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## SPIN-DEPENDENT INTERACTION OF A BAND ELECTRON WITH PARAMAGNETIC IMPURITIES\*

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A band electron interacting with  $N$  electrons in a  $d$ -shell of a paramagnetic impurity in zinc-blende type semiconductors was considered. The hybridization of band and  $d$ -states in the crystal field and the nonorthogonality mechanisms of interaction were examined in addition to the standard direct exchange. The Heisenberg-type form was derived for the interaction between a  $\Gamma_8$ -electron and an impurity with a half-filled  $d$ -shell. The domination of the hybridization mechanism explains the observed sign of the effective exchange constant for  $\Gamma_8$  electrons in diluted magnetic semiconductors (DMS) and the difference in magnitude of this constant for  $\text{Mn}^{2+}$  and  $\text{Fe}^{3+}$  ions in HgSe.

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It is generally believed that the spin-dependent interaction of band electrons with paramagnetic ions in DMS can be described by an effective Heisenberg type Hamiltonian proportional to the product  $\mathbf{S} \cdot \mathbf{J}$ , where  $\mathbf{S}$  is the total spin operator of  $d$ -electrons and  $\mathbf{J}$  is the appropriate quasi-angular momentum for the band electron. Such a Hamiltonian has been rigorously derived from the direct exchange interaction for an electron in  $s$ -like bands (whatever the number of electrons in the  $d$ -shell of the paramagnetic ion) [1] and for an electron in  $\Gamma_8$  bands in zinc-blende type crystals [2], but only in the case of a half filled  $d$ -shell. Incompatible with the direct exchange mechanism, experimental results for DMS gave a negative effective exchange constant  $\beta$  for  $\Gamma_8$  bands.

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Guided by the results of [3], concerning a band electron interacting with an electron from a single  $d$ -orbital hybridized with the band, Bhattacharjee et al. [4] suggested that the antiferromagnetic interaction for a  $p$  band was due to strong  $p$ - $d$  hybridization. In a more recent paper [5] Larson et al. gave support to this idea. They considered several  $d$ -orbitals hybridizing with the  $p$ -type Bloch functions but they used the band with well defined spin and a single hybridization matrix element independent of  $d$ - and  $p$ -orbitals involved. On the other hand, Semenov and Shanina [6] ascribed the antiferromagnetic interaction to the nonvanishing overlaps between the  $p$ -band and localized  $d$ -states.

It can be shown, that  $\Gamma_6$  bands do not hybridize and have vanishing overlaps with  $d$ -orbitals, so that the direct exchange remains the only mechanism for  $s$ - $d$  interaction.

Inspired by the controversy on the origins of the negative exchange constant  $\beta$  and by the new experimental results [7] indicating that  $\beta$  for  $\text{Fe}^{3+}$  ions in  $\text{HgSe}$  is much bigger than that for  $\text{Mn}^{2+}$  in the same compound, we decided to examine in detail the  $p$ - $d$  interaction in zinc-blende type crystals. To account properly for the symmetry of the many-electron wave functions we use the occupation number representation. Our one-electron model Hamiltonian has the form  $H = H_0 + H_1$

$$H_0 = \sum_{m,\sigma} \varepsilon_d \hat{n}_{m\sigma} + \sum_{n,k} \varepsilon_n(k) \hat{n}_{nk} \quad (1)$$

and

$$H_1 = \sum_{n,k,m,\sigma} [L_{m\sigma nk} \hat{a}_{m\sigma}^+ \hat{a}_{nk} + h.c.], \quad (2)$$

where  $m = -2, -1, 0, 1, 2$  and  $\sigma = \pm 1/2$  specify the states of a  $d$ -electron, whereas  $n$  and  $k$  define the Bloch functions of the band electron. The hybridization matrix elements are defined as  $L_{m\sigma nk} = \langle m\sigma | V_{cf} | nk \rangle$ , where  $V_{cf}$  is a crystal field potential, which in the range of localization of the  $d$ -electrons can be expressed as [8]

$$V_{cf} = F(r) \left\{ Y_{40}(\vartheta, \varphi) + \sqrt{5/14} [Y_{44}(\vartheta, \varphi) + Y_{4-4}(\vartheta, \varphi)] \right\} \quad (3)$$

$Y_{lm}(\vartheta, \varphi)$  are the spherical harmonics and  $F(r)$  is the radial function.

We proved that even in the case of a nonorthogonal one-electron basis defining the occupation number representation, the one-particle Hamiltonian has only terms involving one creation and one annihilation operator. All terms proportional to matrix elements of the one-electron potential in [6] which involve two creation and two annihilation operators are incorrect. In fact they resulted from an arbitrary partial selection of terms in the one-electron Hamiltonian artificially expressed in the apparent form of a two particle operator. As a result of the same procedure the hybridization terms  $H_1$  were lost in [6]. We proved that to the first order in overlap integrals the nonorthogonality of the one-electron basis gives only some corrections to hybridization matrix elements without changing the form of the Hamiltonian. Thus, in the following we will neglect these corrections.

$H_1$  couples the states of  $N$   $d$ -electrons and 1 band electron with the states of  $N+1$  (or  $N-1$ )  $d$ -electrons and 0 (or 2) band electrons. To describe properly the

energy differences between the coupled many particle states one should complete  $H_0$  by the Coulomb interaction within the  $d$ -shell. The dominant effect of this interaction, i.e. the repulsion of opposite-spin electrons located on the same orbital, can be accounted for by the Anderson term [8]

$$(1/2) \sum_{m\sigma} U \hat{n}_{m\sigma} \hat{n}_{m-\sigma} \quad (4)$$

We eliminate  $H_1$  to the first order in  $L$  adapting the canonical transformation of [3] to the case of 5  $d$ -orbitals. To the second order in  $L$  the part of the transformed Hamiltonian, which depends on the state of  $d$ -electrons and is responsible for  $\Gamma_8 \rightarrow \Gamma_8$  transitions in the vicinity of the  $\Gamma$  point, has the form

$$H_2 = \sum_{m\sigma, \sigma', m_J, k, m'_J, k'} L_{m\sigma m_J \sigma} L_{m\sigma' m'_J \sigma} \left[ \frac{1}{\varepsilon_p - \varepsilon_d} - \frac{1}{\varepsilon_p - \varepsilon_d - U} \right] \times (\delta_{\sigma\sigma'} \hat{n}_{m-\sigma} + \delta_{-\sigma\sigma'} \hat{a}_{m\sigma}^+ \hat{a}_{m-\sigma}) \hat{a}_{m_J k}^+ \hat{a}_{m'_J k'}. \quad (5)$$

Here we passed to the Luttinger representation for the  $\Gamma_8$  states,  $\varepsilon_p$  is the energy of the  $\Gamma_8$  bands at  $k = 0$ .

To discuss  $sp$ - $d$  interaction it is in the most cases enough to consider only the subspace of the lowest states of the paramagnetic ion given by Hund's rules. For ions like  $\text{Mn}^{2+}$  and  $\text{Fe}^{3+}$  with the half-filled  $d$ -shell (i.e.  ${}^6S$  ground term) the Hund's subspace is spanned by the vectors

$$|0, S\rangle = C \hat{S}_-^{5/2-S} \prod_{m=-2}^2 \hat{a}_{m\uparrow}^+ |0\rangle, \quad S = \mp \frac{5}{2}, \mp \frac{3}{2}, \mp \frac{1}{2}; \quad (6)$$

where  $\hat{S}_- = \sum_{m=-2}^2 \hat{a}_{m\downarrow}^+ \hat{a}_{m\uparrow}$  is the spin lowering operator,  $|0\rangle$  is the vacuum state and  $C$  is the normalization constant depending on  $S$ .

We established that the Hamiltonian (5) leads to the Heisenberg-type  $p$ - $d$  interaction only for the  ${}^6S$  ions. The obtained negative effective exchange constant

$$J_{ex} = -(2/5) L_{2\uparrow, \frac{1}{2}\sigma}^2 \left[ \frac{1}{\varepsilon_p - \varepsilon_d} + \frac{1}{\varepsilon_d + U - \varepsilon_p} \right] \quad (7)$$

has a similar form to that proposed in [4] and [5] but the present formula gives a precise definition for the vaguely defined there hybridization matrix element  $V_{kd}$ .

We notice, that the first energy denominator in formula (7) represents the energy which has to be supplied when one electron is taken from the half-filled  $d$ -shell and put into the band. The second one is the energy required when transferring the band electron into the already half-filled  $d$ -shell. As the latter energy is particularly small for iron impurities in HgSe [10], one can easily conclude that the observed difference in magnitude of the exchange constants for  $\text{Mn}^{2+}$  and  $\text{Fe}^{3+}$  in HgSe can result from the changes in energy denominators in formula (7).

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