low temperatures ($T \ll T_c$). Nevertheless neglecting the surface roughness (i.e. for $\Gamma_0 = 0$ and $\tilde{\omega} k = \omega_k$) the $P(\varepsilon)$ kernel can be rewritten in the form for which the frequency ω_k-summation was performed (cf. e.g. [6, 7] and Eq. (B.1)):

$$P(\varepsilon) = \int_0^{\omega_D} d\xi \frac{\tanh\left(E(\varepsilon)/2T\right)}{E(\varepsilon)}, \tag{24}$$

where $E(\varepsilon) = (\xi^2 + \Delta(\varepsilon)|d|^2)^{1/2}$. At that time in the zero-temperature limit the kernel $P(\varepsilon)$ reduces to the following form:

$$P(\varepsilon) = \log\left(\frac{2\omega_D}{\Delta(\varepsilon)|d|}\right). \tag{25}$$

The detailed consideration of the problem with both specularly reflecting walls, using the latter representation is given in Ref. [6].

4. Solutions

The self-consistent solution of Eqs. (16)–(17) together with self-energy renormalization given by Eq. (23) allows us for determination of the ε parameter in function of the system size L, temperature T, pairing constant f and surface roughness Γ_0:

$$\varepsilon = \varepsilon(L;\ T,\ f,\ \Gamma_0). \tag{26}$$

Further, assuming $\varepsilon = -1/2$ we can derive the critical size L_{c1}, defined as the maximal system size for which the planar state is realized exclusively. Thus, we have:

$$L_{c1} = L_{c1}(T,\ f,\ \Gamma_0). \tag{27}$$

Moreover, employing Eq. (18), by analogy to Ref. [1], we can also derive the critical thickness L_{c2}, below which superfluidity is destroyed (i.e. $\Delta = 0$), for arbitrary state, i.e.

$$L_{c2} = L_{c2}(\varepsilon,\ T,\ f,\ \Gamma_0). \tag{28}$$

It is obvious that the above results (26)–(28) for arbitrary parameters T, f and Γ_0 can be in general found only numerically and some numerical results will be presented in the next section. However, now we examine limited cases for which analytical solution is possible.

4.1. The influence of the surface roughness on the transition temperature T_c

Applying the relations given in Appendix A and assuming that Γ_0 is small ($\Gamma_0 \ll T_{c0}$) from Eq. (18) we can estimate the transition temperature T_c and we get (cf. Appendix B):

$$T_c = T_{c0} - \frac{\pi\Gamma_0}{36} K a^2 \frac{1 - \varepsilon + \frac{9}{5}\varepsilon b}{1 - \varepsilon + \varepsilon a}, \tag{29}$$

where (cf. [9])

$$T_{c0} = T_c^{bulk}\left(1 + f^{-1}\frac{1-\varepsilon}{2l}\right). \tag{30}$$

Hence for large L we have $a = b = K = 1$ and making use of Eq. (10) we obtain:

$$T_c = T_{c0}\left[1 - \frac{5\pi^2\alpha y}{8l}\left(1 + \frac{4}{5}\varepsilon\right)\right], \tag{31}$$

where the ε parameter should be properly fitted for fixed l and y.

4.2. The critical thickness L_{c2}

In order to derive the critical thickness L_{c2} below which superfluidity is destroyed we employ Eq. (18). Then in the limit $T_c \to 0$ and for small Δ we obtain (cf. [1]):

$$\frac{2e^{-C}\Gamma_0\eta}{3T_{c0}\pi}\left[1 + \frac{\Delta^2}{8T_{c0}^2}\frac{1 + \frac{4}{5}\varepsilon^2}{K(1 - \varepsilon + 3\varepsilon a)}\right] = 1, \tag{32}$$

where $C = 0.5772$ is the Euler constant, $\eta = \exp\left(-2\left(1 - \frac{2}{3}\varepsilon\right)\right)$. Note that $\eta(\varepsilon = -1/2) = 0.0695 \leq \eta \leq 0.1353 = \eta(\varepsilon = 0)$. Applying further Eq. (10) with the above accuracy we have:

$$l_{c2} = 15e^{+C}\alpha y\eta\kappa\left[1 + \frac{\Delta^2}{8T_{c0}^2}\left(1 + \frac{4}{5}\varepsilon^2\right)\right], \tag{33}$$

where

$$\kappa = 1 - \frac{1 - \varepsilon}{2l_{c2}}\left[f^{-1} - 2\left(1 - \frac{2}{3}\varepsilon\right)\right].$$

Assuming that $\xi_0 = v_F/\pi\Delta_0$ (the standard BCS coherence length) we can write:

$$\frac{L}{\xi_0} = \frac{le^{-C}}{45\alpha}, \tag{34}$$

and hence ($\Delta = 0$, $\kappa = 1$)

$$L_{c2}(y, \varepsilon) = \frac{1}{3}y\eta\xi_0 \tag{35}$$

is an increasing function of ε which for $\varepsilon = -1/2$ is of the same order as in Ref. [1].

4.3. The critical size L_{c1}

The presented theory permits us to define the equilibrium state which is realized in superfluid ^{3}He with confined slab geometry. Since the equilibrium state tends to the planar one when the system size (thickness) lessens, we can also define the critical size L_{c1} below which the pure planar state ($\varepsilon = -1/2$) is realized exclusively [4–7]. The solution of this problem for the case $\Gamma_0 = 0$ was considered in detail in Ref. [6]. Now, we consider it in two particular cases.

4.3.1. $T = 0$ limit

Restricting ourselves to the case of $T = 0$, small y and large l, Eq. (19) reduces to the form:

$$\frac{f^{-1}}{2l} = Y(\varepsilon) - \pi e^{-C}\frac{\alpha y}{2l}, \tag{36}$$

where

$$Y(\varepsilon) = -\frac{1}{2}\langle(3x^2 - 1)\log(1 - \varepsilon + 3\varepsilon x^2)\rangle,$$

$$Y(0) =), \qquad Y\left(-\frac{1}{2}\right) = \frac{1}{3} \qquad \text{and} \qquad \frac{dY}{d\varepsilon} < 0\,.$$

From the above for $\varepsilon = -1/2$ we obtain:

$$l_{c1} = \frac{3}{2}\left(f^{-1} + \pi e^{C}\alpha y\right). \tag{37}$$

The imposed restrictions induce the equivalent conditions as discussed in the previous section of this paper. Therefore the obtained critical magnitudes can be compared. Employing Eq. (33) we get:

$$l_{c1} - l_{c2} = \frac{3}{2}f^{-1} + l_{c2}\frac{\pi - 10\eta}{10\eta}. \tag{38}$$

Note that the critical size is larger than the critical thickness, $L_{c1} > L_{c2}(\varepsilon)$, for arbitrary ε. It proves that lessening system size always at first the planar $2D$-state is achieved and next, provided that $y > 0$, superfluidity is destroyed. Consequently in previous subsection we can put $\varepsilon = -1/2$ in all obtained results.

4.3.2. $T \approx T_c$ limit

If we assume that $T \to T_c$ and y is small, for large l Eq. (19) reduces to the form:

$$\frac{f^{-1}}{2l} = -\frac{7\xi(3)}{10\pi^2}\frac{\Delta^2}{T_{c0}^2} - \frac{\alpha y}{2l}. \tag{39}$$

Thus,

$$\varepsilon = -\frac{5(f^{-1} + \alpha y)\pi}{8p_F L}\left(1 - \frac{T}{T_{c0}}\right)^{-1} \tag{40}$$

defines the equilibrium state of superfluid ^{3}He in flat geometry, and for $\varepsilon = -1/2$ we obtain the critical size in the form:

$$L_{c1}(y) = \frac{5(f^{-1} + \alpha y)\pi}{4p_F}\left(1 - \frac{T}{T_{c0}}\right)^{-1}, \tag{41}$$

and hence $L_{c1}(y) \geq L_{c1}(0)$.

The obtained results (37) and (41) display that the surface roughness effects enlarge the critical size L_{c1}. On the other hand the y parameter can be treated as a factor which renormalizes the pairing interaction parameter f giving rise to its reduction. Since the parameters f^{-1} and α are decreasing functions of the pressure, $L_{c1}(y)$ also lessens together with pressure enhancement for the fixed roughness y.

5. Numerical results

The results obtained in the previous section allow us to recognize the behaviour and general properties of superfluid ^{3}He in confined slab geometry. We can also roughly estimate the obtained results. Assuming that $p_F \sim 10^8$ cm^{-1} and $3 \leq f \leq 5$ ($f^{-1} = \log(2\omega_D/\Delta_0)$) we get $L_{c1}(0) \sim 10^{-6}$ cm, so $L_{c2}(0) \leq L_{c1}(0) \sim \xi_0$, which is approximated in Refs. [1, 5]. We can observe that the influence of rough (diffusely scattering) substratum exhibits in considerable enhancement of critical parameters.

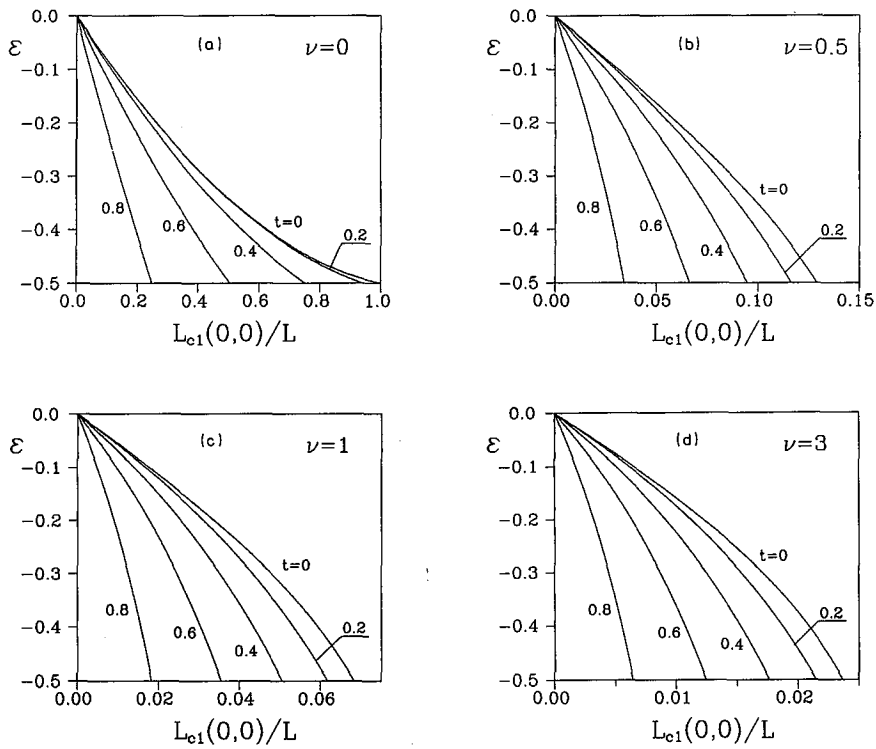


Fig. 1. The equilibrium state structure parameter ε versus reduced inverse size of the system $[L/L_{c1}(0,\,0)]^{-1}$ for chosen reduced temperatures $t = T/T_c = 0$, 0.2, 0.4, 0.6, and 0.8, for distinct values of effective surface roughness parameter: a) $\nu = 0$ — specularly reflecting wall; b) $\nu = 0.5$; c) $\nu = 1.0$; d) $\nu = 3.0$ — diffusely reflecting wall.

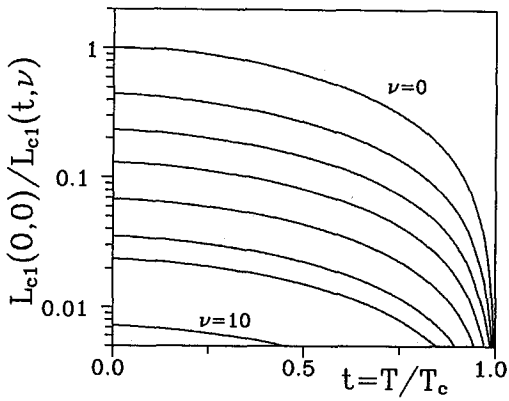


Fig. 2. The inverse reduced critical size $[L_{c1}(t, \nu)/L_{c1}(0, 0)]^{-1}$ versus reduced temperature $t = T/T_c$ for chosen values of ν effective roughness parameter: ν = 0, 0.1, 0.25, 0.5, 1.0, 2.0, 3.0, and 10.0 from top to bottom, respectively. Note that the ordinates are plotted in a logarithmic scale.

In order to solve Eqs. (19)–(21) generally we have to employ numerical techniques. In numerical analysis of the problem it is convenient to introduce dimensionless variables:

$$t = T/T_c, \qquad \nu = \alpha f y, \tag{42}$$

the reduced (dimensionless) temperature and effective roughness parameter, respectively. The T_c temperature is self-consistently determined for appropriate system thickness L and surface roughness ν. The ν parameter varies from $\nu = 0$ for a specularly reflecting surface to $\nu \sim 10$ corresponding to a strongly diffusely scattering boundary. Note also that the size of the system L can be also measured in dimensionless units related to zero-temperature coherence length ξ_0 or equivalently $L_{c1}(0, 0)$ critical size (see above).

The equilibrium states defined univocally by ε parameter are presented in Fig. 1 in function of reduced inverse size of the system, reduced temperature and effective roughness. Fig. 1a corresponds to specularly reflecting boundary, while Figs. 1b–1d to diffusely reflecting wall. For infinite (three-dimensional) system for arbitrary temperature t and surface roughness ν, as we expected, we have sole equilibrium state with $\varepsilon = 0$, i.e. the BW-isotropic state. Note also that for $\nu = 0$ (see Fig. 1a) all plots have convex curvature, in contrast to $\nu \neq 0$ case, where the appropriate dependence of ε on system inverse size $1/L$ exhibits the concave curvature. On the other hand it can be easily noticed that the rough boundaries cause that the equilibrium state is more sensitive to temperature changes (especially close to zero temperature, see Fig. 1). Therefore we claim that the inclusion of roughness of the boundary surface cannot be reduced to simple renormalization of abscissa, i.e. introduction of linearly renormalized length-scale.

The critical size L_{c1}, for which the pure two-dimensionless (planar, for which $\varepsilon = -1/2$) state is realized exclusively, is plotted in Fig. 2, in function of reduced

temperature and effective surface roughness ν. Note that the ordinates corresponding to reduced inverse critical size $L_{c1}(0, 0)/L_{c1}(t, \nu)$ are plotted in logarithmic scale. Let us emphasize that for the surface roughness parameter $\nu \sim 1$ ($\nu \sim 10$) the L_{c1} is approximately enhanced by one (two) order in magnitude with respect to specularly reflecting case. Note that as $t \to 1$ (i.e. $T \to T_c$) the critical size L_{c1} tends to infinity.

The energy gap amplitudes for the equilibrium states are presented in Figs. 3 and 4, in function of reduced inverse system size, reduced temperature and effective surface roughness. In Fig. 3 and Fig. 4 the transversal (polar) Δ_1 and longitudinal (planar) Δ_2 components of gap energy are plotted, respectively. From Fig. 3, for arbitrary ν substrate roughness parameter, it can be seen that while reducing the system size L, $\Delta_1(t, \nu)$ diminishes and finally vanishes at critical size $L_{c1}(t, \nu)$. The above is in contrast to the planar $\Delta_2(t, \nu)$ gap amplitude behaviour, which slightly increases while the system size lessens from infinity to the L_{c1} thickness.

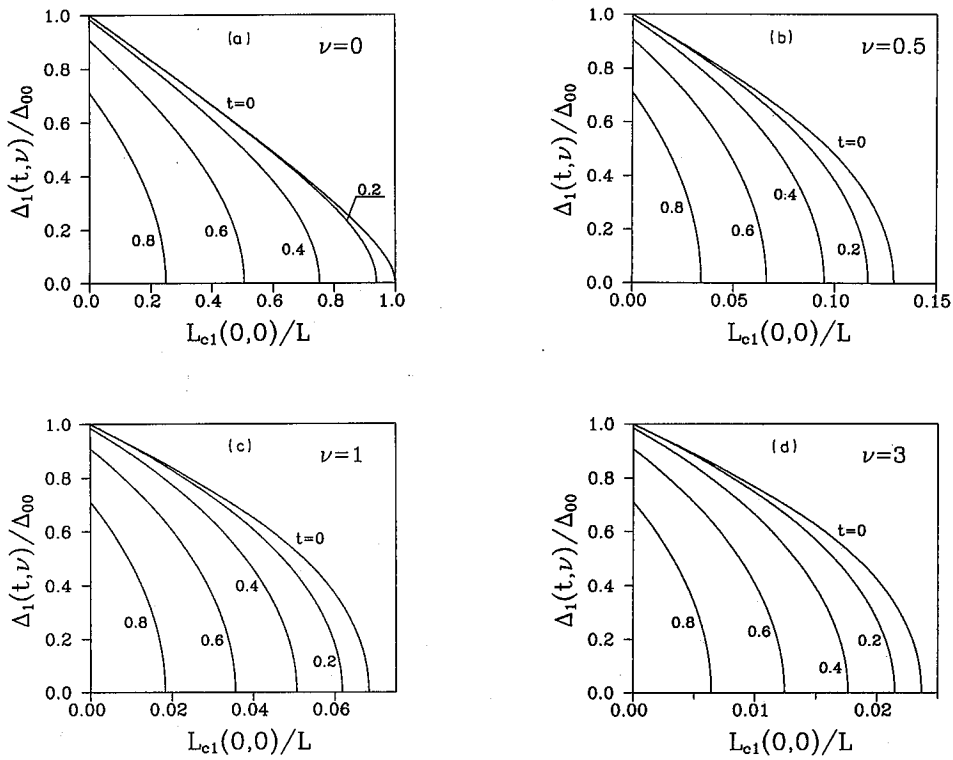


Fig. 3. The reduced transversal (polar) gap amplitude $\Delta_1(t, \nu)/\Delta_{00}$ for equilibrium state versus reduced inverse size of the system $[L/L_{c1}(0, 0)]^{-1}$ for chosen reduced temperatures $t = T/T_c = 0$, 0.2, 0.4, 0.6, and 0.8, for distinct values of effective surface roughness parameter: a) $\nu = 0$ — specularly reflecting wall; b) $\nu = 0.5$; c) $\nu = 1.0$; d) $\nu = 3.0$ — diffusely reflecting wall.

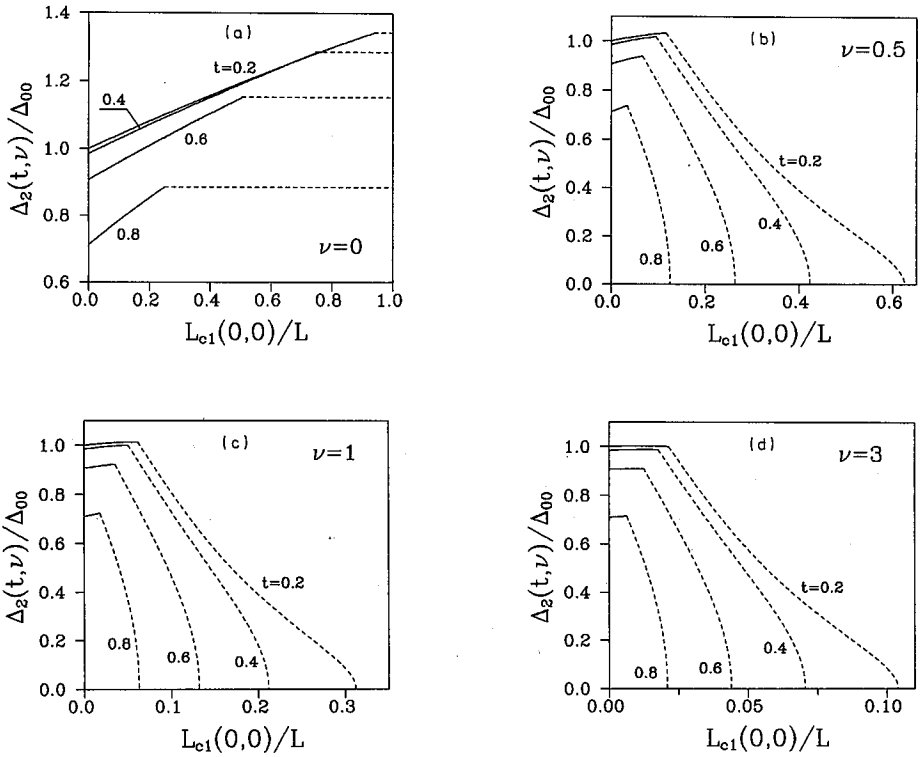
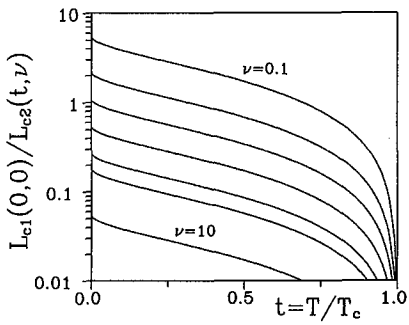


Fig. 4. The reduced transversal (polar) gap amplitude $\Delta_1(t, \nu)/\Delta_{00}$ for equilibrium state versus reduced inverse size of the system $[L/L_{c1}(0, 0)]^{-1}$ for chosen reduced temperatures $t = T/T_c = 0.2$, 0.4, 0.6, and 0.8, for distinct values of effective surface roughness parameter: a) $\nu = 0$ — specularly reflecting wall; b) $\nu = 0.5$; c) $\nu = 1.0$; d) $\nu = 3.0$ — diffusely reflecting wall. The dashed lines correspond to $L_{c2} \leq L \leq L_{c1}$, where the planar state is realized exclusively.

Since at L_{c1} critical size the pure two-dimensional state is realized exclusively, further reduction of the size L alters only the planar Δ_2 gap amplitude, what is depicted in Fig. 4 (the dashed lines). For specularly reflecting boundary the planar state remains unchanged and exists for arbitrary thickness $L \leq L_{c1}(t, \nu)$, including pure two-dimensional system ($L = 0$), cf. Fig. 4a. For the case $\nu \neq 0$ (i.e. diffusely reflecting walls) Δ_2 amplitude diminishes while the system size lessens $L \leq L_{c1}$, and finally vanishes for certain system thickness L_{c2}, i.e. the pure planar state exists exclusively for $L_{c2} \leq L \leq L_{c1}$ (cf. Figs. 4b–4d).

Finally in Fig. 5 and Fig. 6 we present temperature and ν-parameter dependence of L_{c2} critical thickness (below which the superfluidity is destroyed). Note that in Fig. 5 the ordinates $L_{c1}(0, 0)/L_{c2}(t, \nu)$ are plotted in logarithmic scale. From Fig. 6, where we present the L_{c2} dependence on ν roughness parameter for chosen temperatures t, we see that the dependence of L_{c2} on ν for non-zero



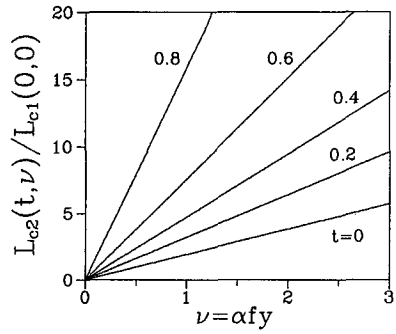


Fig. 5. The inverse reduced critical thickness $[L_{c2}(t, \nu)/L_{c1}(0, 0)]^{-1}$ versus reduced temperature $t = T/T_c$ for chosen values of ν effective roughness parameter: $\nu = 0.1$, 0.25, 0.5, 1.0, 2.0, 3.0, and 10.0 from top to bottom, respectively. Note that the ordinates are plotted in a logarithmic scale.

Fig. 6. The reduced critical thickness $L_{c2}(t, \nu)/L_{c1}(0, 0)$ versus effective surface roughness ν parameter for chosen values of reduced temperature $t = T/T_c = 0$, 0.2, 0.4, 0.6, and 0.8.

temperatures is almost linear, similarly as for the zero temperature case (cf. prediction given by Eq. (35)).

6. Conclusions

The presented formalism allows us to make a thorough study of the behaviour of superfluid ^{3}He in confined geometry when the thickness of the system lessens from infinity (the bulk system) to a size of order of the zero-temperature coherence length (the thin film-like system). According to the obtained results the equilibrium state, parameterized by transversal (perpendicular to the boundary surface) and longitudinal (parallel) gap amplitudes, becomes anisotropic and is altered in a continuous manner finally achieving the two-dimensional structure (i.e. the transversal gap vanishes) at the critical size L_{c1}. Further, provided that the surface roughness is included the amplitude of the planar (longitudinal) energy gap is rapidly reduced and at the critical thickness L_{c2} superfluidity is destroyed. These both critical parameters increase if effects connected with surface roughness become more intensive. Moreover, the same effects modify equilibrium state. Simplifying the problem one can say that these effects lead to the renormalization of the pairing interaction parameter f causing its diminution. Since the influence of the confined geometry becomes more extensive if the coupling parameter is small, the rough substratum intensifies properties of superfluid ^{3}He involved by the confined slab geometry.

Concluding we emphasize that the presented results are high-rigour for sufficiently large thickness L, thereby the surface roughness makes them more precise. Moreover, if the system size satisfies condition $L_{c2} \leq L \leq L_{c1}$, the equilibrium

state is two-dimensional and can be identified with the ABM (axial) state similarly as in Refs. [1, 2].

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Appendix A

In order to simplify the forms of the presented results we introduce the following symbols:

$$x = \frac{n}{N_0} = \frac{p_z}{p_F} = \hat{p}_z = \cos\vartheta_n, \tag{A.1}$$

which is the discrete variable if $L < \infty$.

Moreover, we apply the following formulas (cf. Eq. (1)):

$$\langle 1\rangle = K, \qquad \langle x^2\rangle = \frac{1}{3}aK,$$

$$\langle 3x^2 - a\rangle = 0, \qquad \langle x^4\rangle = \frac{1}{5}abK, \tag{A.2}$$

where

$$a = K^2\left(1 - \frac{1}{N}\right)\left(1 - \frac{1}{2N}\right), \qquad K = \left(1 - \frac{\beta}{N}\right)^{-1},$$

$$b = K^2\left(1 - \frac{1}{N} - \frac{1}{3N^2}\right). \tag{A.3}$$

The above formulae after the macroscopic averaging over β for large N (or equivalently l) reduce to the form:

$$a = 1 - \frac{1}{2l}, \qquad b = 1 \quad \text{and} \quad K^j = 1 + \frac{j}{2l}, \tag{A.4}$$

where we consistently replaced N by l (cf. [6]). Moreover, for large N (or l) and $j > 0$ we can write (cf. [14]):

$$\langle x^{2j}\rangle = \frac{K^{2j+1}}{2j+1}\left(1 - \frac{2j+1}{2N}\right) \tag{A.5}$$

and hence averaging over β (Eq. (A.4)) and with the assumed accuracy we get

$$\langle x^{2j}\rangle_\beta = \frac{1}{2j+1}. \tag{A.6}$$

Thus, we can formulate the following relation (valid for arbitrary j):

$$\langle x^{2j}\rangle_\beta = \frac{\delta_{0j}}{2l} + \langle x^{2j}\rangle_{\text{bulk}}, \tag{A.7}$$

which allows us to simplify our calculations. Note that in the presented approach we consistently neglect all terms of the order l^{-2} and higher.

Appendix B

In order to obtain the presented results we employ the following relations:

$$\tanh\left(\frac{\pi z}{2}\right) = \frac{4z}{\pi}\sum_{k=1}^{+\infty}\left((2k-1)^2+z^2\right)^{-1}, \tag{B.1}$$

$$\sum_{k=1}^{+\infty}\left(\frac{1}{2k-1+z}-\frac{1}{2k-1}\right) = \frac{1}{2}\left(\psi\left(\frac{1}{2}\right)-\psi\left(\frac{1+z}{2}\right)\right), \tag{B.2}$$

$$\sum_{k=1}^{+\infty}\frac{1}{(k+z)^2} = \frac{\mathrm{d}}{\mathrm{d}z}\psi(z), \tag{B.3}$$

where $\psi(z)$ is the digamma function and $\psi\left(\frac{1}{2}\right) = -\log(4\mathrm{e}^C)$ Moreover we use the well known digamma asymptotics [14]:

$$\psi\left(\frac{1+z}{2}\right) = \psi\left(\frac{1}{2}\right) + \frac{1}{2}\pi^2 z, \qquad \text{for} \quad z \ll 1, \tag{B.4}$$

$$\psi(z) = \log(z) - \frac{1}{2z} - \frac{1}{12z^2}, \qquad \text{for} \quad z \gg 1, \tag{B.5}$$

and

$$\sum_{k=1}^{k_{\mathrm{D}}}\frac{1}{2k-1} = \frac{1}{2}\log(4\mathrm{e}^C k_{\mathrm{D}}) + \frac{1}{48k_{\mathrm{D}}^2}, \tag{B.6}$$

for sufficiently large k_{D}.

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