THE SYMMETRY AND OPTICAL PHENOMENA

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(Received June 18, 1990)

The explicit forms of the broken (by a electric field E, magnetic field Hand spatial dispersion of wave vector k) point symmetry groups of a crystal are given. For these groups the dielectric permeability tensors: $\varepsilon_{ij}(\omega, E)$, $\varepsilon_{ij}(\omega, H)$ and $\varepsilon_{ij}(\omega, k)$ — the particular cases of $\varepsilon_{ij}(\omega, E, H, k)$ — are written out (the susceptibility tensors $\chi_{ij}(\omega, E)$, $\chi_{ij}(\omega, H)$ and $\chi_{ij}(\omega, k)$ take the same forms). In order to illustrate the results obtained the electrooptical phenomena (connected with the tensors $\varepsilon_{ij}(\omega, E)$) are discussed.

PACS numbers: 75.30.Cr, 78.20.Bh, 78.20.Jq

1. Introduction

The electrooptical phenomena obey the same group-theoretical rules as the previously discussed magnetooptic ones [1] and as the phenomena related to the spatial dispersion of the wave vector k [2, 3].

The aim of this paper is to present the complete results allowing us to describe the electrooptic phenomena (the optical properties of medium in the external electric field E), the magnetooptic ones (the optical properties of medium in the external magnetic field H) as well as the optical properties of the medium in which there is a spontaneous dispersion of wave vector k. In Table I the broken symmetry groups K(F) for fields F = E, H, k are given for all point groups K and all orientations of the fields F with respect to the symmetry elements of the crystals. In Table II the explicit forms of the groups K(F) = J(F) + bJ(F) are given, where J(F) denotes the subgroup consisting of the elements preserving the direction of the field $F(J(F): F \rightarrow F)$, while bJ(F) is a set of elements of the group K which reverse the direction of the field $F(bJ(F): F \rightarrow -F)$. For each group K(F) — the number is given, which shows where, in Table III one can read off the explicit form of the real part of tensor ε_{ij} , or, more precisely, its even part $\varepsilon_{ij}^e(\varepsilon_{ij}^e(\omega, F) = \varepsilon_{ij}^e(\omega, -F))$ and odd part $\varepsilon_{ij}^o(\varepsilon_{ij}^o(\omega, F) = -\varepsilon_{ij}^o(\omega, -F))$. The imaginary parts $\varepsilon_{ij}^{\prime e}$ and $\varepsilon_{ij}^{\prime o}$ have exactly the same forms as ε_{ij}^e and $\varepsilon_{ij}^{\circ o}$, respectively; the Kramers-Kronig relations

say that if e.g. $\varepsilon_{12}^{e} \neq 0$ then also $\varepsilon'_{12}^{e} \neq 0$ but it does not imply that $\varepsilon_{12}^{e} = \varepsilon'_{12}^{e}$. In Table III we give also the form of the real part of the tensor $\tilde{\varepsilon}_{ij}$ for all 122 point groups (cp. Table I). They are written out in the crystalographic frame of axes. If we need to determine the difference $\tilde{\varepsilon}_{ij}(\omega, F) - \tilde{\varepsilon}_{ij}$, we must transform the latter tensor to the local frame of axes in which the tensor $\tilde{\varepsilon}_{ij}(\omega, F)$ is written out.

The susceptibility tensor $\tilde{\chi}_{ij}$ satisfies exactly the same Onsager relation [4]:

$$\tilde{\chi}_{ij}(\omega, \boldsymbol{E}, \boldsymbol{H}, \boldsymbol{k}) = \tilde{\chi}_{ji}(\omega, \boldsymbol{E}, -\boldsymbol{H}, -\boldsymbol{k})$$
(1)

as the dielectric parmeability tensor:

$$\tilde{\varepsilon}_{ij}(\omega, \boldsymbol{E}, \boldsymbol{H}, \boldsymbol{k}) = \tilde{\varepsilon}_{ji}(\omega, \boldsymbol{E}, -\boldsymbol{H}, -\boldsymbol{k}).$$
⁽²⁾

Therefore, the tensor $\tilde{\chi}_{ij}$ has exactly the same form as $\tilde{\epsilon}_{ij}$. The identity of the forms of tensors $\tilde{\chi}_{ij}$ and $\tilde{\epsilon}_{ij}$ does not mean their equivalence which would imply that for each two arbitrarily chosen components the conditions $\epsilon_{ij}/\epsilon_{kl} = \chi_{ij}/\chi_{kl}$ had to be fulfilled. So, as far as the forms are concerned, the Tables I-III can be also referred to the $\tilde{\chi}_{ij}(\omega, E, H, k)$ tensors. The analogies in the behaviour of the tensors ϵ_{ij} and χ_{ij} in the most essential aspects of our theory (i.e. in the vicinity of the Néel, Curie temperatures and the point where the group K is broken to K(F) group) are depicted in elegant form on Figs. 2 and 3 of the paper [5]. On the figures the angle of rotation of polarization plane of the light and the magnetic susceptibility are plotted as functions of the magnetic field.

We can obtain the information about the transport properties of the crystal also by measuring the χ_{ij} tensor [6]. In the paper [6] the tensors χ_{ij} are measured on the left- and right-hand side of Néel points. A different behaviour has been obtained. It is obvious that for two different point groups K we obtain the different K(H) groups (see: Table I and Table II) and consequently the different tensors $\tilde{\chi}_{ij}$ (and similar $\tilde{\epsilon}_{ij}$) are measured.

The investigations of the phase transitions by parallel optical and magnetical methods have been announced for some time [7, 8]. There is a great deal of hope that the Tables I-III will appear useful in such experiments.

Each medium is less or more dispersive. Therefore, while discussing for example the electrooptical effects it won't do any harm to treat the gyration tensor, $\tilde{\varepsilon}_{ij}(\omega, \mathbf{k})$, as the correction to the $\tilde{\varepsilon}_{ij}(\omega, \mathbf{k})$ tensor [9]. Table II allows to discuss these and similar effects as magnetoelectric optical effects [5, 7, 8, 10], the magnetooptical effects in dispersive medium, and so on. The theoretical analyses [9, 11] and the interpretations of experimental data given by experimenters [5, 12, 13] have been based up to the present on the tables given in the monographs [14] (see: Refs. [9, 11–13]) and [15] (see: Ref. [5]).

In this paper we will discuss only the electrooptical effects. The discussion will be short as we have some analogies with previously discussed magnetooptical effects [1] and the effects following from the dispersion of wave vector k in medium [2, 3]. Since the time the papers [1, 2] have appeared, there have been no new theoretical and experimental ideas, so there is no need to supplement the previous discussions [1, 2].

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2. The electrooptical effects

The tensors $\tilde{\varepsilon}_{ij}(\omega, E)$ (i.e. $\tilde{\varepsilon}_{ij}^{e}(\omega, E)$ and $\tilde{\varepsilon}_{ij}^{o}(\omega, E)$ — the particular cases of the tensor (2) — are obtained in exactly the same way as the tensor $\tilde{\varepsilon}_{ij}(\omega, H)$ [1] and $\tilde{\varepsilon}_{ij}(\omega, k)$ [2]. When adopting the previously described group-theoretical procedure we must remember that the field E is invariant under the action of the following groups J(E) = 1, 2, m, mm2, 3, 3m, 4, 4mm, 6, 6mm, 11', 21', m1', mm21', 31', 3m1', 41', 4mm1', 61', 6mm1', 2', m', m'm2', m'm'2, 3m', 4', 4'mm', 4m'm', 6', 6'mm', 6m'm'. The results obtained in this way are not in contradiction to experimental data [5-8, 10, 12, 13, 16, 17].

The nonreciprocal rotation of the plane of light polarization discussed in the papers [5, 7, 8, 10], which is induced by electric field in Cr_2O_3 crystal (the symmetry group $K = \overline{3}'m'$) for geometry $k \parallel E \parallel z$ (then $K(E) = \overline{3}'m'$) can be observed not only for $K(E) = \overline{3}'m'$ but also when $K(E) = \overline{3}' 32$, $\overline{3}'m'$, 4/m', 422, $4/m'm'm', \overline{6}', 6/m', 622, \overline{6}'m'2, 6/m'm'm'$; the reciprocal rotation can be observed for $K(E) = \overline{3}, 32', \overline{3}m', 4/m, 42'2', 4/mm'm', \overline{6}, 6/m, 62'2', \overline{6}2'm', 6/mm'm'$. In the former case the angle of rotation of polarization plane satysfies the condition:

$$\varphi(\mathbf{E}) = -\varphi(-\mathbf{E}), \qquad i.e. \ \varphi(\mathbf{E}) \sim |\mathbf{E}| \operatorname{sign}(\mathbf{E}), \qquad (3)$$

while in the latter case — the condition

$$\varphi(\mathbf{E}) = \varphi(-\mathbf{E}), \qquad i.e. \ \varphi(\mathbf{E}) \sim \mathbf{E}^2.$$
 (4)

The remark: for nonreciprocal rotation the relation (4) but not (3) should be valid, while for reciprocal one — vice versa, i.e. the relation (3) but not (4) is valid; note that if the left-hand side of the relations (3) and (4) corresponds to the geometry $\pm k \parallel E \parallel z$ then the right-hand side of this relations corresponds to the geometry $\pm k \parallel -E \parallel z$. In turn, the experimental definitions are based on the geometries $\pm k \parallel E \parallel z$ and $\pm k \parallel E \parallel z$, respectively. We see that the common elements of theoretical suggestion and experimental definitions are the mutual relation between the vector k and E.

It follows from our results that the nonreciprocal rotation in $Y_3Fe_5O_{12}$ for the geometry $k \parallel E \parallel H \parallel z$, observed in experiments [5, 7, 8], is generated by the magnetic field H and not by the electric one E. Remark: it follows from theoretical definition of nonreciprocal and reciprocal rotations of the plane of light polarization that the external magnetic field H can only generate the reciprocal rotation, while the electric field E and the spatial dispersion of wave vector k can generate both rotations, i.e. the nonreciprocal and reciprocal ones (cp. Table II and III).

For groups K(E) different from the above mentioned ones (e.g. K(E) = m'm'm, m'mm', mm'm', m'm'm', mm'm, mmm', and so on) or for k/E we are dealing with more complicated functions for $\varphi(E)$ than those given by formulae (3) and (4); however, also for them the first term of power series can sometimes take the form (3) or (4). The form of the above function depends not only on the symmetry but also on the relations between the components of the tensor

 $\tilde{\varepsilon}_{ij}(\omega, E)$. For the same group K(E) and two media we can obtain two different functions $\varphi(E)$. It is not useful to discuss the all possible relations between the elements of $\tilde{\varepsilon}_{ij}(\omega, E)$. It is more easy to interpret the experimental data than to predict them. The group-theoretical method does not allow us to estimate which of the two functions allowed by the symmetry is more likely to be a proper one.

If J(E) = 3m, 4mm, 6mm, 31', 3m1', 41', 4mm1', 61', 6mm1', 4', 6', 4'mm', 6'mm' or $K(E) = \overline{3}m$, 4/mmm, 6/mmm, $\overline{6}2m$, $\overline{3}1'$, 321', $\overline{3}m1'$, 4/m1', 4221', 4/mm1', $\overline{6}1'$, 6/m1', 6221', $\overline{6}2m1'$, 6/mmm1', $\overline{3}'m$, 4'/m, 4'/m', 4'22', 4/m'mm, 4'/mmm', 4'/m'm'm, 6'/m', 6'/m, 6'22', $\overline{6}'m2'$, 6/m'mm, 6'/mmm', 6'/m'm'm then the z-axis is, similarly as in the case E = 0, the optical axis; it means that birefringence $\Delta n = n_1 - n_2$ equals zero:

$$\Delta n = 0. \tag{5}$$

If $k \mid E$ then we have

$$\Delta n = f(\boldsymbol{E}^2); \tag{6}$$

and the first term of power series can take the form

$$\Delta n \sim |E| \tag{7}$$

or

$$\Delta n \sim E^2. \tag{8}$$

For the groups $K(E) = 2_x/m_x$, $2'_x/m'_x$, $2_x/m_x 1'$, $2_y/m_y$, $2'_y/m'_y$, $2_y/m_y 1'$, $2_z/m_z$, $2'_z/m'_z$, $2_z/m_z 1'$, mmm, mmm', mmm1' the function (6), and consequently (7) or (8) will also appear for any geometry of vector k with respect to the field E. The quadratic [16, 17] and linear [12, 13] dependences of the field E belong to the most frequently measured.

The first term of power series for $K(E) = \overline{4}, 4', \overline{4}1', \overline{4}2m, \overline{4}2m1', \overline{4}2'm', \overline{4}'2m', \overline{4}'m2'$ takes the form

$$\Delta n \sim |\mathbf{E}| \text{sign}(\mathbf{E}) \tag{9}$$

for $k \perp E \parallel z$, but in the case $K(E) = \overline{4}' m 2'$ — also for $k \parallel E \parallel z$.

In the case of other groups K(E) or for arbitrary geometry of the vector k with respect to the field E the form of the function $\Delta n(E)$ is determined not only by symmetry but also by the material constants. These functions are more complicated than (7), (8) or (9). For such a case it is more easy to consider the concrete experimental data than to discuss it in general form.

Acknowledgments

I wish to thank Drs P. Kosiński and M. Majewski for usueful discussions and reading the manuscript.

TABLE I

The symmetry group K(F) broken by a field F, where F stands for electric field (E), magnetic field (H) or spatial dispersion of the wave vector (k) for all directions: parallel (||), perpendicular (\bot) and arbitrarily oriented (a.o.) relative to the axes and planes of the K-symmetric crystal. For these K groups the explicit expressions of dielectric permittivity tensors ε_{ij} are also given (details in Table III).

The indices m = x, y, z; r = x, y; s = a, b; i = 1, 2, 3; j = 1, 2, 3, 4 and p = a, b, c, d, e, f. The Θ is the element if the time inversion. The symbol $|| C_{2m}$ is equivalent to: $F || C_{2x}$ or $F || C_{2y}$ or $F || C_{2z}$. The similar abbreviations are used in other cases. The number put in the column (3) enables us to reconstruct the explicit form of ε_{ij} (via Table III). The groups K(F) are always written in the local frame of axes such that z || F.

445	(a)	(2)		(=)
(1)	(2)	(3)	(4)	(5)
No	Group K	Tensor ε_{ij}	Directions of \boldsymbol{F}	Broken symmetry
				group K(<i>F</i>)
1	1	78	a.o.	1
2	ī	78	a.o.	1
3	2	79	$\ C_{2z} \ $	2_z
			$\perp C_{2z}$	2 _r ′
			a.o.	1
4	m	79	$\perp \sigma_z$	m_z
			σ_z	m _r
			a.o.	1
5	2/m	79	$\parallel C_{2z}$	$2_z/m_z$
-	_,		$\perp C_{2z}$	$2_r/m_r$
			a.o.	ī
6	222	80	$\parallel C_{2m}$	222
Ū			$\perp C_{2m}$	2_r
			a.o.	1
7	mm2	80	$\parallel C_{2z}$	mm2
•			$\downarrow C_{2z}$	2 _r
			$\perp \sigma_r$	m2m, 2mm
	1		$\ \sigma_r\ $	m_r
			a.o.	1
8	mmm	80	$\parallel C_{2m}$	mmm
-			$\perp C_{2m}$	$2_r/m_r$
			a.o.	1
9	4	81	II C4z	4
	-		$\perp C_{4z}$	2_r
			a.o.	1
10	6	81	II Cez	6
. .	Ĭ		$\perp C_{6z}$	2 _r
		· ·	a.o.	1
	1	1	1	i i

(1)	(2)	(3)	(4)	(5)
11	$\frac{1}{4}$	81		4
				2
			a.o.	
12	6	81	C _{3z}	6
			$\perp C_{3z}$	mr
		ĺ	a.o.	1
13	4/m	81	$ C_{4z} $	4/m
			$\perp C_{4z}$	$2_r/m_r$
		· ·	a.o.	Ī
14	6/m	81	$\ C_{6z}$	6/m
			$\perp C_{6z}$	$2_r/m_r$
			a.o.	Ī
15	422	82	$\ C_{4z}$	422
			$\perp C_{2m}, C_{2s}$	2 _r
			$\ C_{2s}, C_{2r} \ $	222
			a.o.	1
16	622	82	$\ C_{6z}$	622
			$\perp C_{6z}, C'_{2i}, C''_{2i}$	2 _r
			$\ C'_{2i}, C''_{2i} \ $	222
			a.o.	1
17	4mm	82	$\ C_{4z}$	4mm
			$\perp C_{4z}$	2 _r
			$\perp \sigma_r, \sigma_{ds}$	$m2m, \ 2mm$
			$\ \sigma_r, \sigma_{ds} \ $	m _r
			a.o.	1
18	6mm	82	C _{6z}	6mm
1			$\perp C_{6z}$	2 _r
			$\perp \sigma_{vi}, \sigma_{di}$	$m2m,\ 2mm$
			$\ \sigma_{vi}, \sigma_{di} \ $	m _r
			a.o.	1
19	$\overline{4}2m$	82	$ S_{4z}$	$\overline{4}2m$
			$\perp C_{2m}$	2 _r
			$\ C_{2r}$	222
			$\perp \sigma_{ds}$	$m2m, \ 2mm$
			σ_{ds}	m_r
	_		a.o.	1
20	62m	82	C _{3z}	62m
			$\perp C_{3z}; \parallel \sigma_{vi}$	m_r
			C' ₂₁	mm2
			$\perp C'_{2i}$	2 _r
			$\perp \sigma_{vi}$	$m2m,\ 2mm$
			a.o.	1

TABLE I cont.

			T	ABLE I cont.
(1)	(2)	(3)	(4)	(5)
21	4/mm	82	$\parallel C_{4z}$	4/mmm
			$\perp C_{2m}, C_{2s}$	$2_r/m_r$
		•	$\parallel \mathbf{C}_{2r}, \ \mathbf{C}_{2s}$	mmm
			a.o.	$\overline{1}$
22	6/mmm	82	C _{6z}	6/mmm
			$\perp C_{6z}, C'_{2i}, C''_{2i}$	$2_r/m_r$
			$\ C'_{2i}, C''_{2i} \ $	mmm
			a.o.	1
23	3	81	$\ C_{3z}$	3
			$\perp C_{3z}$; or a.o.	1
24	3	81	$\parallel C_{3z}$	3
			$\bot C_{3z}$; or a.o.	ī
25	32	82	$\parallel C_{3z}$	32
			$\perp C_{3z}$; or a.o.	1
			$\parallel C'_{2i}$	2_z
			$\perp C'_{2i}$	2_r
26	3m	82	C _{3z}	3 <i>m</i>
			$\perp C_{3z}$; or a.o.	1
			$\perp \sigma_{di}$	m_z
	_		σ_{di}	$\frac{m_r}{\overline{a}}$
27	3m	82	$ C_{3z} $	$\frac{3m}{7}$
		1	$\perp C_{3z}$; or a.o.	
			$ C'_{2i}$	$2_z/m_z$
			$\perp C'_{2i}$	$\left \frac{2_r}{m_r} \right $
28	23	83	$ C_{2m}$	222
			$\perp U_{2m}$	$\frac{2r}{2}$
			$\ C_{3j} \ $	1
00	9	0.9	$1 \pm C_{3j}$; or a.o.	1
29	mə	00	$ \cup_{2m}$	9 /m
			$1 C_{2m}$	$\frac{2r}{2}$
			$ C_{3j}$	$\left \begin{array}{c} \mathbf{J} \\ \mathbf{T} \end{array} \right $
20	420	02	$\square \square C_{3j}$, or a.o.	1
30	432	00	$ C_{4m}$	122
			$\parallel C_{2p}$	222
			$ \ C_{2p} $	32
			$ C_{a4}$: or a.o.	1
31	$\overline{4}3m$	83		$\overline{4}2m$
UT.	10/16	ॅॅ	LSar	2 _r
		1.		3m
			$\perp C_{34}$; or a.o.	1
			$\downarrow \sigma_{dn}$	m2m, 2mm
			Πσen	2.
	1	1	1 11 ~ ap	1 -7

TABLE I cont.

(1)	(2)	(3)	(4)	(5)
32	$\overline{m3m}$	83	II C _{4m}	4/mmm
			$\perp C_{4m}, C_{2n}$	$\frac{1}{2r}/m_r$
			C _{2n}	mmm
				$\overline{3}m$
			$\perp C_{3i}$; or a.o.	Ī
33	ī'	84	a.o.	17
34	2'	85		2'
•-	-			2'
			a.o.	1
35	m'	85	$\perp \Theta \sigma_{\sigma}$	m'.
			$\parallel \Theta \sigma$	<i>m</i> ′_
			a.o.	1
36	2/m'	86	C ₂₇	$\frac{1}{2}$, m'_{2}
	'		$\perp C_{2z}$	$2r/m'_{r}$
			a.o.	1'
37	2'/m	86	II OC22	$\frac{1}{2'_{2}}/m_{2}$
•••	- /		$ \Theta C_2 $	$\frac{2}{2'}/m_{\pi}$
			20022	
38	21/m	85	1.0. 0.C.	$\frac{1}{2'/m'}$
00	<i>2 11</i>	00		$\frac{2z}{m'}$
			1002z	$\frac{2r}{1}$
39	2'2'2	87	и.с. Съ.	2/2/2
			$\perp C_{2\pi}$	2.
			$\parallel \Theta C_{2r}$	22'2'. 2'22'
			$\perp \Theta C_{2r}$	2'
			a.o.	1
40	m'm'2	87	$\ C_{2z}$	m'm'2
			$\bot C_{2z}$	2_r
			$\bot \Theta \sigma_r$	m'2m', 2m'm'
			$ \Theta \sigma_r$	m_r'
			a.o.	1
41	m'm2'	88	$ \Theta C_{2z}$	m'm2'
			$\perp \Theta C_{2z}$	2'r
			$\perp \Theta \sigma_x$	m2'm'
			$\perp \sigma_y$	2'm'm
1			$\Theta \sigma_x$	m'_r
			$\ \sigma_y\ $	m_r
			a.o.	1
42	m'm'm'	80	$\ C_{2m}\ $	<i>m'm'm</i>
			$\pm C_{2m}$	$\frac{2_r}{m'_r}$
			a.o.	1

(1)	(2)	(3)	(4)	(5)
43	mmm'	80	$\ C_{2z}$	mmm'
			$\perp C_{2z}$	$2_r/m'$
			$\perp \Theta C_{2r}$	$2_r'/m_r$
		ł	$\parallel \Theta C_{2r}$	m'mm, mm'm
			a.o.	$\overline{1}'$
44	m'm'm	87	$\ C_{2z}$	m'm'm
			$\perp C_{2z}$	$2_r/m_r$
			$ \Theta C_{2r}$	m'mm', mm'm'
			$\perp \Theta C_{2r}$	$\frac{2'_r}{m'_r}$
			a.o.	1
45	4′	82	$\parallel \Theta C_{4z}$	4'
			$\perp \Theta C_{4z}$	2_r
10	<i>a</i> 1	0.0	a.o.	
46	6'	82		6', 0'
			$\pm \Theta C_{6z}$	2_r
	/		a.o.	
47	4	82	$ \Theta S_{4z}$	4
			$\pm \Theta S_{4z}$	Z_r
40	7	00	a.o.	 /
48	0	82		0
			$\pm 0.5_{3z}$	$\frac{m_r}{1}$
40	49/9/	Q1		1 19/9!
49	44 4	01	$ C_{4z}$	-12 Z 9
			$\ \Theta C_{2} - \Theta C_{2}$	2/22/ 22/2/
			$\perp \Theta C_{2r}, \Theta C_{2s}$	2'
			a.o.	1
50	62'2'	81	$\ C_{6z}$	62'2'
			$\perp C_{6z}$	2 _r
			$\parallel \Theta C'_{2i}, \Theta C'_{2i}$	2'22', 22'2'
			$\perp \Theta C'_{2i}, \Theta C''_{2i}$	2' _r
			a.o.	1
51	4'22'	82	$\ \Theta C_{4z}$	4'22'
			$\perp C_{2m}$	2_r
			$\ C_{2r}$	222
			$\parallel \Theta C_{2s}$	2'22', 22'2'
			$\perp \Theta C_{2s}$	2'
			a.o.	
52	6'22'	82	$ \Theta C_{6z} $	6'22'
			$ \bot \Theta \cup_{2z}, \Theta \cup_{2i} $	27 1011 1011
				2 44 , 44 4 91919
	1		$ \cup_{2i}$	222
			$1 \pm 0_{2i}$	1
	1			, -

TABLE I cont.

TABLE I cont.

(1)	(2)	(3)	(4)	(5)
53	4/m'	82	$\parallel C_{4z}$	4/ <i>m</i> ′
			$\perp C_{4z}$	$\frac{2_r}{m'_r}$
.			a.o.	1
.54	0/ <i>m</i> '	82	$ C_{6z} $	0/m
			$\pm C_{6z}$	$\frac{2r}{1}$
55	4'/m'	82	α.ο. ∥ ΘC₄-	$\frac{1}{4'/m'}$
00	-,		$\perp \Theta C_{4z}$	$2_r/m'_r$
			a.o.	1
56	6'/m	82	$\ \Theta C_{6z}$	6'/m
			$\perp \Theta C_{6z}$	$2_r'/m_r$
			a.o.	ī
57	4'/m	82	$\parallel \Theta C_{4z}$	4'/m
			$\pm \Theta C_{4z}$	$\frac{2_r}{1}$
58	6'/m'	82		$\frac{1}{6'/m'}$
00	0,		$\perp \Theta C_{6z}$	$2'_{r}/m'_{r}$
			a.o.	ī
59	4m'm'	81	$\ C_{4z}$	4m'm'
			$\perp C_{4z}$	2_r
			$ \Theta \sigma_r, \Theta \sigma_{ds}$	m'_r
			$\pm \Theta \sigma_r, \Theta \sigma_{ds}$	<i>m 2m</i> , <i>2m m</i>
60	6 <i>m'm</i>	81	$\parallel C_{6\tau}$	6 <i>m'm</i>
			$\perp C_{6z}$	2_r
			$\ \Theta \sigma_{vi}, \Theta \sigma_{di} \ $	m'_r
			$\perp \Theta \sigma_{vi}, \Theta \sigma_{di}$	m'2m', 2m'm'
			a.o.	1
61	4' <i>mm</i> '	82	$ \Theta C_{4z} $	4' <i>mm</i> '
			$\ \sigma_n \ $	m_r
			$\perp \sigma_r$	2mm, m2m
			$\ \Theta \sigma_{ds}$	m'_r
			$\perp \Theta \sigma_{ds}$	m'2m', 2m'm'
40			a.o.	1
62	0' <i>m</i> 'm	82		0' <i>m</i> 'm 9/
			$\parallel \sigma_{y}$	4 _r
			$\perp \sigma_{di}$	2'm'm, m'2'm
		1	$\ \Theta \sigma_{vi}$	m'_r
			$\bot \Theta \sigma_{vi}$	2'mm', m2'm'
			a.o.	1

$\overline{(1)}$	(0)			
(1)	$\frac{(2)}{1-(2)}$	$\left(3\right)$	(4)	(5)
63	42' <i>m</i> '	81	$ S_{4z}$	42'm'
			$\perp S_{4z}$	2 _r
		1	$ \Theta C_{2r}$	2'22', 22'2'
			$\perp \Theta C_{2r}$	2'r
			$ \Theta \sigma_{ds}$	m'_r
			$\perp \Theta \sigma_{ds}$	2m'm', m'2m'
			a.o.	1
64	6m'2'	81	S _{3z}	<u>6</u> m'2'
			$\bot S_{3z}$	m _r
			$ \Theta C'_{2i}$	mm'2', m'm2'
			LOC'	2'.
			$\ \Theta\sigma_{vi}$	<i>m</i> '.
			$\bot \Theta \sigma_{vi}$	2'mm', m2'm'
			a.o.	1
65	$\overline{4}'2m'$	82	II OS4-	$\overline{4}'2m'$
			$\perp C_{2m}$	2.
			$\ C_{2n}$	-r 222
		·	$\ \Theta_{\sigma_{2}}$	m'
			$ \Theta \sigma_{s} $	m'9m' 9m'm
				1
66	<u></u> <u></u> <u></u> <u></u>	09		1
00	0711'2	04		0 m 2
			$\perp \bigcirc \Im_{3z}; \parallel \bigcirc \sigma_{vi} \mid$	m_r
				2m'm', m'2m'
			10 ² i	2 _r
			a.o.	$\frac{1}{\overline{1}}$
67	4 m2'	82	$ \Theta S_{4z}$	4 m2'
			$\perp \Theta S_{4z}$	2 _r
			$ \Theta C_{2r}$	2'22', 22'2'
			$\perp \Theta C_{2r}$	$2'_r$
			σ_{ds}	m_r
			$\perp \sigma_{ds}$	$2mm,\ m2m$
			a.o.	1
68	$\overline{6}'m2'$	82	$ \Theta S_{3z}$	$\overline{6}'m2'$
			$\perp \Theta S_{3z}$	m'_r
			$\ \Theta C'_{2i}\ $	mm'2', m'm2'
			LOC,	2'r
			$\ \sigma_{vi}$	m _r
			$\perp \sigma_{ni}$	m'2'm, 2'm'm
			a.o.	1
	·	•		-

TABLE I cont.

TABLE I cont.

(1)	(2)	(3)	(4)	(5)
69	$\frac{(-)}{4/m'm'm'}$	82	C _{4z}	$\frac{1}{4/m'm'm'}$
	,		$\perp C_{2m}, C_{2s}$	$2_r/m_r'$
			$\ C_{2r}, C_{2s} \ $	m'm'm'
			a.o.	1'
70	6/m'm'm'	82	$\ C_{6z}$	6/m'm'm'
	•		$\perp C_{6z}, C'_{2i}, C''_{2i}$	$2_r/m'_r$
			$\ C'_{2i}, C''_{2i} \ $	m'm'm'
			a.o.	1
71	4/m'mm	82	$\ C_{4z} \ $	4/m'mm
			$\perp C_{4z}$	$2_r/m'_r$
			$\ \Theta C_{2r}, \Theta C_{2s}$	mm'm, m'mm
			$\perp \Theta C_{2r}, \Theta C_{2s}$	$2_r'/m_r$
			a.o.	1
72	6/m'mm	82	$\ C_{6z}$	6/m'mm
			$\perp C_{6z}$	$2_r/m'_r$
			$ \Theta C'_{2i}, \Theta C''_{2i} $	mm'm, m'mm
			$\perp \Theta C'_{2i}, \Theta C''_{2i}$	$\frac{2'_r}{\pi'}$
			a.o.	
73	4'/m'm'm	82	$ \Theta C_{4z}$	4'/m'm'm
			$\perp C_{2m}$	$2_r/m_r$
			$ C_{2r} $	m m m
				mmm, mmm
			10028	$\frac{2r}{1'}$
71	61 Immm	09		6'/mmm'
14		02	$\begin{bmatrix} OS_{6z} \\ OS_{6z} \\ $	$\frac{2'}{m_{-}}$
			$\parallel \Theta C_{2i}''$	mm'm, $m'mm$
		ļ	$\ C_{0}^{\prime}$	mmm'
			$\perp C'_{2i}$	$2_r/m'_r$
			a.o.	$\overline{1}'$
75	4'/mmm'	82		4'/mmm'
10	- /		$\perp C_{2m}$	$2_r/m_r$
			$\ C_{2r}\ $	mmm
			$ \parallel \Theta C_{2s}$	<i>m'mm'</i> , <i>mm'm'</i>
			$\square\Theta C_{2s}$	$2_r'/m_r'$
	· · ·		a.o.	1
76	6'/m'm'm	82	ӨС _{6z}	6'/m'm'm
			$\perp \Theta C_{6z}, \Theta C_{2i}''$	$\left \frac{2_r'}{m_r'} \right $
	ļ		$ \Theta C_{2i}''$	$\mid m'mm', mm'm'$
		1	$ \ C'_{2i}$	<i>m'm'm</i>
			$\perp C'_{2i}$	$\left \frac{2_r}{m_r}\right $
			a.o.	1

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(1)	(2)	(3)	(4)	(5)
77	4/mm'm'	81	$\ C_{4z}$	4/ <i>mm</i> ′ <i>m</i> ′
			$\perp C_{4z}$	$2_r/m_r$
			$ \Theta C_{2r}, \Theta C_{2s}$	m'mm', mm'm'
			$\perp \Theta C_{2r}, \Theta C_{2s}$	$\frac{2'_r}{m'_r}$
			a.o.	1
78	6/ <i>mm'm</i>	81	$\ C_{6z}$	6/mm'm'
				$2_r/m_r$
			$ \Theta C_{2i}, \Theta C_{2i}$	2' / m'
			$\pm 0.02i$, $0.02i$	$\frac{2r}{1}$
79	32′	81	II Ca.	32'
	02	0-	$\perp C_{3z}$; or a.o.	1
			∥ ΘC′ ₂ ,	2'_
			$\bot\Theta C'_{2i}$	$2\tilde{r}$
80	3m'	81	$\ C_{3z}$	3m'
			$\perp C_{3z}$; or a.o.	1
		1	$ \Theta \sigma_{di}$	m'_r
			$\perp \Theta \sigma_{di}$	m'_z
81	3	82	$ \Theta S_{6z}$	3
			$\perp \Theta S_{6z}$; or a.o.	1
82	<u>3</u> m'	81	$ S_{6z}$	3m'
			$\perp S_{6z}$; or a.o.	
		1997 - 19	$ \Theta C'_{2i} $	$2'_{z}/m'_{z}$
			$\bot \Theta C_{2i}$	$\frac{2_r}{n}$
83	3 m	82	$ \Theta S_{6z}$	$\int \frac{3m}{\tau'}$
			$\perp \Theta S_{6z}$; or a.o.	
			$ \Theta C_{2i}$	$2_z/m_z$
<u>.</u>			$\square \square $	$\frac{2_r}{7'm'}$
84	3 m'	82		$\left \begin{array}{c} 3 \\ 1 \\ 1 \end{array} \right $
	1		$\bot \Theta S_{6z}$; or a.o	1 $2/m'$
			$\ C_{2i}$	$2z/m_z$
95	m/2	83	$\parallel C_{2i}$	m'm'm'
90	mo		$ C_{2m}$	$\frac{2r}{m'}$
				$\overline{3}'$
				ד'
00	T'2ml	02		$\frac{1}{\overline{a}'}2m'$
80	4 311	00		2.
		.	$ C_{2i}$	$\frac{-r}{3m'}$
			$\perp C_{3i}$; or a.o.	1
			$\ \Theta\sigma_{dp}$	m_r'
			$\square \Theta \sigma_{dp}$	m'2m', 2m'm'

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(1)	(2)	(3)	(4)	(5)
87	4'32'	83	$\parallel \in C_{4m}$	4'22'
			$\perp \Theta C_{4m}$	2_r
			C _{3j}	32'
			$\perp C_{3j}$; or a.o.	1
			$\ \Theta C_{2p}$	22'2', 2'22'
			$\perp \Theta C_{2p}$	$2_r'$
88	m'3m'	83	$\parallel C_{4m}$	4/m'm'm'
			$\perp C_{4m}, C_{2p}$	$2_r/m'_r$
			$\parallel C_{2p}$	m'm'm'
				3'm'
			$\perp C_{3j}$; or a.o.	1
89	m'3m	83	$ S_{4m}$	4'/m'm'm
			$\perp S_{4m}$	$2_r/m'_r$
			$\parallel C_{3i}$	$\overline{3}'m$
			$\perp C_{3i}$; or a.o.	1
			$\ \sigma_{dn} $	m'mm, mm'm
			$\perp \sigma_{dp}$	$2'_{r}/m_{r}$
90	m3m'	83	$ \Theta C_{4m}$	4'/mmm'
			$\perp \Theta C_{4m}$	$2_r/m_r$
			$\ C_{3j}$	<u>3</u> m'
			$\perp C_{3j}$; or a.o.	1
			$ \Theta C_{2p}$	m'mm', mm'm'
		l	$\perp \Theta C_{2p}$	$2_r'/m_r'$
91	11'	84	a.o	11'
92	11'	84	a.o.	11'
93	21'	86	$\ C_{2z}$	$2_{z}1'$
			$\perp C_{2z}$	$2_r 1'$
			a.o.	111
94	m1′	86	$\perp \sigma_z$	$m_z 1'$
		1	$\ \sigma_z$	$m_r 1'$
			a.o.	11'
95	2/m1'	86	$ \parallel \mathbf{C}_{2z}$	$2_z/m_z 1'$
			$\perp C_{2z}$	$\frac{2_r}{m_r}$
			a.o.	11'
96	2221'	80	$\ C_{2m}$	2221/
	ļ		$\perp C_{2m}$	$2_r 1'$
			a.o.	11'
97	mm21'	80	$ C_{2z}$	mm21'
			$\perp C_{2z}$	$ 2_r 1' $
			$ \perp \sigma_r$	m2m1', 2mm1'
	}		$ \sigma_r$	$ m_r 1' $
			a.o.	11′

				TABLE I cont.
(1)	(2)	(3)	(4)	(5)
98	mmm1'	80	$\ C_{2m}$	mmm1'
		1	$\perp C_{2m}$	$2_r/m_r 1'$
• •		1	a.o.	11'
99	41'	82	$ C_{4z} $	41'
			$\perp C_{4z}$	$2_{r}1'$
			a.o.	11'
100	61'	82	$\ C_{6z}$	61'
		1	$\perp C_{6z}$	$2_{r}1'$
	_		a.o.	11'
101	41'	82	$ S_{4z}$	41'
			$\perp S_{4z}$	$2_{r}1'$
		İ	a.o.	11'
102	61'	82	$\ C_{3z} \ $	61'
	Í		$\perp C_{3z}$	$m_r 1'$
			a.o.	11'
103	4/m1'	82	$\ C_{4z}$	4/m1'
			$\perp C_{4z}$	$2_r/m_r 1'$
			a.o.	11'
104	6/m1'	82	$\ C_{6z}$	6/m1'
		Ì	$\perp C_{6z}$	$2_r/m_r 1'$
			a.o.	11'
105	4221'	82	$\ C_{4z}$	4221'
			$\perp C_{2m}, C_{2s}$	$2_{r}1'$
			$\parallel \mathbf{C}_{2r}, \mathbf{C}_{2s}$	2221'
100			a.o.	11'
106	6221′	82	$\ C_{6z}$	6221'
			$\perp \mathbf{C}_{6z}, \mathbf{C}'_{2i}, \mathbf{C}''_{2i}$	$2_r 1'$
			$\ C'_{2i}, C''_{2i}\ $	2221/
107	A 17	00	a.o.	11'
107	4 <i>mm</i> 1'	82	$\ C_{4z}\ $	4mm1'
			$\perp C_{4z}$	2 _r 1'
			$\perp \sigma_r, \sigma_{ds}$	m2m1', 2mm1'
			$\ \sigma_r, \sigma_{ds} \ $	$m_r l'$
100	6		a.o.	11'
TUN	0mm1'	82	$\ C_{6z}\ $	6mm1'
	ĺ	ľ	$\perp C_{6z}$	$2_r l'$
			$\perp \sigma_{vi}, \sigma_{di}$	m2m1', 2mm1'
			$\ \sigma_{vi}, \sigma_{di} \ $	$m_r 1'$
			a.o.	11'

TABLE I cont.

(1)	(2)	(3)	(4)	(5)
109	$\overline{42m1'}$	82	S _{4z}	$\overline{4}2m1'$
	-		$\perp C_{2m}$	$2_{r}1'$
			σ_{dr}	$m2m1', \ 2mm1'$
			$\perp \sigma_{dr}$	$m_r 1'$
			$\ \mathbf{C}_{2n}\ $	2221'
			11 ° 21	11'
110	701/	0.0	u.o. II Ca	$\overline{6}2m1'$
110	02111	02	$\ C_{3z} \ \sigma$	m-1'
			1032, 1001	mm21'
			$ O_{2i}$	2.1
			$\perp \cup_{2i}$	$m^{2m1'}$ $2mm1'$
		Į.	$\perp \sigma_{vi}$	11/
			a.o.	$\frac{11}{4/mmm1'}$
111	4/mmm1'	82	$ C_{4z}$	$\frac{4}{100000000000000000000000000000000000$
		1	$\perp C_{2m}, C_{2s}$	$\frac{2r}{mr}$
			$\ C_{2r}, C_{2s}\ $	$\frac{1}{1}$
	1.5		a.o.	
112	6/ <i>mmm</i> 1'	82	$ C_{6z} $	$6/mmm^{1}$
			$\perp C_{6z}, C'_{2i}, C''_{2i}$	$2_r/m_r 1$
	1		$\ C'_{2i}, C''_{2i} \ $	
	· ·	-	a.o.	11'
113	31'	82	$\ C_{3z}$	31'
			$\perp C_{3z}$; or a.o.	11′
114	31'	82	C _{3z}	31'
			$\perp C_{3z}$; or a.o.	111
115	321'	82	$\ C_{3z}\ $	321'
110	0		$\perp C_{3z}$; or a.o.	11'
	1			$2_{z}1'$
		-	$\perp C'_{2i}$	$2_r 1'$
116	3m1/	82	$\ \mathbf{C}_{3r} \ $	3m1'
110	5///1		$1 C_{2}$; or a.o.	11'
			$\parallel \sigma_{n}$	$m_r 1'$
				m.1'
	5 1/	0.0		$\frac{3m1'}{3m1'}$
117	$3m^{1}$	02	$\ C_{3z} \ $	111
			$\pm C_{3z}$, or a.o.	$\frac{11}{2}$ /m 1/
				$2_z/m_z^{-1}$
			$\perp C_{2i}$	2r/mr1
118	231'	83	$\ C_{2m}$	2221
			$ \perp C_{2m}$	$\frac{2r1}{21}$
			$\ C_{3j}$	J 31'
			$\perp C_{3j}$; or a.o.	111'
119	m31'	83	$\ C_{2m}$	mmm1'
			$\perp C_{2m}$	$\frac{2_r}{m_r}$
	1		C _{3j}	31'
	1		$\perp C_{3j}$; or a.o.	11′
	I	1		•

		~		TABLE I cont.
(1)	(2)	(3)	(4)	(5)
120	4321'	83	C _{4m}	4221'
			$\perp C_{4m}, C_{2p}$	2 _r 1'
			$\parallel C_{2p}$	2221'
			C _{3j}	321′
			$\perp C_{3j}$; or a.o.	11'
121	$\overline{4}3m1'$	83	$ S_{4m}$	$\overline{4}2m1'$
			$\perp S_{4m}$	$2_{r}1'$
			$\ C_{3j}\ $	3m1'
			$\perp C_{3j}$; or a.o.	11'
			$\perp \sigma_{dp}$	m2m1', 2mm1'
			$ \sigma_{dp}$	$m_r 1'$
122	m3m1'	83	$\ C_{4m}$	4/ <i>mmm</i> 1′
			$\perp C_{4m}, C_{2p}$	$2_r/m_r 1'$
			$\parallel C_{2p}$	mmm1'
			$\ C_{3j}$	$\overline{3}m1'$
			$\perp C_{3j}$; or a.o.	11'

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TABLE II

The explicit form of the broken groups K(F)=J(F)+b(F)J(F)for F = E, k, H, where J(F) contains the elements preserving field direction F, $F \to F$, and b(F) J(F)reversing the field F to opposite, $F \to -F$. For each group K(F) the forms of the tensors ε_{ij}^{e} and ε_{ij}^{e} are also given (via table III).

(1)	(0)	(1)			(0)	(7)	(0)		(10)
(1)	(2)	$\left \begin{pmatrix} 3 \end{pmatrix} \right $	(4)	(0)		(0)	(0)	$\left(\frac{9}{1} \right)$	(10)
$\frac{\mathbf{K}(\mathbf{F})}{1}$	J(E)	b(E)	ε_{ij}	J(k)	O(k)	ε _{ij}	J(H)		ε _{ij}
1 1		— т	10		— т	19			
1			10			10	1		
2_x		C_{2x}	19		C_{2x}	19		C_{2x}	19
2_y		C_{2y}	20	1	U_{2y}	20		U _{2y}	20
2_z	2_z		2	2_z	-	2	2_z	-	
m_{x}	m_x	-	3	m_x] -	3		σ_x	19
m_y	m_y	-	4	m_y		4	1	σ_y	20
m_{z}	1	σ_z	21	1	σ_z	21	$\frac{m_z}{T}$	-	2
$2_x/m_x$	m_x	I	22	m_x	I	22	$ \underline{1} $	C_{2x}	19
$2_y/m_y$	m_y	I	23	m_y		23	1	C_{2y}	20
$2_z/m_z$	2_z	I	24	2_z	I	24	$2_z/m_z$	-	2
222	2_z	C_{2x}	25	2_z	C_{2x}	25	2_z	C_{2x}	25
mm2	mm2	-	5	mm2	- -	5	2_z	σ_x	25
m2m	m_x	σ_z	26	m_x	σ_z	26	m_z	C_{2y}	25
2mm	m_y	σ_z	27	m_y	σ_z	27	m_z	C_{2x}	25
mmm	mm2	I	28	mm2	I	28	$2_z/m_z$	C_{2x}	25
3	3	— ·	6	3	-	6	3	-	6
3	3	I	29	3	I	29	3	-	6
32	3	C_{2x}	30	3	C_{2x}	30	3	C_{2x}	30
3m	3 <i>m</i>	_	7	3 <i>m</i>	_	7	3	σ_x	30
$\overline{3}m$	3 <i>m</i>	I	31	3m	I	31	3	σ_x	30
4	4	-	6	4	_ ·	6	4	-	6
$\overline{4}$	2_z	S_{4z}	32	2_z	S_{4z}	32	4	_	6
4/m	4	Ι	29	4	I	29	4/m	_	6
422	4	C_{2x}	30	4	C_{2x}	30	4	C_{2x}	30
4mm	4mm	-	7	4mm	-	7	4	σ_x	30
$\overline{4}2m$	mm2	S_{4z}	33	mm2	S_{4z}	33	4	C_{2x}	30
4/mmm	4mm	Ι	31	4 <i>mm</i>	I	31	4/m	C_{2x}	30
6	6	-	6	6	_	6	6	-	6
<u>6</u>	3	σ_z	29	3	σ_z	29	6	_	6
6/m	6	I	29	6	Ī	29	6/m	_	6
622	6	C_{2x}	30	6	C_{2x}	30	6	C_{2x}	30
6mm	6mm	-	7	6mm	_	7	6	σ_x	30
$\overline{6}2m$	3 <i>m</i>	σ_z	31	3m	$C_{2\pi}$	31	6	$\tilde{C_{2\pi}}$	30
6/mmm	6mm	Ĩ	31	6mm	I	31	6/m	C_{2r}	30
11'	11′	_	8	1	Θ	41	1	Θ	41

.

TABLE II cont.

(1)	1 (2)	TT	-						
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
11'	11'	I	40	1	Θ	40	1	Θ	41
$2_x 1'$	11'	C_{2x}	42	$2'_x$	Θ	54	$ 2'_x$	Θ	54
$2_y 1'$	11'	C_{2y}	43	$2'_y$	Θ	55	$2\tilde{y}$	Θ	55
$2_{z}1'$	$2'_z$	-	9	2_z	Θ	56	2_z	Θ	56
$m_x 1'$	$m_x 1'$	-	10	m_x	Θ	57	m'_x	Θ	54
$m_y 1'$	$m_y 1'$	-	11	m_y	Θ	58	m'_{y}	Θ	55
$m_z 1'$	11'	σ_z	44	m'_z	Θ	59	m_z	Θ	56
$2_x/m_x 1'$	$m_x 1'$	I	47	$2'_x/m_x$	Θ	47	$2'_{x}/m'_{x}$	Θ	54
$2_y/m_y' 1'$	$m_y 1'$	I	46	$2_y'/m_y$	Θ	46	$2'_{y}/m'_{y}$	Θ	55
$2_z/m_z 1'$	$2_{z}1'$	I	45	$2_z/m'_z$	Θ	45	$2_z/m_z$	Θ	56
2221′	2,1'	C_{2x}	48	2'2'2	Θ	72	2'2'2	Θ	72
mm21'	mm21'	-	5	mm2	Θ	28	m'm'2	Θ	72
m2m1'	$m_x 1'$	C_{2y}	49	m2'm'	Θ	73	m'2'm	Θ	72
2mm1'	$m_y 1'$	C_{2x}	50	2'mm'	Θ	74	2'm'm	Θ	72
mmm1'	mm21'	I	28	mmm'	Θ	28	m'm'm	Θ	72
31'	31'	-	7	3	Θ	30	3	Θ	30
31'	31'	I	31	3	Θ	31	3	Θ	30
321'	31'	C_{2x}	31	32'	Θ	30	32'	Θ	30
3m1'	3m1'	-	7	3m	Θ	31	3m'	Θ	30
$\overline{3}m1'$	3m1'	I	31	3'm	Θ	31	$\overline{3}m'$	Θ	30
41′	41'	_	7	4	Θ	30	4	Θ	30
4 1′	2,1'	S₄,	34	4	Θ	31	4	Θ	30
4/m1'	41'	I	31	4/m'	Θ	31	$\frac{1}{4/m}$	Θ	30
4221'	41'	C_{2r}	31	42'2'	Θ	30	42'2'	Θ	30
4mm1'	4mm1'	-	7	4mm	Θ	31	4m'm'	Θ	30
$\overline{4}2m1'$	mm21'	S₄,	33	$\overline{4}'2'm$	Θ	31	$\overline{4}2'm'$	Θ	30
$\frac{1}{4/mmm1'}$	4mm1'	I	31	4/m'mm	Θ	31	4/mm'm'	Θ	30
61'	61'	_	7	6	Θ	30	6	Θ	30
<u>6</u> 1'	31'	σ_{\star}	31	6	Θ	31	6	Θ	30
6/m1'	61'	Ī	31	6/m'	Θ	31	6/m	Θ	30
6221'	61'	\mathbf{C}_{2r}	31	62'2'	Θ	30	62'2'	Θ	30
6 <i>mm</i> 1'	6mm1'	- 21	7	6 <i>mm</i>	Θ	31	6m'm'	Θ	30
$\overline{6}2m1'$	3m1'	σ.	31	$\overline{6}'2'm$	θ	31	$\overline{6}2'm'$	A	30
6/mmm1'	6mm1'	T	31	6/m'mm	Ă	31	6/mm'm'	Ă	30
$\overline{1}'$	1	1 1/	11	ο <i>γ π. π.</i> π. Τ'		8	1	τ/	<i>4</i> 1
1 2/	1	Ċ/	51	⊥ 2/		13	<u>-</u> 9/	1 _	13
~ <i>x</i> 9'	1	C_{2x}	52	~x 9/		14	2x 9/	_	14
~y ?'	- 9/	2y	12	1	C'	52	<i>²y</i>	C!	53
$\frac{z}{m'}$	$\frac{z}{m'}$	_	13	1	σ'_{2z}	51	- m'	\bigcirc_{2z}	00 19
m'	m'_{x}	_	14	1	$\frac{\sigma}{\pi}$	59	m'	_	10
'''y	'''y	-	17	+	U Y	04	'''y	-	1.4

TABLE II cont.

$\begin{array}{c c c c c c c c c c c c c c c c c c c $	(9)	(10)
m'_z 1 σ'_z 53 m'_z - 12 1 σ'_z	σ'_z	53
$2_x/m'_x$ m'_x C_{2x} 54 $\vec{1}'$ C_{2x} 42 m'_x 1	ľ	54
$2_{y}/m'_{y}$ m'_{y} C_{2y} 55 $\overline{1}'$ C_{2y} 43 m'_{y} 1	1′	55
$2_z/m'_z$ 2_z σ'_z 56 $2_z/m'_z$ - 9 2_z I	I′	56
$2'_{r}/m_{x}$ m_{x} C'_{2r} 57 $2'_{r}/m_{z}$ - 10 $2'_{x}$ 1	I′	54
$2'_{y}/m_{y}$ m_{y} C'_{2y} 58 $2'_{y}/m_{y}$ - 11 $2'_{y}$ I	ľ	55
$2'_z/m_z$ $2'_z$ σ_z 59 $\overline{1}'_z$ σ_z 44 m_z I	I′	56
$2'_{r}/m'_{r}$ m'_{r} I 60 $2'_{r}$ I 60 $2'_{r}$ I 60 $2'_{r}/m'_{r}$ -	-	13
$2'_{r}/m'_{u} m'_{u} I 61 2''_{u} I 61 2''_{u} J 61 2''_{u}/m'_{u} -$	-	14
$2'_z/m'_z$ $2'_z$ I 62 m'_z I 62 $\overline{1}$	C'_{2z}	53
$2^{'}2^{'}2^{'}$ $2_{z}^{'}$ $C_{2x}^{'}$ 63 $2^{'}2^{'}2$ $ 15$ $2^{'}2^{'}2$ $-$	-	15
$2'22'$ $2'_{z}$ C'_{2y} 64 $2'_{x}$ C'_{2z} 66 $2'_{x}$ 0	C'_{2z}	66
$22'2'$ $2'_{z}$ C_{2x} 65 $2'_{y}$ C'_{2z} 67 $2'_{y}$ $0'_{z}$	C'_{2z}	67
$m'm'2$ $m'm'2$ - 15 2_z σ'_x 63 $m'm'2$ -	-	15
$m'2m' \mid m'_x \mid C_{2y} \mid 66 \mid m'_z \mid C_{2y} \mid 64 \mid m'_x \mid C_{2y}$	σ'_z	66
$2m'm'$ m'_y C_{2x} 67 m'_z C_{2x} 65 m'_y c	σ'_z	67
$m'2'm$ m'_x σ_z 68 $2'_y$ σ_z 69 $m'2'm$ $-$	-	15
$2'm'm$ m'_y σ_z 69 $2'_x$ σ_z 68 $2'm'm$ $-$	-	15
$m'm2'$ $m'm2'$ – 16 m_y C'_{2z} 70 m'_x C	C'_{2z}	66
$2'mm' \mid m_y \mid \sigma'_z \mid 70 \mid 2'mm' \mid - \mid 16 \mid 2'_x \mid c$	σ'_z	66
$m2'm'$ m_x σ'_z 71 $m2'm'$ $ 17$ $2'_y$ σ'_z	σ'_z	67
$mm'2' \mid mm'2' \mid - \mid 17 \mid m_x \mid C'_{2z} \mid 71 \mid m'_y \mid 0$	C'_{2z}	67
$m'm'm' \mid m'm'2 \mid I' \mid 72 \mid 2_z/m'_z \mid C_{2x} \mid 48 \mid m'm'2 \mid I$	I′	72
mmm' $mm2$ I' 28 mmm' - 5 2'2'2 I	ľ	72
$mm'm \mid mm'2' \mid I' \mid 73 \mid 2'_x/m_x \mid C_{2y} \mid 49 \mid 2'm'm \mid 1$	ľ	72
$m'mm$ $m'm2'$ I' 74 $2'_y/m_y$ C_{2x} 50 $m'2'm$ 1	ľ	72
$m'm'm \mid m'm'2 \mid I \mid 75 \mid 2'2'2 \mid I \mid 75 \mid m'm'm \mid -$	~	15
$m'mm' \mid m'm2' \mid I \mid 76 \mid 2'mm' \mid I \mid 76 \mid 2'_x/m'_x \mid 0$	C'_{2z}	66
$mm'm' mm'2' 1 77 m2'm' 1 77 2'_y/m'_y 0$	C'_{2z}	67
$\overline{3}'$ 3 I' 30 $\overline{3}'$ - 7 3 I	I'	30
$32'$ 3 C'_{2x} 29 $32'$ - 6 $32'$ -	-	6
$3m'$ $3m'$ - 6 3 σ'_x 29 $3m'_z$ -	-	6
$\overline{3}m'$ $3m'$ I 29 $32'$ I 29 $3m'$ -	-	6
$\overline{3}'m$ $3m$ I' 31 $\overline{3}'m$ - 7 $32'$ I	I'	30
$\overline{3}'m'$ $3m'$ $1'$ 30 $\overline{3}'$ σ'_x 31 $3m'$ 1	I′	30
$4'$ $ 4'$ $ 7 $ $ 2_z$ $ C'_{4z} $ $ 35 $ $ 2_z$ $ C'_{4z} $	C'_{4z}	35
$\overline{4}'$ 2 _z S' _{4z} 35 $\overline{4}'$ - 7 2 _z S	S' _{4z}	35
4/m' 4 1' 30 $4/m'$ - 7 4 1	I'	30

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TA	BLE	Π	cont.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
4'/m	4'	I	31	$\overline{4}'$	Ι	31	$\frac{1}{2_{r}/m_{r}}$	C'	35
4'/m'	4'	ľ	31	$2_z/m'_z$	C'4,	34	$ \overline{4}''$	I'	30
42'2'	4	C'_{2x}	29	42'2'	- "	6	42'2'		6
4'22'	4'	C_{2x}	31	2'2'2	C'_{4z}	36	2'2'2	C'4.	36
4m'm'	4m'm'	-	6	4	σ'_x	29	4m'm'	- 14	6
4' <i>mm</i> '	4' <i>mm</i> '	-	7	mm2	C'_{4z}	38	$m'_a m'_b 2$	C'_{4z}	36
$\overline{4}2'm'$	m'm'2	S_{4z}	37	2'2'2	S_{4z}	39	$\overline{4}2'm'$	-	6
$\overline{4}'2m'$	m'm'2	S'_{4z}	36	4	C_{2x}	31	$m_a^\prime m_b^\prime 2$	S'_{4z}	36
$\overline{4}'m2'$	mm2	S'_{4z}	38	$\overline{4}'m2'$	-	7	$2'_{a}2'_{b}2$	S'_{4z}	36
4/m'm'm'	4m'm'	I'	30	4/ <i>m</i> '	C_{2x}	31	4m'm'	I′	30
4/m'mm	4mm	I'	31	4/m'mm	-	7	42'2'	ľ	30
4' <i>/mmm</i> '	4'mm'	I	31	$\overline{4}'m2'$	I	31	$m'_a m'_b m$	C'_{4z}	36
4'/m'm'm	4' <i>mm</i> '	I'	31	$m_a m_b m'$	S_{4z}	33	$\overline{42'}m'$	I′	30
4/mm'm'	4m'm'	Ι	29	42'2'	I	29	4/mm'm'	-	6
6'	6′	-	7	3	C'_{2z}	30	3	C'_{2z}	30
$\overline{6}'$	3	σ'_z	30	<u></u> 6'	-	7	3	σ'_z	30
6/m'	6	Ι'	30	6/ <i>m</i> ′	-	7	6	ľ	30
6'/m'	6'	Ι	31	<u></u> 6'	I	31	3	σ'_z	30
6'/m	6'	I'	31	3'	σ_z	31	<u>6</u>	I′	30
62'2'	6	C'_{2x}	29	62'2'	_	6	62'2'	-	6
6'22'	6'	C_{2x}	31	32'	C'_{2z}	30	32'	C'_{2z}	30
6m'm'	6 <i>m'm</i>	-	6	6	σ'_x	29	6 <i>m'm</i> '	-	6
6'mm'	6' <i>mm</i> '	-	7	3m	C'_{2z}	31	<u>3</u> m'	C'_{2z}	30
$\overline{6}2'm'$	3 <i>m</i> ′	σ_z	29	32'	σ_z	29	<u>6</u> 2' <i>m</i> '		6
$\overline{6}'m2'$	3 <i>m</i>	σ'_z	31	$\overline{6}'m2'$		7	32'	σ'_z	30
$\overline{6}'m'2$	3m'	σ'_z	30	<u></u> 6'	C_{2x}	31	3 <i>m</i> ′	σ'_z	30
6/m'm'm'	6 <i>m' m'</i>	I'	30	6/m'	C_{2x}	31	6 <i>m'm</i>	I′	30
6/m'mm	6 <i>mm</i>	I′	31	6/m'mm	-	7	62'2'	I'	30
6'/mmm'	6' <i>mm</i> '	I′	31	$\overline{3}'m$	σ_z	31	$\overline{6}2'm'$	I′	30
6'/m'm'm	6' <i>mm</i> '	Ι	31	$\overline{6}'2'm$	I	31	$\overline{3}m'$	C'_{2z}	30
6/mm'm'	6 <i>m' m'</i>	Ι	29	62'2'	Ι	29	6/mm'm'	-	6

TABLE III

The forms of the even $\varepsilon_{ij}^{e}(\varepsilon_{ij}^{e}(F) = \varepsilon_{ij}^{e}(-F))$ and odd $\varepsilon_{ij}^{o}(\varepsilon_{ij}^{o}(F) = -\varepsilon_{ij}^{o}(-F))$ parts of the tensor $\tilde{\varepsilon}_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{o} + i(\varepsilon_{ij}^{ie} + \varepsilon_{ij}^{io})$. The forms of the imaginary parts ε_{ij}^{ie} and ε_{ij}^{io} are exactly the same as ε_{ij}^{e} and ε_{ij}^{o} , respectively. The components 11, 22, 33, 12, 21, 13, 31, 23, 32 are denoted by symbols a, b, c, d, e, f, g, h, i, respectively. The numbers 78-88 refer to the tensors $\tilde{\varepsilon}_{ij} = \varepsilon_{ij} + i\varepsilon_{ij}^{i}$ of the groups K.

	The numbers 18-88						refer to the tensors $\varepsilon_{ij} = \varepsilon_{ij}$.					+ $i\varepsilon_{ij}$ of the groups K.					
	ε_{ij}^{e}			ε_{ij}^{o}			$\varepsilon^{\rm e}_{ij}$			ε_{ij}°			ε_{ij}^{e}			ε_{ij}°	
~ 1						2						3					
a	d	f	a	d	f	a	d	0	a	d	0	a	0	0	a	0	0
е	ь	h	е	Ь	h	е	Ъ	0	е	b	0	0	Ъ	h	0	Ь	h
g	i	с	g	i	С	0	0	С	0	0	С	0	i	С	0	i	С
4			ĸ			5						6					
a	0	f	a	0	f	a	0	0	a	0	0	a	d	0	a	d	0
0	b ,	0	0	ь	0	0	b	0	0	b	0	-d	a	0	-d	a	0
g	0	с	g	0	С	0	0	С	0	0	С	0	0	C	0	0	С
7						8						9					
a	0	0	a	0	0	a	d	f	a	d	f	a	d	0	а	d	0
0	a	0	0	a	0	d	ь	h	d	b	h	d	ь	0	d	b	0
0	0	с	0	0	с	f	h	С	f	h	с	0	0	с	0	0	с
10						- 11						12					
a	0	0	a	0	0	a	0	f	a	0	f	a	d	f	a	d	f
0	Ъ	h	0	b	h	0	ь	0	0	Ъ	0	d	ь	h	d	Ь	h
0	h	С	0	h	с	f	0	С	f	0	с	-f	-h	С	-f	-h	C
13						14						15					
a	d	f	a	d	f	a	d	f	a	d	f	a	d	0	a	d	0
-d	Ъ	h	-d	Ъ	h	-d	ь	h	-d	b	h	-d	Ъ	0	-d	Ь	0
-f	h	С	-f	h	с	f	-h	с	f	-h	С	0	0	С	0	0	С
16						17						18					
a	. 0	f	a	0	ſ	a	. 0	0	a	0	0	a	d	f	0	0	0
0	b	0	0	Ь	0	0	Ь	h	0	b	հ	e	Ь	h	0	0	0
-f	0	С	-f	0	С	0	-h	С	0	-h	С	g	i	С	0	0	0
19						20						21					
a	0	0	0	d	f	a	0	f	0	d	0	a	d	0	0	0	f
0	b	h	е	0	0	0	ь	0	e	0	h	e	b	0	0	0	h
0	i	С	g	0	0	g	0	С	0	i	0	0	0	С	g	i	0
22						23						24					
a	0	0	0	0.	0	a	0	f	0	0	0	a	d	ο	0	0	0
0	b	h	0	0	0	0	b	0	0	0	0	e	b	0	0	0	0
0	i	С	0	0	0	g	0	С	0	0	0	0	0	С	0	0	0
25						26						27					
a	0	0	0	d	0	a	0	0	0	0	0	a	0	0	0	0	f
0	Ь	0	e	0	0	0	b	0	0	0	հ	0	Ь	0	0	0	0
0	0	С	0	0	0	0	0	С	0	i	0	0	0	С	g	0	0

														TA.	RLE	111 co	ont.
28						29						30					
a	0	0	0	0	0	a	d	0	0	0	0	a	0	0	0	d	0
0	Ь	0	0	0	0	-d	a	0	0	0	0	0	a	0	-d	0	0
0	0	С	0	0	0	0	0	С	0	0	0	0	0	с	0	0	0
31						32						33					
a	0	0	0	0	0	a	d	0	a	d	0	a	0	0	0	d	0
0	a	0	0	0	0	-d	a	0	d	-a	0	0	a	0	d	0	0
0	0	с	0	0	0	0	0	с	0	0	0	0	0	с	0	0	0
34						35						36					
a	0	0	a	d	0	a	0	0	a	d	0	a	0	0	0	d	0
0	a	0	d	-a	0	0	a	0	е	-a	0	0	a	0	е	0	0
0	0	с	0	0	0	0	0	С	0	0	0	0	0	с	0	0	0
37						38						39					-
a	d	0	0	d	0	a	0	0	a	0	0	a	d	0	a	0	0
-d	a	0	d	0	0	0	a	0	0	-a	0	-d	a	0	0	-a	0
0	0	С	0	0	0	0	0	С	0	0	0	0	0	с	0	0	0
40						41						42					
a	d	f	0	0	0	a	d	f	0	d	f	a	0	0	0	d	f
d	Ь	h	0	0	0	d	Ь	h	-d	0	h	0	Ь	h	d	0	0
f	h	С	0	0	0	f	h	С	-f	-h	0	0	h	С	f	0	0
43						44	,					45					
a	0	f	0	d	0	a	d	0	0	0	ſ	a	d	0	0	0	0
0	Ъ	0	d	0	h	d	b	0	0	0	h	d	b	0	0	0	0
f	0	С	0	h	0	0	0	С	f	հ	0	0	0	С	0	0	0
46						47						48					
a	0	f	0	0	0	a	0	0	0	0	0	a	0	0	0	\mathbf{d}	0
0	ь	0	0	0	0	0	ь	h	0	. 0	0	0	b	0 -	d	0	0
f	0	С	0	0	0	0	h	С	0	0	0	0	0	С	0	0	0
49						50					_	51	_			_	_
a	0	0	0	0	0	a	0	0	0	0	f	a	d	f	0	d	f
0	Ь	0	0	0	h	0	Ь	0	0	0	0	-d	b	h	d	0	h
0	0	С	0	h	0	0	0	С	f	0	0	-f	h	С	f	-h	0
52	-	-	_	-	-	53		-	-			54	_	_	_		
a	d	f	0	d	f	a	d	f	0	d	t	a	0	0	0	d	1
-d	b	h	d	0	h	d	b	h	-d	0	h	0	b	h	-d	0	0
f	-h	С	-f	h	0	-f	-h	С	f	h	0	0	h	C	-f	0	0
55		-		_		56	-			-		57			_	_	
a	0	f	0	d	0	a	d	0	0	d	0	a	0	0	0	0	0
0	b	0	-d	0	h	d	b	0	-d	0	0	0	b	h	0	0	h
f	0	С	0	-h	0	0	0	С	0	0	0	0	h	С	0	-h	0

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TABLE III cont.

58						59						60					
а	0	f	0	0	f	a	d	0	0	0	f	a	d	f	0	0	0
0	Ъ	0	0	0	0	d	Ь	0	0	0	h	-d	b	h	0	0	0
f	0	с	-f	0	0	0	0	с	-f	-h	0	-f	h	С	0	0	0.
61						62						63					
a	d	f	0	0	0	a	d	f	0	0	0	a	d	0	0	d	0
-d	ь	h	0	0	0	d	b	h	0	0	0	-d	Ъ	0	d	0	0
f	-h	с	0	0	0	-f	-h	с	0	0	0	0	0	С	0	0	0
64						65						66					
a	0	f	0	d	0	a	0	0	0	d	f	a	0	f	0	d	0
0	ь	0	d	0	h	0	ь	h	d	0	0	0	b	0	-d	0	h
-f	0	с	0	-h	0	0	-h	С	-f	0	0	-f	0	С	0	h	0
67						68						69					
a	0	0	0	d	f	a	d	0	0	0	f	a	d	0	0.	0	f
0	b	h	-d	0	0	-d	Ъ	0	0	0	h	-d	Ь	0	0	0	h
0	-h	с	f	0	0	0	0	С	-f	h	0	0	0	С	f	-h	0
70						71						72					
a	0	f	0	0	f	a	0	0	0	0	0	a	0	0	0	d	0
0	b	0	0	0	0	0	Ъ	h	0	0	h	0	b	0	-d	0	0
-f	0	С	f	0	0	0	-h	С	0	h	0	0	0	С	0	0	0
73						74						75					
a	0	0	0	0	0	a	0	0	0	0	f	a	d	0	0	0	0
0	b	0	0	0	h	0	Ъ	0	0	0	0	-d	b	0	0	0	0
0	0	с	0	-h	0	0	0	с	-f	0	0	0	0	С	0	0	0
76						77						78					
a	0	ſ	0	0	0	a	0	0	0	0	0	a	d	f			
0	b	0	0	0	0	0	Ъ	h	0	0	0	е	b	h			
-f	0	С	0	0	0	0	-h	С	0	0	0	g	i	С			
79						80						81					
a	d	0				a	0	0				a	d	0			
е	b	0				0	b	0				-d	a	0			
0	0	с				0	0	С				0	0	С			
82						83						84					
a	0	0				a	0	0				a	d	f			
0	a	0				0	a	0				d	b	h			
0	0	С				0	0	a				f	h	С			

			 					ГАВ	LE I	II co	nt.
85			86		••••••		87				
a	d	f	a	d	0		a	d	0		
d	Ь	h	d	Ь	0	-	d	Ъ	0		
-f	-h	с	0	0	С		0	0	с		
88											
a	0	f									
0	b	0									
-f	0	с									
			•			•					

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