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TRANSMISSION COEFFICIENT FOR A DOUBLE-BARRIER QUANTUM WELL STRUCTURE

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In this paper the transmission coefficient for a double-barrier quantum well (DBQW) structure as a function of applied voltage is calculated, for the first time, using WKB approximation. This approach allows to discuss a dependence of several quantities characteristic of the system (e.g. the value of the coefficient, resonance voltage, charge stored in the well) on the barrier and the well parameters.

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Recent advances in the technology of growth of semiconductor heterostructures have stimulated a great deal of interest, both experimental and theoretical, in the phenomenon of resonant tunneling through double-barrier quantum well (DBQW) structures. Elementary understanding of the resonant tunneling usually bases on the consideration of an one-dimensional potential energy profile created by the bottom of conduction band of the DBQW structure. The general analysis of phenomena occurring in such systems has been given by Ricco and Azbel [1]. In spite of the simplicity of their approach these authors did not avoid repeating the mistake which had already appeared in the early paper by Kane [2] (see the expression for the global transmission coefficient given by Eq. (1) in [1]).

In the present work we have calculated the transmission coefficient T for a DBQW structure given in Fig. 1, using quasi-classical WKB approximation. The calculation is restricted to the electron energies which do not exceed the height of any barrier. The DBQW profile consists of three regions, namely of two barriers with the heights of V_1 and V_2 and the widths of b_1 and b_2, respectively, and of the potential well with the width of w confined between the barriers. The applied voltage V results in the electric field F which is assumed being constant over any particular region. The physical quantities in the respective regions are referred by the indices $b1$, w, $b2$ in the structure and outside the structure — by the indices

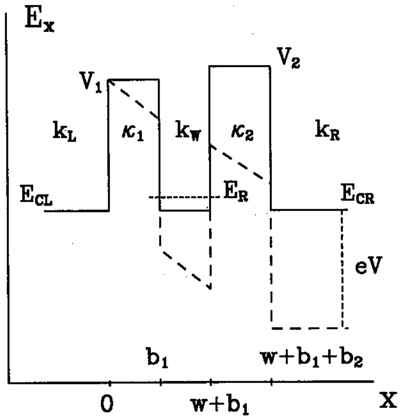

Fig. 1. Profile of the potential energy of an electron in the DBQW structure without (solid-) and with an applied bias (dashed- lines).

L and R for the left and right side, respectively. Distinguishing explicitly between kinetic energies E_i and the effective masses m_i of an electron in various regions allows for direct use of the results of our calculations for modeling structures with a variety of band offsets and crystal compositions.

The transmission coefficient for a DBQW structure under bias takes the following form:

$$T = \frac{4\sin 2\beta_1 \sin 2\beta_2 \sin 2\beta_3 \sin 2\beta_4}{M} \tag{1}$$

where the denominator M is given as

$$M = S_1^2 e^{2(\gamma_1+\gamma_2)} + S_2^2 e^{2(\gamma_2-\gamma_1)} + S_3^2 e^{2(\gamma_1-\gamma_2)} + S_4^2 e^{-2(\gamma_1+\gamma_2)}$$

$$+ S_5 e^{2\gamma_1} + S_6 e^{2\gamma_2} + S_7 e^{-2\gamma_1} + S_8 e^{-2\gamma_2} + S_9. \tag{2}$$

The prefactors S_i are given by trigonometric functions:

$$S_1 = \sin[\alpha(b_1+w) - \beta_2 - \beta_3],$$

$$S_2 = \sin[\alpha(b_1+w) + \beta_2 - \beta_3],$$

$$S_3 = \sin[\alpha(b_1+w) - \beta_2 + \beta_3],$$

$$S_4 = \sin[\alpha(b_1+w) + \beta_2 + \beta_3],$$

$$S_5 = -2S_1S_3\cos 2\beta_4,$$

$$S_6 = -2S_1S_2\cos 2\beta_1,$$

$$S_7 = -2S_2S_4\cos 2\beta_4,$$

$$S_8 = -2S_3S_4\cos 2\beta_1,$$

$$S_9 = 2S_1S_4\cos[2(\beta_1+\beta_4)] + 2S_2S_3\cos[2(\beta_1-\beta_4)],$$

where the components of the arguments are:

$$\alpha(x) = \frac{2}{3}\left(\frac{2m_w}{\hbar}\right)^{1/2}\left\{[E_w + eF_w x]^{3/2} - [E_w + eF_w b_1]^{3/2}\right\}\frac{1}{eF_w},$$

$$\beta_1 = \mathrm{arctg}\frac{m_\mathrm{L}\kappa_1(0)}{m_{b1}k_\mathrm{L}},$$

$$\beta_2 = \mathrm{arctg}\frac{m_w\kappa_1(b_1)}{m_{b1}k_w(b_1)},$$

$$\beta_3 = \mathrm{arctg}\frac{m_w\kappa_2(b_1+w)}{m_{b2}k_w(b_1+w)},$$

$$\beta_4 = \mathrm{arctg}\frac{m_\mathrm{R}\kappa_2(b_1+b_2+w)}{m_{b2}k_\mathrm{R}},$$

and

$$k_\mathrm{L} = \left[\frac{2m_\mathrm{L}}{\hbar^2}E_\mathrm{L}\right]^{1/2}, \qquad k_w(x) = \left[\frac{2m_w}{\hbar^2}(E_w - eF_w x)\right]^{1/2},$$

$$k_\mathrm{R} = \left[\frac{2m_\mathrm{R}}{\hbar^2}E_\mathrm{R}\right]^{1/2}, \qquad \kappa_1(x) = \left[\frac{2m_{b1}}{\hbar^2}(V_1 - eF_{b1}x - E_{b1})\right]^{1/2},$$

$$\kappa_2(x) = \left[\frac{2m_{b2}}{\hbar^2}(V_2 - eF_{b2}x - E_{b2})\right]^{1/2}.$$

The exponents γ_i in Eq. (2) are given by

$$\gamma_1 = \frac{2}{3}\left(\frac{2m_{b1}}{\hbar^2}\right)^{1/2}\left[(V_1 - E_{b1})^{3/2} - (V_1 - E_{b1} - eF_w b_1)^{3/2}\right]\frac{1}{eF_{b1}},$$

$$\gamma_2 = \frac{2}{3}\left(\frac{2m_{b2}}{\hbar^2}\right)^{1/2}\left\{[V_2 - E_{b2} - eF_{b2}(b_1+w)]^{3/2} - [V_2 - E_{b2} - eF_{b2}(b_1+b_2+w)]^{3/2}\right\}\frac{1}{eF_{b2}}.$$

As the simplest approximation we assume no space charge within the DB structure which leads to the following expressions for electric fields:

$$F_{b1} = F_{b2} = F_b = \frac{V}{b_1 + b_2 + (\varepsilon_b/\varepsilon_w)w}, \qquad F_w = \frac{\varepsilon_b}{\varepsilon_w}\cdot F_b \tag{3}$$

where ε_b and ε_w are the dielectric constants in the barriers and in the well, respectively. For a given bias voltage the transmission coefficient has a sharp maximum at

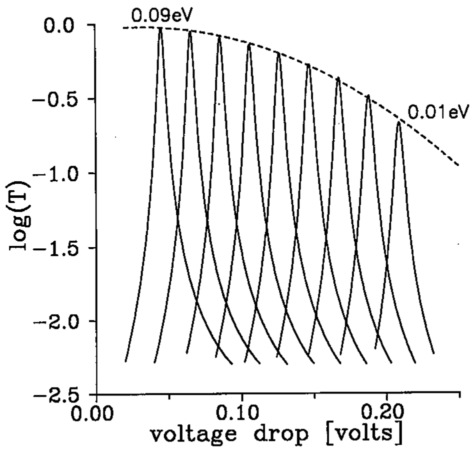

Fig. 2. Transmission coefficient as a function of applied bias for various energies of tunneling electrons (from 0.01 to 0.09 eV in 0.01 eV steps). Calculation for the DBQW structure with $b_1 = b_2 = w = 40$ Å, $V_1 = V_2 = 0.4$ eV and $m_i = 0.07m_0$.

the resonance energy $E = E_{\mathrm{RES}}$ which is calculated as the solution of the equation $S_1 = 0$, i.e., the resonant shape of $T(E, V)$ is due to the denominator $M(E, V)$. The dependence of the transmission coefficient on the bias voltage is shown in Fig. 2 for various energies of tunneling electrons. The dashed line represents the dependence of the maximal transmission on the bias voltage for a given DBQW structure.

The simplified approach given here presents the advantage of an analytical solution and hence it can be used for discussion of various aspects of resonant tunneling. In this calculation the wave functions of an electron incident upon the structure, transmitted, reflected and trapped in the well have been calculated. They can be used for analysis of the charge of electrons "stored" in the well which is to be the subject of another paper.

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