SOLITONS IN A ONE-DIMENSIONAL
DEGENERATE HUBBARD MODEL

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An effective magnon-lattice Hamiltonian for the degenerate Hubbard chain
with electron-phonon interaction is derived and a formalism for the description
of solitary magnons is presented.

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1. Introduction

Over the past few years many papers have been published on the one-dimen-
sional systems dealing with the question whether equation of motion for them admit
the existence of soliton solutions (e.g.[1-9]). One of the possible sources of non-
linearity is the electron-phonon coupling. Pushkarov et al. [4] considered a soliton
formation in a ferromagnetic Heisenberg chain caused by the magnon-phonon cou-
pling. The present paper extends the problem into itinerant system. The following
discussion is stimulated by a recent discovering of quasi one-dimensional organic
ferromagnets [10] which are believed to be well described by the one-dimensional
doubly degenerate Hubbard model (DDH) [11]. It is well-known that molecular
 crystals are good candidates for the occurrence of solitons. The widely discussed
 Davydov soliton [12] describing the transport of intramolecular energy and elec-
 trons along α-helical protein molecules is the best known example.

Molecular crystals are mechanically soft (small elasticity coefficient) and due
to the sensitivity of π-orbital overlap on the local distortion a strong electron-phonon
coupling is expected.

The aim of the present paper is to give a general schema of looking for
the soliton solution in itinerant ferromagnets and DDH is chosen only because it
is the simplest itinerant model having a ferromagnetic Hartree-Fock ground state
[13, 14]. A complex question of whether these solutions represent stable
2. Model

We consider the doubly degenerate Hubbard chain coupled to the harmonic lattice

\[ H = H_e + H_L + H_{e-L}, \]  

where \[ H_e = \sum_{i\lambda\sigma} t(c_{i+1,\lambda\sigma}^+ c_{i\lambda\sigma} + \text{h.c.}) + \sum_{i\lambda} U n_{i\lambda+} n_{i\lambda-} + \sum_{i\sigma\sigma'} V n_{i\sigma} n_{i\sigma'}, \]

\[ -\sum_{i\sigma\sigma'} J c_{i\sigma}^+ c_{i+1\sigma} c_{i+1\sigma'} c_{i\sigma'}, \]  

where \( c_{i\lambda\sigma}^+ \) is the electron creation operator at the site \( i \) in the \( \lambda \) orbital \((\lambda = 1, 2)\) with spin \( \sigma \). The parameter \( t \) is a hopping integral between nearest-neighbor sites, and we assume that the two \( \lambda \) orbitals are not mixed by the hopping. The second term, proportional to \( U \), represents the intraorbital Coulomb repulsion. The third term, proportional to \( V \), describes the interorbital Coulomb repulsion and the final term containing \( J \) represents the exchange interaction.

In the degenerate model it is possible to have an ordering of the orbital states besides the magnetic order \([14-18]\). From now on, we will consider the average electron density per site \( n \) not much different from unity, what corresponds to the occupation in the earlier mentioned ferromagnetic organic systems \([10, 11]\). In the strong coupling limit at \( T = 0 \) the ground state for \( n \neq 1 \) is ferromagnetic with simultaneous orbital order of "antiferromagnetic" type \([14, 15]\). For \( n \neq 1 \) ferromagnetic ordering with no orbital order is also possible (high values of \( J \) are not favourable to an orbital superlattice \([19]\)). The lattice part of the Hamiltonian \( H_L \) is described in the harmonic approximation by:

\[ H_L = \frac{m}{2} \sum (u_i)^2 + \frac{m v_0^2}{2} \sum (u_{i+1} - u_i)^2, \]  

where \( m \) is the mass of the molecule in the chain, \( u_i \) is the molecule displacement from the equilibrium position and \( v_0 \) is the sound velocity. The lattice constant is chosen as unity.

The interaction between the electron subsystem and the lattice, in the linear approximation with respect to small deviations \( u_i \), reads

\[ H_{e-L} = \sum_{i\lambda\delta = \pm 1} t_1(\delta)(u_{i+\delta} - u_i) c_{i\lambda\sigma}^+ c_{i+\delta,\lambda\delta}, \]

\[ t_1(\delta) = -t_1(-\delta). \]

In general the above coupling can influence both the ground state and the excitation spectrum. In the preliminary discussion presented here we neglect the former,
assuming that the strength of the electron–lattice coupling compared with the correlation and exchange energies is still not large enough to introduce a static distortion. The problem of magnons and solitons in the distorted lattice will be published elsewhere [19].

3. Magnons

In this Section we define the magnon operators for the two ground states, ferromagnetic without orbital order \( |F\rangle \),

\[
|F\rangle : \quad n_{i\uparrow} = n_{i\downarrow} = \frac{1}{2} n_{i}\tag{5a}
\]

and ferromagnetic with orbital order of "antiferromagnetic" type \( |F0\rangle \). For the latter case it is more convenient to introduce the double labeling of the sites distinguishing between the sublattices \((l,1)\) if \( i = 2l \) and \((l,2)\) if \( i = 2l + 1 \). The electron site occupations for this state are:

\[
|F0\rangle : \quad n_{l1\uparrow} = n_{l2\downarrow} = n_{11\uparrow}\nonumber
\]

\[
n_{l1\downarrow} = n_{l2\uparrow} = n_{12\uparrow}\tag{5b}
\]

3.1. Magnons for the ground state \( |F\rangle \)

A general form of the magnon creation operator is

\[
\beta_{\nu}^{+} = \sum_{p\lambda\lambda'} b_{\lambda\lambda'}^{\nu}(p + q, p) c_{p+q,\lambda+\rho\lambda'}^{+},
\]

where \( \nu \) labels the magnon branches:

\[
c_{p\lambda\rho} = N^{-\frac{1}{2}} \sum_{j} c_{j\lambda\rho} e^{ipj},
\]

\( N \) is the number of lattice sites. For low temperatures only acoustic branch is relevant. In RPA one gets the acoustic magnon amplitude in a form:

\[
b_{\lambda\lambda'}(p + q, p) = b(p + q, p) \delta_{\lambda\lambda'},
\]

where

\[
b(p + q, p) = \frac{d_q}{\varepsilon_{p+q} - \varepsilon_p + \Delta - E_q},
\]

\[
\varepsilon_p = 2t \cos(p),
\]

\[
\Delta = \frac{1}{2} (U + J)(n_- - n_+).
\]
$E_q$ is the magnon energy calculated from RPA equation of motion:

$$\frac{U + J}{2N} \sum_p \frac{n_{p-} - n_{p+q,+}}{\varepsilon_{p+q} - \varepsilon_p + \Delta - E_q} = 1$$

(9)

and the normalization factor $d_q$ is

$$d_q = \left[ \sum_p \frac{n_{p-} - n_{p+q,+}}{\varepsilon_{p+q} - \varepsilon_p + \Delta - E_q} \right]^{-\frac{1}{2}}$$

(10)

In the following we will use the Fourier transforms of the magnon operators:

$$\beta_j^+ = N^{-\frac{1}{2}} \sum_q e^{-iqj} \beta_q^+ = \sum_{ll_1\lambda} b(l-j,l_1-j)c_{l\lambda+}^+ c_{l_1\lambda-} ,$$

(11)

where

$$b(j,j_1) = \sum_{pq} e^{i(p+q)\lambda} e^{-ipj_1} b(p+q,p) .$$

(12)

### 3.2 Magnons for the $|F0\rangle$ ground state

Before defining the magnon operator let us first write electron band energies for $|\overline{F0}\rangle$ given in the Hartree-Fock approximation:

$$E_{kt\lambda\sigma} = \frac{1}{2} (E_{1\lambda\sigma} + E_{2\lambda\sigma}) + (-1)^t \sqrt{\Delta(k,\sigma)},$$

(13)

where $t = 1, 2$ labels the bands:

$$\Delta(k,\sigma) = (E_{1\lambda\sigma} - E_{2\lambda\sigma})^2 + 4\varepsilon_k^2,$$

(14)

$$E_{w\lambda\sigma} = U n_{w\lambda,-\sigma} + V (n_{w\lambda+} + n_{w\lambda-}) - J n_{w\overline{\lambda}\sigma},$$

(15)

$\overline{\lambda} = 2$ if $\lambda = 1$ and $\overline{\lambda} = 1$ if $\lambda = 2$.

Upstrokes used anywhere in the text have always the meaning explained above.

The Brillouin zone is half as large as the original. The orbital degeneracy (energy does not depend on $\lambda$) is a consequence of the neglection of interorbital hopping (2). The transformation diagonalizing the Hartree–Fock approximation of Hamiltonian (2) reads

$$a_{kt\lambda\sigma} = \left( \frac{2}{N} \right)^{\frac{1}{4}} \sum_{jw} e^{ikj} S_{t\lambda}^w (k,\sigma)c_{jw\lambda\sigma} ,$$

(16)

$w = 1, 2$ labels sublattices and

$$S_{ww_1}^1 = \begin{pmatrix} u & v \\ -v & u \end{pmatrix}$$

(17a)
Matrix $S^2_{ww_1}$ has $u$ and $v$ reversed. The acoustic magnon operator for the considered case takes the form:

$$\beta^+_q = \sum_{p\lambda w_1} b^\lambda_{ww_1}(p + q, p) c^+_{p+q, w_1} c_{p w_1 \lambda^-}.$$  \hspace{1cm} (18)

The RPA equation of motion for $\beta^+_q$ is equivalent to the following set of equations for the magnon amplitudes:

$$\left(\Omega^\lambda_{ww_1} - E_q\right) b^\lambda_{ww_1}(p + q, p) + \sum_{r \neq w_1} \epsilon_{p+q} b^\lambda_{rw_1}(p + q, p) - \sum_{r \neq w_1} \epsilon_p b^\lambda_{wr}(p + q, p)$$

$$+ \delta_{ww_1} \frac{2}{N} \sum_k A^\lambda_w(k + q, k) b^\lambda_{ww_1}(k + q, k) = 0,$$  \hspace{1cm} (19)

where

$$\Omega^\lambda_{ww_1} = U(n_{w\lambda^-} - n_{w_1 \lambda^+}) + V \sum_\sigma (n_{w\lambda\sigma} - n_{w_1 \lambda\sigma})$$

$$+ J(n_{w_1 \overline{\lambda}^-} - n_{w_\overline{\lambda}^+}),$$  \hspace{1cm} (20)

$$A^\lambda_w(k + q, k) = U(n_{k+q, w\lambda^+} - n_{kw\lambda^-}) + J(n_{k+q, w\lambda^+} - n_{kw_1 \overline{\lambda}_-}),$$  \hspace{1cm} (21)

where

$$n_{kw\lambda\sigma} = \langle c^+_{kw\lambda\sigma} c_{kw\lambda\sigma} \rangle,$$  \hspace{1cm} (22a)

$$n_{w\lambda\sigma} = \frac{2}{N} \sum_k n_{kw\lambda\sigma}.$$  \hspace{1cm} (22b)

Using (17) one can express (22) explicitly

$$n_{k11\sigma} = n_{k22\sigma} = u^2 \Theta(E_{k11\sigma} - E_F) + v^2 \Theta(E_{k21\sigma} - E_F)$$

$$n_{k12\sigma} = n_{k21\sigma} = v^2 \Theta(E_{k11\sigma} - E_F) + u^2 \Theta(E_{k21\sigma} - E_F),$$  \hspace{1cm} (23)

where $E_F$ is a Fermi level and $\Theta$ is a step function. It is easy to check that the magnon amplitudes satisfy the following, intuitively clear symmetry relations:

$$b^\lambda_{ww_1}(p + q, p) = b^\overline{\lambda}_{ww_1}(p + q, p).$$  \hspace{1cm} (24)
From (19) using (24) one gets the equation (25) determining the acoustic magnon energy spectrum:

$$\det[\Gamma_{w_1}(q, E_q) - \delta_{w_1}] = 0,$$

where

$$\Gamma_{w_1}(q, E_q) = \frac{-2}{N} \sum_p L_{w_1}(p + q, p, E_q) A_{w_1}^q(p + q, p)$$

with

$$L_{w_1}(p + q, p, E) = \frac{(-1)^{w + w_1} M_{w_1}(p + q, p, E_q)}{\det[M_{ji}(p + q, p, E_q)]},$$

where

$$M_{11} = \Omega_{11} - E_q + \frac{\varepsilon_{p+q}^2}{\Omega_{21} - E_q} + \frac{\varepsilon_p^2}{\Omega_{12} - E_q},$$

$$M_{22} = \Omega_{22} - E_q + \frac{\varepsilon_{p+q}^2}{\Omega_{12} - E_q} + \frac{\varepsilon_p^2}{\Omega_{21} - E_q},$$

$$M_{12} = M_{21} = \varepsilon_{p+q} \varepsilon_p \left( \frac{1}{\Omega_{21} - E_q} + \frac{1}{\Omega_{12} - E_q} \right).$$

The magnon amplitudes corresponding to the solution of equation (25) are

$$b^1_{w_1}(p + q, p) = D_q B_0(p + q, p),$$

where $D_q$ is a normalization factor and

$$B_0(p + q, p) = \frac{2}{N} \left[ L_{w_1}(p + q, p, E_q) + L_{w_2}(p + q, p, E_q) \frac{\Gamma_{11}(q, E_q) - 1}{\Gamma_{12}(q, E_q)} \right]$$

and for $w \neq w_1$

$$b^1_{w_1}(p + q, p) = \frac{\varepsilon_{p+q} b^1_{w_1}(p + q, p)}{\Omega_{w_1}^1 - E_q} - \varepsilon_p b^1_{w_0}(p + q, p).$$

The amplitudes for $\lambda = 2$ are given by (24). Later we shall need the Fourier transforms of magnon operators:

$$\beta_j^+ = \left( \frac{2}{N} \right)^{\frac{1}{2}} \sum_q e^{-iqj} \beta_q^+ = \sum_{l r_1 r_1 \lambda} b^\lambda_{r r_1} (l r - j, l r_1 - j) c_{l r_1 \lambda}^+ c_{l_1 r_1 \lambda}^-,$$

where

$$b^\lambda_{r r_1} (l, l_1) = \sum_{p q} e^{i(p + q)l} e^{-iql_1} b^\lambda_{r r_1} (p + q, p).$$

To make the definition (32) unambiguous let us allow $j$ to run over the sites of sublattice $1$, i.e. $j = (j_1, 1)$. 
4. Magnon–lattice coupling

A very useful tool to study magnon–magnon or magnon–phonon interactions is the effective magnon Hamiltonian [20] being in analogy to the well-known Holstein–Primakoff boson representation of the Heisenberg Hamiltonian [21], an expansion of the Hubbard Hamiltonian in terms of magnon operators $\beta^+_q, \beta_q$. Up to terms of fourth order $H_{\text{eff}}$ reads

$$H_{\text{eff}} = \sum_{q_1} A(q, q_1) \beta^+_q \beta^+_{q_1} + \sum_{k, k_1, q} W(k, k_1, q) \beta^+_k \beta^+_{k+q_1} \beta^{-}_{k-q_1} \beta_{k_1} \beta_k,$$

where

$$A(q, q_1) = \langle 0 | [H_e + H_{e-L}, \beta^+_{q_1}] | 0 \rangle$$

$$W(k, k_1, q) = \frac{1}{4} \langle 0 | [\beta_{k+q} \beta_{k_1-\Delta}, \beta^+_{k_1}] | 0 \rangle.$$

As it is easy to check, an analogous expression to (35) but with $H_e$ replaced by $H_{e-L}$ vanishes.

Contrary to the 3D systems the magnon interaction term for the linear chain cannot be neglected even at low temperatures. The reason is that the magnon bound states, as was shown by Bethe [22], exist for all wavevectors and thus some of them have arbitrary small energy. The itinerant picture discussion of nonlinear effects resulting from magnon–magnon interactions is under consideration and will be published elsewhere [19]. Here we concentrate only on the nonlinear effects having as a source a coupling of magnetization and lattice fluctuations. The magnon–lattice coupling does not mix the states with different magnon occupations and therefore in a first approximation one can discuss the coupling of magnon with the lattice independently from the interaction of magnon bound states with the lattice.

In the following we consider only the former problem. We use the free magnon representation since at low temperatures magnons are not strongly influenced by the occurrence of magnon bound states due to the small occupation of all possible excitations. Choosing as the ground state $|\psi\rangle = |F\rangle$ and restricting to the one-magnon subspace one can rewrite $H_{\text{eff}}$ in the more convenient for the present purposes site representation as follows:

$$H_{\text{eff}} = \sum_{j, \Delta} A(\Delta) \beta^+_j \beta^+_j \Delta \beta \beta^+_j \beta^+_j \Delta + \sum_{i, j, \Delta} X_{i, \Delta} (u_i + \delta - u_i) \beta^+_j \beta^+_j \Delta,$$

where

$$A(\Delta) = \sum_q 2 E_q \cos(q \Delta)$$

$$X_{i, \Delta} = t_1(\delta)(n_- - n_+) \sum_s [b(r, s)b^*(r + \delta - \Delta, s - \Delta)$$

$$- b(s, r + \delta)b^*(s - \Delta, r - \Delta)].$$
and the indices run over: $\delta = \pm 1; \Delta = 0, \pm 1, \ldots; r = i - j = 0, 1, \ldots$

From (4b) and the obvious symmetry relation:

$$b(j, j_1) = b(-j, -j_1)$$  \hspace{1cm} (39)

results

$$X_r^{\Delta \delta} = -X_{-r}^{-\Delta, \delta}.$$  \hspace{1cm} (40)

A similar effective Hamiltonian for $|0\rangle = |F0\rangle$ has the form:

$$H_{\text{eff}} = \sum_{j, \Delta} A(\Delta)\beta_j^+ \beta_{j+\Delta} + \sum_{i, j, \Delta, \delta} X_{ii-j}^{\Delta \delta} (u_{it+\delta} - u_{it}) \beta_j^+ \beta_{j+\Delta}$$  \hspace{1cm} (41)

$$X_{ii-j}^{\Delta \delta} = t_1(\delta) \sum_{sw, \lambda} \left[ b_{iw}^\lambda (it - j, sw - j) b_{iw}^{\lambda*} (it - j + \delta - \Delta, sw - j - \Delta) - b_{wi}^\lambda (sw - j, it - j + \delta) b_{wi}^{\lambda*} (sw - j - \Delta, it - j - \Delta) \right]$$  \hspace{1cm} (42)

where: $\Delta = 0, \pm 2, \ldots; \delta = \pm 1; i_1 - j = 0, \pm 2, \ldots; i_2 - j = \pm 1, \pm 3, \ldots$

5. Solitary magnons

The following derivation of the nonlinear Schrödinger equation describing the long wavelength dynamics of the considered system is only a simple generalization of the derivation given by Davydov [1] for molecular systems and adapted by Pushkarov [4] for Heisenberg model. The differences which the reader will see in some intermediate steps are a consequence of the different space extension of the "magnon hopping term" ($A(\Delta)\beta_j^+ \beta_{j+\Delta}$) and magnon lattice coupling. In the present considerations they extend beyond the nearest neighbors.

5.1. Solitons for the ferromagnetic ground state $|F\rangle$

Using (40) one can rewrite the effective Hamiltonian (36) extended by a lattice part as follows:

$$H = H_L + \sum_{j, \Delta} A(\Delta)\beta_j^+ \beta_{j+\Delta} + \frac{1}{2} \sum_{i, \Delta r, \delta} X_{ir}^{\Delta \delta} \left[ (u_{i+r+\delta} - u_{i+r}) \beta_j^+ \beta_{j+\Delta} - (u_{i-r-\delta} - u_{i-r}) \beta_j^+ \beta_{j-\Delta} \right].$$  \hspace{1cm} (43)

We look for the solution of the Schrödinger equation

$$i \hbar \frac{\partial}{\partial t} |\Psi\rangle = H |\Psi\rangle$$  \hspace{1cm} (44)

in the form:

$$|\Psi(t)\rangle = \sum_j C_j(t) \beta_j^+ |F\rangle$$  \hspace{1cm} (45)
with the normalization condition:

$$\langle \Psi(t)|\Psi(t) \rangle = \sum_j |C_j(t)|^2 = 1.$$  \hfill (46)

The set of equations for the amplitudes $C_j$ corresponding to (44) takes the form:

$$i\hbar \frac{\partial C_j}{\partial t} = \left[ H_L + A(0) + 2 \sum_{\Delta > 0} A(\Delta) \right] C_j + \sum_{\Delta > 0} A(\Delta) (C_{j+\Delta} + C_{j-\Delta} - 2C_j) + \frac{1}{2} \sum_{r,\Delta} X_r^{\Delta\delta} [(u_{j+r+\delta} - u_{j+r})C_{j+\Delta} - (u_{j-r-\delta} - u_{j-r})C_{j-\Delta}].$$  \hfill (47)

To discuss the lattice dynamics the elastic terms should be extended by the magnon–lattice coupling contribution. Following [1] we construct the functional $F(t) = \langle \Psi(t)|H|\Psi(t) \rangle$ playing the role of the Hamilton function in terms of $u_j$ and $m u_j$ with $C_j$ fixed. From the Hamilton equations:

$$\dot{p}_j = -\frac{\partial F}{\partial u_j},$$  \hfill (48)

$$\dot{u}_j = \frac{p_j}{m}$$  \hfill (49)

follows

$$m\ddot{u}_j = m v_0^2 (u_{j+1} + u_{j-1} - 2u_j) + \frac{1}{2} \sum_{r,\Delta} X_r^{\Delta\delta} (C_j^{\ast} + C_{j+r+\delta} - C_j^{\ast} - C_{j-r-\delta})$$

$$- C_j^{\ast} C_{j+r-\Delta} - C_j C_{j-r+\Delta} + C_j^{\ast} C_{j-r+\Delta} + C_j C_{j-r-\Delta}).$$  \hfill (50)

We are interested in the case when the deformation region is much larger than the lattice constant. We can consider $u_j$ and $C_j$ as smooth functions of the position, and going over to a continuum approximation:

$$u_j(t) \Rightarrow u(\zeta, t), \quad C_j(t) \Rightarrow C(\zeta, t),$$

(50) and (47) take the form:

$$\frac{\partial^2 u(\zeta, t)}{\partial t^2} = v_0^2 \frac{\partial^2 u(\zeta, t)}{\partial \zeta^2} + \frac{\Gamma}{m} \frac{\partial}{\partial \zeta} |C(\zeta, t)|^2,$$  \hfill (51)

where

$$\Gamma = \sum_{r,\Delta} X_r^{\Delta\delta},$$  \hfill (52)

$$i\hbar \frac{\partial C(\zeta, t)}{\partial t} = B_1 C(\zeta, t) - B_2 \frac{\partial^2 C(\zeta, t)}{\partial \zeta^2} + \frac{\Gamma}{m} \frac{\partial u(\zeta, t)}{\partial \zeta} C(\zeta, t),$$  \hfill (53)

where

$$B_1 = H_L,$$  \hfill (54a)
We attempt to find the stationary profile solutions:

\[ u = u(\zeta - vt), \]

\[ C = C_0(\zeta - vt) \exp[i\vartheta(\zeta, t)]. \]  \hspace{1cm} (55)

It is easy to check that the lattice energy \( B_1 \) does not depend on time

\[ B_1 = \frac{1}{2} m v_0^2 \left( 1 + \frac{v^2}{v_0^2} \right) \int \left( \frac{\partial u(\xi)}{\partial \xi} \right)^2 d\xi, \]

\[ \xi = \xi - vt. \]  \hspace{1cm} (56)

The substitution of (55) into (51) gives

\[ \frac{\partial u}{\partial \zeta} = \frac{-\Gamma}{m(v_0^2 - v^2)} |C(\zeta, t)|^2. \]  \hspace{1cm} (57)

Putting now (57) into (53) one gets the well-known nonlinear Schrödinger equation for the magnetization amplitude \( C \):

\[ i\hbar \frac{\partial C(\zeta, t)}{\partial t} - B_1 C(\zeta, t) + B_2 \frac{\partial^2 C(\zeta, t)}{\partial \zeta^2} + B_3 |C(\zeta, t)|^2 C(\zeta, t) = 0, \]  \hspace{1cm} (58)

where

\[ B_3 = \frac{\Gamma^2}{m(v_0^2 - v^2)}, \quad v < v_0. \]  \hspace{1cm} (59)

The one-soliton solution to (58) normalized to unity is given by:

\[ C(\zeta, t) = (2L)^{-\frac{1}{4}} \exp \left\{ i \left[ \frac{\hbar v}{2B_2} (\zeta - \zeta_0) - \Omega t \right] \right\}, \]  \hspace{1cm} (60)

where

\[ L(v) = \frac{4B_2}{B_3(v)}, \]  \hspace{1cm} (61a)

\[ \hbar \Omega = B_1 + \frac{\hbar v^2 - B_3^2/4}{4B_2}. \]  \hspace{1cm} (61b)

The variable \( \zeta_0 \) appears as a result of the translational invariance of the problem and can be determined by the initial conditions. It should be noted that (60) describes properly only slow moving solitons \( v < v_0 \). In the \( v \to v_0 \) limit anharmonic effects in the lattice oscillations must be allowed for (for detail see [8]). The low velocity soliton is the more localized the larger the electron-phonon coupling is and the smaller are the elasticity and spin wave stiffness constants. It is easy
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to see that for a vanishing electron–lattice coupling ($\Gamma \to 0$) (60) takes the plane
wave form.

The soliton-like magnetization deviation (60) is accompanied by the lattice
distortion:

$$\frac{\partial u}{\partial \xi} = \frac{-\Gamma}{2mL(v^2_0 - v^2_s)} \frac{1}{\cosh^2 \left(\frac{\xi - \xi_0}{L} - \psi\right)}.$$ (62)

5.2. Solitons for |F0\rangle ground state

The general form of magnon–lattice coupling for the two sublattice case
(43) is far more complicated than the one discussed above. Fortunately we are
interested here as in the whole paper only in the low energy excitations extending
over many lattice sites. In this case it is justified to restrict a discussion to the
coupling of magnons with the long wavelength acoustic lattice modes only. Having
this in mind one can introduce a reasonable simplifying approximation:

$$u_{i1+\delta} - u_{i1} = u_{i2+\delta} - u_{i2} ,$$ (63)

which transforms the coupling term (43) into the form discussed in the previous
Section:

$$H_{m-L} = \sum_{i,j,\Delta} X_{i-j}^{\Delta\delta}(u_{it+\delta} - u_{it})\beta^+_j \beta_{j+\Delta},$$ (64)

where

$$X_{i-j}^{\Delta\delta} = \frac{1}{2} (X_{i1-j}^{\Delta\delta} + X_{i2-j}^{\Delta\delta}).$$ (65)

In this way the present problem is mapped into the one previously discussed. The
one-soliton solution has a form (60) with the coupling constants defined by (65)
and $\Delta$ running over 0, ±2, . . .

Summarizing, the present paper gives a formal background for a discussion of
solitary bound states of magnons and lattice deformations in the itinerant ferro-
magnets. In a subsequent paper [19] we will discuss the soliton like magnon bound
states and their interaction with the lattice.

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References