

PARAMETRIC AMPLIFICATION IN MAGNETIZED NONDEGENERATE SEMICONDUCTING PLASMAS*

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Using the straightforward coupled-mode theory, the parametric amplification is analytically investigated in magnetized piezoelectric as well as non-piezoelectric semiconductors. The origin of nonlinear interaction is taken to be in the second-order optical susceptibility $\chi^{(2)}$ arising from the nonlinear induced current density. The threshold value of the pump electric field E_{0th} is obtained for crystals. E_{0th} is found to decrease with the rise in magnetic field and fall with the rise in scattering angle. Parametric gain constants are obtained for different situations of practical interest, i.e. (i) for piezoelectric coupling only g_p , (ii) for deformation potential coupling only g_d , (iii) for both the couplings g_b . Numerical investigations reveal that the magnetic field increases the gain. Maximum gains are obtained for backward scattered mode. g_d is always found to be less than g_p and g_b . It is also found that g_p and g_b both are identical up to $k \approx 3 \times 10^7 \text{ m}^{-1}$ and for $k \geq 3 \times 10^7 \text{ m}^{-1}$, g_b exceeds g_p .

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1. Introduction

Optical nonlinearity is manifested by changes in the optical parameters of a medium as the intensity of the impressed light wave is increased or when one or more light waves are introduced. The phenomenon of parametric interaction of coupled waves exhibit a vital role in nonlinear optics. The motivation which led to a rapid development in the field of nonlinear optics is the possibility of exploiting

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the nonlinear behaviour in various solid state devices such as frequency converters, parametric amplifiers and oscillators, optical phase conjugators etc. [1-3]. It is a well-known fact that the origin of parametric interaction lies in the second-order nonlinear optical susceptibility $\chi^{(2)}$ of the medium. Earlier Flytzanis [4] has reviewed the progress made in the study of $\chi^{(2)}$ in different frequency regimes in solids as well as in diluted media. $\chi^{(2)}$, the lowest order susceptibility, is a third rank tensor which is nonvanishing in a medium which lacks inversion symmetry. The polarization which is quadratic in nature in terms of field amplitude leads to the optical phenomena of second harmonic generation (SHG), sum and difference frequency generation etc. In general, the terms in $\chi^{(2)}$ provide a coupling between the set of three electromagnetic waves; each of which is characterised by frequency ω_i , wave number k_i , state of polarization ϵ_i , as well as complex amplitude $E_i = A_i \exp(i\omega_i t)$. Attempts were made to explain the experimentally observed behaviour of $\chi^{(2)}$ on the basis of multiple valence and conduction bands, but the agreement was not very encouraging [5, 6]. All these studies [4-6] were mainly covered with the induced polarizations arising from bound-electron nonlinearities. The sum rules for nonlinear susceptibilities in solids have been studied by Peiponen [7].

It is observed that the crystalline nonlinear media offer the greatest device potential. This is because $\chi^{(2)}$ is nonzero for noncentrosymmetric (NCS) crystals and secondly the birefringence of a crystalline medium could be used to phase match velocities of fundamental and harmonic radiations by compensating the material dispersion. However, for nonlinear optical applications, nonlinear crystals, should satisfy four basic criteria, namely adequate nonlinearity, optical transparency, proper birefringence for phase matching and sufficient resistance to optical damage by intense optical irradiation. The doped semiconductors are found to be transparent to photons of energy below their energy band gap and thereby prove to be advantageous hosts. The properties of nonlinear materials are better understood when discussed with reference to nonlinear devices and the theory of nonlinear interactions.

The parametric interaction of acoustic waves with microwave electric field in piezoelectric semiconductors was discovered by Economou and Spector [8]. The importance of the effect of a d.c. magnetic field on parametric action was very well emphasized by Cohen [9]. The parametric excitation of hybrid mode has been studied by Ghosh and Agarwal [10]. Recently Aghamkar et al. [11] have reported the parametric dispersive as well as absorptive characteristics while calculating $\chi^{(2)}$ originating from the finite induced current density produced in a semiconducting medium. Motivated by the intense interest in the field of study of parametric interaction based on $\chi^{(2)}$, in the present paper the authors have made an attempt to investigate parametric amplification process originating from $\chi^{(2)}$, in a non-degenerate *n*-InSb crystal of NCS nature when a magnetostatic field is applied perpendicular to the direction of the pump wave propagation. Since Aghamkar et al. [11] have concluded that the highly doped semiconductors are best suited for observing parametric interactions, the authors in the present paper have confined themselves to the highly doped crystals only.

2. Theoretical formulations

We consider the hydrodynamic model of a homogeneous, nondegenerate n -type semiconductor plasma having both piezoelectric as well as deformation potential couplings and the medium is of infinite extent with electrons as carriers. This model restricts the validity of the analysis to the limit $kl \ll 1$, where k is the wave number and l is the mean free path of the electrons. In order to study parametric interaction processes originating from the effective nonlinear optical susceptibility (χ_{EN}) the medium is subjected to the magnetic field B_0 (along z -axis) perpendicular to the propagation direction (x -axis) of spatially uniform high frequency pump electric field $E_0 \exp(-i\omega_0 t)$. The scattered waves are propagating along a direction making an arbitrary angle θ with the pump wave propagation direction, i.e. propagating in x - z plane making an angle θ with x -axis. Thus θ is the scattering angle, i.e. the angle between k_0 and k_1 . We apply the coupled mode theory [12] to obtain a simplified expression for the acoustic waves via density perturbation.

The basic equations used are as follows:

$$\frac{\partial v_0}{\partial t} + \nu v_0 = -\frac{e}{m} [E_0 + v_0 \times B_0], \quad (1)$$

$$\frac{\partial v_1}{\partial t} + \nu v_1 + \left(v_0 \frac{\partial}{\partial x} \right) v_1 = -\frac{e}{m} [E_1 + v_1 \times B_0], \quad (2)$$

$$v_0 \frac{\partial n_1}{\partial x} + n_0 \frac{\partial v_1}{\partial x} = -\frac{\partial n_1}{\partial t}, \quad (3)$$

$$\frac{\partial E_s}{\partial x} + \frac{\beta}{\epsilon} \frac{\partial^2 u}{\partial x^2} - \frac{C_d}{e} \frac{\partial^3 u}{\partial x^3} = -\frac{n_1 e}{\epsilon}, \quad (4)$$

$$\rho \frac{\partial^2 u}{\partial t^2} + 2\gamma_s \rho \frac{\partial u}{\partial t} + \beta \frac{\partial E_s}{\partial x} + \frac{C_d \epsilon}{e} \frac{\partial^2 E_s}{\partial x^2} = C \frac{\partial^2 u}{\partial x^2}. \quad (5)$$

Equations (1) and (2) represent the zeroth- and first-order momentum transfer equations, respectively, in which v_0 and v_1 are the zeroth- and first-order oscillatory fluid velocities having effective mass m and charge $-e$ and ν is the phenomenological electron collision frequency. Equation (3) represents the continuity equation for electrons, where n_0 and n_1 are the equilibrium and perturbed electron densities, respectively. The Poisson equation (4) gives the space charge field E_s in which the second and the third terms on the left hand side give the piezoelectric and deformation potential contribution to polarization, respectively. ϵ , β and C_d are the scalar dielectric, piezoelectric and deformation potential constants of the semiconductor, respectively. Equation (5) describes the motion of the lattice in a crystal having piezoelectric and deformation potential couplings both. In this equation ρ , u , γ_s and C being the mass density of the crystal, displacement of the lattice, phenomenological damping parameter of acoustic mode and crystal elastic constant, respectively. In (2) we have neglected the effect due to $v_0 \times B_1$ by assuming that the shear acoustic wave is propagating along such a direction of the crystal that it produces a longitudinal electric field, e.g. in n -InSb, if k is

taken along (011) and the lattice displacement u is along (100) the electric field induced by the wave is a longitudinal field [13].

In a highly doped semiconductor the low frequency acoustic wave (ω_s) as well as the pump electromagnetic wave (ω_0) produce density perturbations (n_1) at the respective frequencies in the medium which can be obtained by using the standard approach [14]. Considering the low frequency perturbations (n_s) to be proportional to $\exp[i(k_s x - \omega_s t)]$, while v_0 varies as $\exp(-i\omega_0 t)$ and neglecting the Doppler shift under the assumption that $\omega_0 \gg \nu > kv_0$, we get from Eqs. (1-4) as follows:

$$\frac{\partial^2 n_1}{\partial t^2} + \nu \frac{\partial n_1}{\partial t} + n_1 \bar{\omega}_p^2 + \frac{n_0 e \beta}{m \epsilon} \frac{\partial^2 u}{\partial x^2} - \frac{n_0 C_d}{m} \frac{\partial^3 u}{\partial x^3} = -\bar{E} \frac{\partial n_1}{\partial x}, \quad (6)$$

where $\bar{E} = -[(e/m)E_0 + \omega_c v_{0y}]$, and $\bar{\omega}_p^2 = \omega_p^2 \nu^2 / (\nu^2 + \omega_c^2)$, in which $\omega_p^2 = n_0 e^2 / (m \epsilon)$ is the electron plasma, ν being the electron collision and $\omega_c = (eB_0/m)$ being the cyclotron frequencies, respectively.

The density perturbations associated with the phonon mode (viz. n_s) and the scattered electromagnetic waves (n_f) arising due to the three wave parametric interaction will propagate at the generated frequencies ω_s and $\omega_0 \pm \omega_s$ respectively. For these modes the phase matching condition $\omega_0 = \omega_1 + \omega_s$ and $k_0 = k_1 + k_s$, i.e. the energy and momentum conservation relations should be satisfied. Now since θ is the angle between k_1 and k_0 , thus in writing the conservation equations we have assumed $k_{1y} = 0$, i.e. the scattered wave to propagate in the x - z plane. It must be mentioned here that these conservation equations could be satisfied over a wide range of scattering angle. Now for spatially uniform laser irradiation $|k_0| \approx 0$ and one obtains $|k_1| = |k_s| = |k|$ (say). On resolving equation (6) into two components (fast and slow) by denoting $v = v_f + v_s$ and $n = n_f + n_s$ under rotating wave approximation (RWA) one obtains:

$$\frac{\partial^2 n_f}{\partial t^2} + \nu \frac{\partial n_f}{\partial t} + \bar{\omega}_p^2 n_f = -\bar{E} \frac{\partial n_s^*}{\partial x} \quad (7a)$$

and

$$\frac{\partial^2 n_s}{\partial t^2} + \nu \frac{\partial n_s}{\partial t} + \bar{\omega}_p^2 n_s + \frac{n_0 e \beta}{m \epsilon} \frac{\partial^2 u}{\partial x^2} - \frac{n_0 C_d}{m} \frac{\partial^3 u}{\partial x^3} = -\bar{E} \frac{\partial n_f^*}{\partial x}. \quad (7b)$$

In the above analysis we have restricted ourselves only to the Stokes component ($\omega_0 - \omega_s$) of the scattered electromagnetic waves. One can easily infer from (7) that the slow and fast components of the density perturbations are coupled to each other via the pump electric field. Thus the presence of the pump electric field is the fundamental necessity for the parametric interactions to occur. From Eqs. (5), (7a) and (7b) one obtains the expression for n_s as:

$$n_s = \frac{i e n_0 k_x^3 E_s (\beta^2 + C_d^2 \epsilon^2 k_x^2 / e^2)}{m \rho \epsilon (\omega_s^2 - k_x^2 v_s^2 + 2i \gamma_s \omega_s)} \times \left[(\bar{\omega}_p^2 - \omega_s^2) - i \omega_s \nu - \frac{k_x^2 |\bar{E}|^2}{(\bar{\omega}_p^2 - \omega_1^2 + i \omega_1 \nu)} \right]^{-1}, \quad (8)$$

where $k_x = k \cos \theta$ and $v_s^2 = (C/\rho)$; v_s being the velocity of the acoustic wave. In the present report in order to study the effect of nonlinear current density on the induced polarization in a magnetized highly doped semiconductor, the effect of the transition dipole moment is neglected while analysing parametric interaction in the crystal. It is evident from the above expression that n_s depends upon the various powers of pump intensity, $I = \frac{1}{2} \eta \epsilon_0 c_0 |E_0|^2$; η and c_0 being the background refractive index of the crystal and the velocity of light in vacuum, respectively. This produced density perturbation thus affecting the propagation characteristics of the scattered waves, which can be studied by employing the electromagnetic wave equation:

$$\nabla \times \nabla \times \mathbf{E}_1 = \frac{-1}{C_L^2} \frac{\partial^2 \mathbf{E}_1}{\partial t^2} - \mu_0 \frac{\partial \mathbf{J}_1}{\partial t}, \quad (9)$$

where $C_L = (\mu_0 \epsilon_0 \epsilon_L)^{-1/2}$ is the velocity of light in the medium and \mathbf{J}_1 is the perturbed current density and $\epsilon_L = \epsilon/\epsilon_0$.

The Stokes component of the induced current density is given by

$$\mathbf{J}_1 = -n_s^* e v_0. \quad (10a)$$

Using Eqs. (8) and (10a) one gets:

$$\mathbf{J}_1 = \frac{-iek_x^3 v_s^2 \epsilon \omega_p^2 \omega_0 E_s^* E_0 (\mathcal{K}^2 + L^2 k_x^2)}{2m\gamma_s \omega_s (\omega_c^2 - \omega_0^2)} \left[\delta'^2 + i\omega_s \nu - \frac{k_x^2 |\bar{E}|^2}{(\delta^2 - i\omega_1 \nu)} \right]^{-1} \quad (10b)$$

where $\mathcal{K}^2 = \beta^2/\epsilon C$, $L^2 = (C_d/e)^2 (\epsilon/C)$, $\delta'^2 = \bar{\omega}_p^2 - \omega_0^2$, $\delta^2 = \bar{\omega}_p^2 - \omega_1^2$ and $\omega_1 = \omega_0 - \omega_s$. In deriving Eqs. (10) we have used the expression for the components of v_0 (along x- and y-directions) which is the oscillatory electron fluid velocity in the presence of the pump and the magnetostatic fields. Using Eq. (1), these expressions are obtained as:

$$v_{0x} = \frac{\bar{E}}{(\nu - i\omega_0)} \quad \text{and} \quad v_{0y} = -(e/m) \frac{\omega_c E_0}{(\omega_c^2 - \omega_0^2)} \quad (11)$$

Henceforth treating the induced polarization \mathbf{P}_1 as the time integral of the current density \mathbf{J}_1 , one may write:

$$\mathbf{P}_1 = \int \mathbf{J}_1 dt. \quad (12)$$

Now the second order susceptibility in the coupled mode scheme obtained by using equations (10) to (12), on neglecting the induced polarization due to transition dipoles, comes out to be as follows:

$$\chi_{EN}^{(2)} = \frac{ek_x^3 v_s^2 \epsilon_L \omega_p^2 \omega_0 (\mathcal{K}^2 + L^2 k_x^2)}{2m\gamma_s \omega_s (\omega_c^2 - \omega_0^2) \omega_1} \left[\delta'^2 + i\omega_s \nu - \frac{k_x^2 |\bar{E}|^2}{(\delta^2 - i\omega_1 \nu)} \right]^{-1}. \quad (13)$$

As susceptibility is a complex quantity, it can be expressed as:

$$\chi_{EN}^{(2)} = \chi_{EN_r}^{(2)} + i\chi_{EN_i}^{(2)}. \quad (14)$$

From Eqs. (13) and (14) one may infer that the total crystal susceptibility is influenced by the free carrier concentration through plasma frequency $\omega_p \neq 0$ and by magnetostatic field through cyclotron frequency $\omega_c \neq 0$. The dispersion characteristics of the scattered wave in a parametric process from $(\chi_{EN_i}^{(2)})$ and the parametric gain through $\chi_{EN_i}^{(2)}$ can be seen from Eq. (13).

Here in the present paper we will confine ourselves to the study of parametric gain in the presence of a magnetostatic field and deformation potential coupling.

The parametric amplification in a doped semiconductor is given by:

$$\alpha_{EN} = \left(\frac{\omega_1}{\eta c_0} \right) \chi_{EN_i}^{(2)}, \quad (15)$$

where α_{EN} is the effective nonlinear absorption coefficient. The nonlinear growth of the signal $\omega_1 = \omega_0 - \omega_s$ as well as idler (ω_s) is possible only if α_{EN} obtained from Eq. (15) is negative.

3. Parametric growth

Restricting ourselves to the analytical investigations and numerical estimations of the parametric growth in isotropic and magnetostatic semiconductors in heavily doped regime, i.e. $\bar{\omega}_p \approx \omega_0 (\approx \omega_1)$ with $\bar{\omega}_p \gg \nu (\approx \omega_s)$, Eq. (13) can be solved for complex χ_{EN} to get:

$$\chi_{EN_i}^{(2)} = \frac{e\epsilon_L A k_x \omega_p^2 \omega_0 (1 + L^2 k_x^2 / \mathcal{K}^2) (\bar{\omega}_p^2 \omega_1^2 \nu^2 - \delta^2 k_x^2 |\bar{E}|^2)}{2m\gamma_s \omega_1 \omega_s (\omega_c^2 - \omega_0^2) \left[(\delta^2 \bar{\omega}_p^2 - k_x^2 |\bar{E}|^2)^2 + (\omega_1 \bar{\omega}_p^2 \nu)^2 \right]} \quad (16)$$

and

$$\chi_{EN_i}^{(2)} = \frac{-e\epsilon_L A k_x \omega_p^2 \omega_0 (1 + L^2 k_x^2 / \mathcal{K}^2) (\omega_s \omega_1 \nu^2 - k_x^2 |\bar{E}|^2)}{2m\gamma_s \omega_1 \omega_s (\omega_c^2 - \omega_0^2) \left[(\delta^2 \bar{\omega}_p^2 - k_x^2 |\bar{E}|^2)^2 + (\omega_1 \bar{\omega}_p^2 \nu)^2 \right]} \quad (17)$$

where $A = \mathcal{K}^2 k_x^2 v_x^2$.

For obtaining parametric amplification it is necessary to determine the threshold value of the pump amplitude for the onset of the parametric process. For this setting $\chi_{EN_i} = 0$. The necessary conditions for magnetoactive and isotropic crystals yields:

(i) In the presence of a magnetostatic field, i.e. $\omega_c \neq 0$

$$[(E_{0,th})]_{B_0 \neq 0} = \frac{m\nu}{ek_x} (\omega_1 \omega_s)^{1/2} \left[1 - (\omega_c / \omega_0)^2 \right]. \quad (18a)$$

(ii) In the absence of a magnetostatic field, i.e. $\omega_c = 0$

$$[(E_{0,th})]_{B_0 \neq 0} = \frac{m\nu}{ek_x} (\omega_1 \omega_s)^{1/2}. \quad (18b)$$

A comparison of the threshold fields in the presence and absence of the magnetic field B_0 , yields:

$$\frac{[(E_{0th})]_{B_0 \neq 0}}{[(E_{0th})]_{B_0 = 0}} = \left[1 - (\omega_c/\omega_0)^2 \right]. \quad (18c)$$

From the above equations one may infer that the application of a magneto-static field considerably reduces the required value of the threshold pump field. It may also be inferred that the expressions for the threshold fields are independent of the material parameters viz. piezoelectric and/or deformation potential coupling coefficients. Thus it will remain the same for any material belonging to NCS group.

The parametric growth (i.e. χ_{EN_i} negative) can be achieved in the two different regions under the following conditions:

$$(i) \quad \text{if } k_x^2 |\bar{E}|^2 > \omega_1 \omega_s \nu^2, \quad \omega_c^2 < \omega_0^2; \quad (19a)$$

$$(ii) \quad \text{if } k_x^2 |\bar{E}|^2 < \omega_1 \omega_s \nu^2, \quad \omega_c^2 > \omega_0^2. \quad (19b)$$

It can be seen that under condition (19b) the expression for the growth becomes nearly independent of the pump field amplitude and thus it is of no practical interest.

Now we shall discuss the different aspects of χ_{EN} for different situations of practical interest.

(1) Piezoelectric couplings ($\beta \neq 0, C_d = 0$):

For this case, using Eqs. (17) and (19b) one obtains the growth rate as:

$$g_p = \frac{e^3 \epsilon_L A k_x^3 \omega_p^2 \omega_0^5 \omega_1 \nu |E_0|^2}{2m^3 \gamma_s \omega_s \eta c_0 (\omega_c^2 - \omega_0^2)^3} [(\delta^2 \bar{\omega}_p^2 - k_x^2 |\bar{E}_0|^2)^2 + (\bar{\omega}_p^2 \omega_1 \nu)^2]^{-1}. \quad (20a)$$

The suffix p is used for piezoelectric coupling.

(2) Deformation potential coupling ($\beta = 0, C_d \neq 0$):

For this case using Eqs. (17) and (19a) one obtains as:

$$g_d = \frac{e^3 \epsilon_L \nu_s^2 k_x^2 \omega_p^2 \omega_0^5 \omega_1 L^2 \nu |E_0|^2}{2m^3 \gamma_s \omega_s \eta c_0 (\omega_c^2 - \omega_0^2)^3} [(\delta^2 \bar{\omega}_p^2 - k_x^2 |\bar{E}_0|^2)^2 + (\bar{\omega}_p^2 \omega_1 \nu)^2]^{-1}. \quad (20b)$$

The suffix d is used for deformation potential coupling.

(3) Both piezoelectric and deformation potential couplings ($\beta \neq 0, C_d \neq 0$):

In this case using the same pair of equations as above, the gain constant becomes:

$$g_b = \frac{e^3 \epsilon_L A k_x^3 \omega_p^2 \omega_0^5 \omega_1 \nu |E_0|^2 [1 + L^2 k_x^2 / K^2]}{2m^3 \gamma_s \omega_s \eta c_0 (\omega_c^2 - \omega_0^2)^3} [(\delta^2 \bar{\omega}_p^2 - k_x^2 |\bar{E}_0|^2)^2 + (\bar{\omega}_p^2 \omega_1 \nu)^2]^{-1}. \quad (20c)$$

The suffix b stands for both the couplings.

4. Results and discussions

We now proceed to the numerical analysis of the parametric gain in a NCS semiconducting crystal, viz. *n*-InSb at 77K irradiated by pulsed $10.6\ \mu\text{m}$ CO_2 lasers. The material parameters for this crystal are taken as: $m = 0.0150 m_0$, $\epsilon_1 = 15.8$, $v_s = 4 \times 10^3\ \text{m s}^{-1}$, $\beta = 0.054\ \text{C m}^{-2}$, $C_d = 4.5\ \text{eV}$, $\rho = 5.8 \times 10^3\ \text{kg m}^{-3}$, $\omega_s = 2 \times 10^{11}\ \text{s}^{-1}$, $n_0 = 2 \times 10^{24}\ \text{m}^{-3}$, $\nu = 4 \times 10^{11}\ \text{s}^{-1}$ and $\omega_0 = 1.78 \times 10^{14}\ \text{s}^{-1}$.

Using the above parameters, we have studied the effect of wave number k , magnetic field (in terms of ω_c) and the scattering angle (θ) on the threshold electric field. Figure 1 shows that the threshold electric field ($E_{0,\text{th}}$) decreases with the increase in the wave number k . It may be inferred from Fig. 2 that $E_{0,\text{th}}$ remains constant up to $\omega_c \approx 2 \times 10^{13}\ \text{s}^{-1}$; if one applies magnetic field for which $\omega_c > 2 \times 10^{13}\ \text{s}^{-1}$, the $E_{0,\text{th}}$ starts decreasing with increment in ω_c . Figure 3 depicts the dependence of $E_{0,\text{th}}$ on scattering angle θ . It is observed that for a constant magnetic field, $E_{0,\text{th}}$ increases gradually with increasing value of θ .

Considering the pump intensities well above the threshold, the gain constants for all the three types of couplings (i.e. piezoelectric g_p , deformation g_d and both g_b) can be estimated. All the three gain constants increase linearly with the laser pump amplitude $E_0 (> E_{0,\text{th}})$ (Fig. 4).

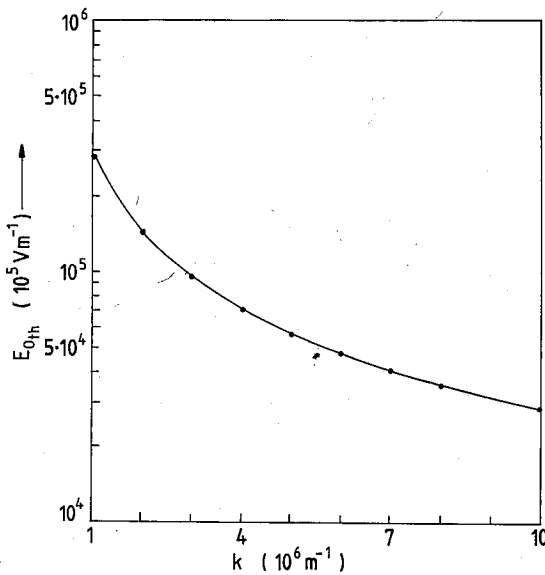


Fig. 1. Variation of threshold field $E_{0,\text{th}}$ with wave number k at $\theta = 45^\circ$ and $\omega_c = 10^{19}\ \text{s}^{-1}$.

It is also found that the values of g_b and g_p are identical and nearly 10^4 order higher than the values of g_d for $k = 2 \times 10^6\ \text{m}^{-1}$, $\omega_c \approx 2 \times 10^{12}\ \text{s}^{-1}$ and

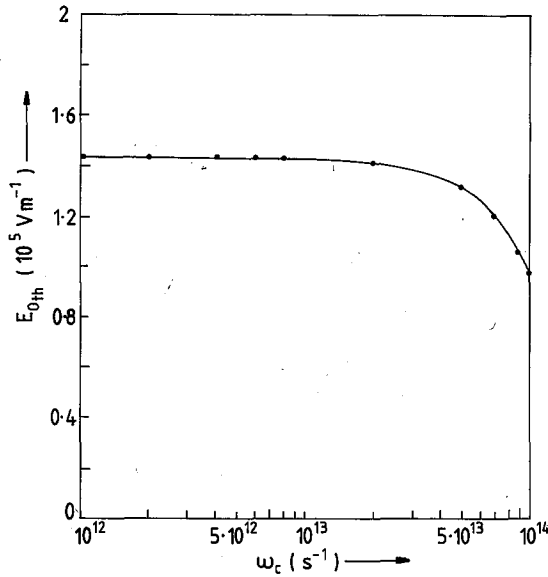


Fig. 2. Variation of threshold field E_{0th} with magnetic field as measured by ω_c at $\theta = 45^\circ$ and $k = 2 \times 10^6 \text{ m}^{-1}$.

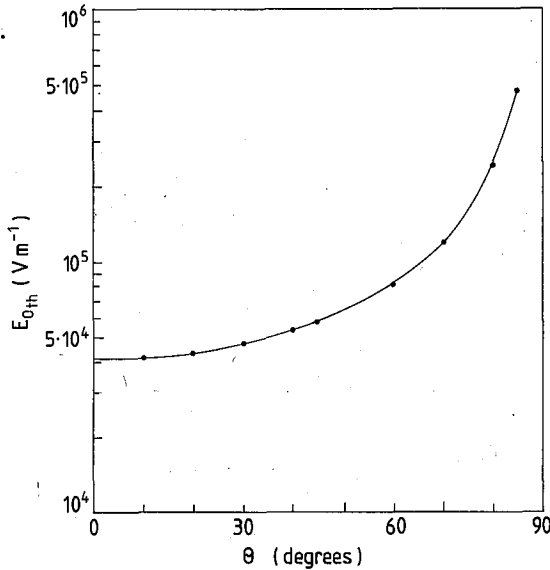


Fig. 3. Variation of threshold field E_{0th} with angle θ at $k = 5 \times 10^6 \text{ m}^{-1}$ and $\omega_c = 10^{13} \text{ s}^{-1}$.

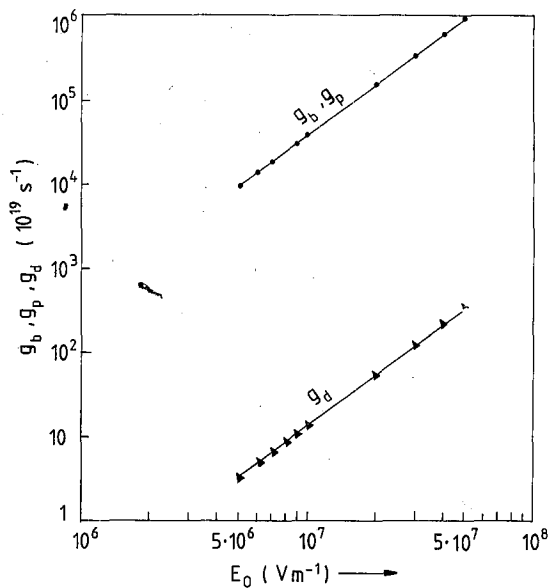


Fig. 4. Variation of parametric gain constants (g_b , g_p and g_d) with threshold field $E_{0,th}$ at $k = 2 \times 10^6 \text{ m}^{-1}$, $\omega_c = 2 \times 10^{12} \text{ s}^{-1}$ and $\theta = 45^\circ$.

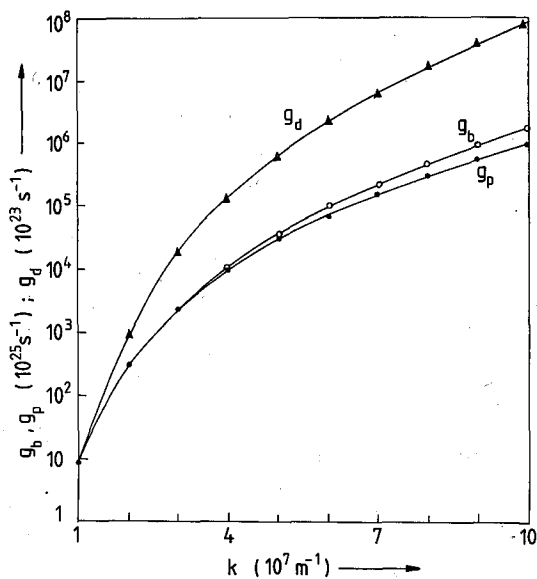


Fig. 5. Variation of parametric gain constants (g_b , g_p and g_d) with wave number k at $\theta = 45^\circ$, $\omega_c = 10^{12} \text{ s}^{-1}$ and $E_0 = 10^7 \text{ Vm}^{-1}$.

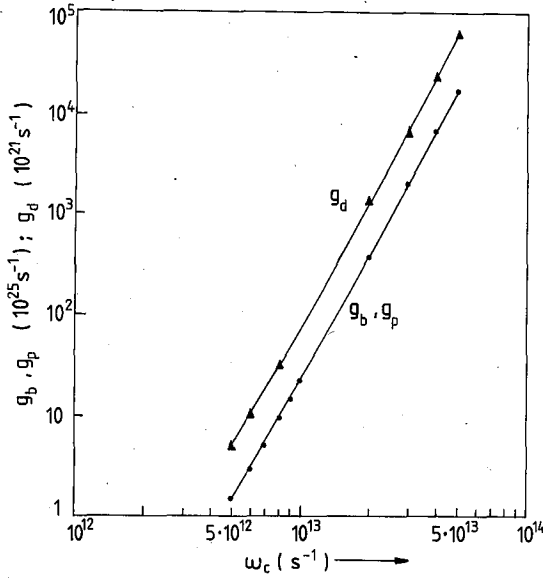


Fig. 6. Variation of parametric gain constants (g_b , g_p and g_d) with ω_c at $k = 2 \times 10^6 \text{ m}^{-1}$, $\theta = 45^\circ$ and $E_0 = 10^7 \text{ Vm}^{-1}$.

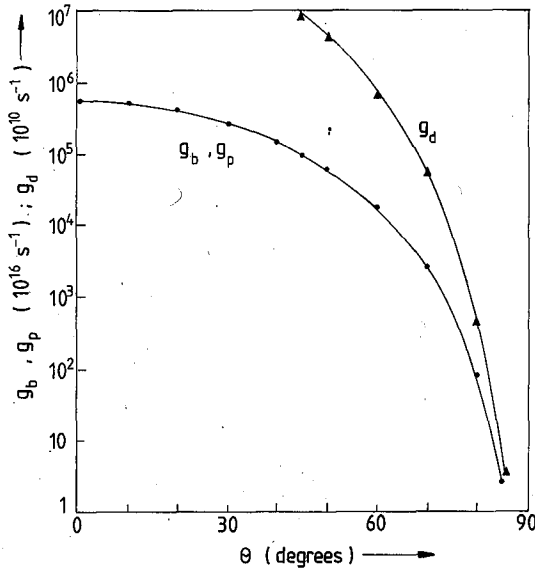


Fig. 7. Variation of parametric gain constants (g_b , g_p and g_d) with angle θ at $k = 10^6 \text{ m}^{-1}$, $\omega_c = 10^{12} \text{ s}^{-1}$ and $E_0 = 10^7 \text{ Vm}^{-1}$.

$\theta = 45^\circ$. We have plotted the dependence of gain constants on k which varies from 10^7 to 10^8 m^{-1} for $E_0 = 10^7 \text{ V m}^{-1}$, $\omega_c \approx 2 \times 10^{12} \text{ s}^{-1}$ and $\theta = 45^\circ$ in Fig. 5. In this wavelength region, i.e. $10^7 > k > 10^8 \text{ m}^{-1}$, we found that all the three gain constants increase parabolically with k . It is also found that g_b and g_p are equal up to $k = 3 \times 10^7 \text{ m}^{-1}$ and for $k > 3 \times 10^7 \text{ m}^{-1}$, g_b becomes higher than g_p . Thus it may be inferred from this figure that the deformation potential coupling becomes comparable to piezoelectric interactions in the wavelength region where $k > 3 \times 10^7 \text{ m}^{-1}$. Below this wavelength region contribution of deformation potential is negligible compared to the contribution of piezoelectricity and thus in this region g_p and g_b are identical. The magnetic field (in terms of ω_c) increases all the gain constants (Fig. 6). But we cannot increase ω_c indefinitely because of the restriction $\omega_c^2 < \omega_0^2$ (Eq. (19a)). Figure 7 shows the variation of gain constants with the scattering angle θ . It can be inferred from Fig. 7 that the gain constants decrease monotonically with scattering angle, however there is a more rapid decrease for $\theta > 30^\circ$ than for $\theta < 30^\circ$.

From the above discussion, it is clear that the required range of threshold electric field $E_{0_{th}} = 2.4 \times 10^4 \text{ V m}^{-1}$ for $k = 10^7 \text{ m}^{-1}$ is corresponding to excitation intensity $4.36 \times 10^2 \text{ W cm}^{-2}$. Such pump intensities are easily obtained by using frequency doubled CW $10.6 \mu\text{m}$ CO_2 lasers. The most fascinating results obtained in the study reported above are as follows:

- (1) The applied magnetic field increases the values of gain constants, while it reduces the required pump intensities.
- (2) The gains are maximum whereas the required threshold intensity is minimum for zero scattering angle, i.e. one gets maximum gain for minimum threshold for backward scattered mode.
- (3) It is found that the gains with piezoelectricity g_p and with both couplings g_b are identical up to the wave number $k = 3 \times 10^7 \text{ m}^{-1}$ and beyond this value of k , g_b becomes more than g_p .
- (4) The gain with deformation potential g_d is always found to be less than that of g_p and g_b .

Thus one may conclude from the above discussion that the magnetoactive, highly doped semiconductors, having both the coupling mechanisms are the best choice for the fabrication of parametric backward wave amplifiers.

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