Lorentz Transformations in 1+1 Dimensional Spacetime: Mainly the Superluminal Case

B.S. Damski*

Jagiellonian University, Faculty of Physics, Astronomy and Applied Computer Science, Łojasiewicza 11, 30-348 Kraków, Poland

Received: 19.05.2025 & Accepted: 29.07.2025

Doi: 10.12693/APhysPolA.148.22 *e-mail: bogdan.s.damski@student.uj.edu.pl

We discuss the most general form of the Lorentz transformation in 1+1 dimensional spacetime, focusing mainly on its superluminal branch. For this purpose, we introduce the 2-velocity of a reference frame and the clockwork postulate. Basic special relativity effects are discussed in the proposed framework. Different forms of the superluminal Lorentz transformation, which were studied in the literature, are critically examined from the perspective of our formalism. Counterintuitive features of the superluminal Lorentz transformation are identified both in our approach and in earlier studies.

topics: special theory of relativity, superluminal Lorentz transformation, inertial reference frame

1. Introduction

The topic of superluminal particles, which are nowadays called tachyons (a term coined by Feinberg [1]), remains controversial among physicists for two reasons. The first one is that the existence of tachyons carrying information is commonly believed to lead to violations of causality [2]. The second one is that no experimental evidence has been found for their existence. On the theoretical side, different attempts to construct a legitimate theory were made. However, none of them was accepted by the whole community. On the experimental side, some physicists claimed that they registered tachyons, but either the result of their experiment was not repeatable or it was later shown that a mistake was made in the measurement process (see [3] for a review of such efforts).

The recent publication of Dragan and Ekert [4] has once again renewed interest in tachyon-related physics. In their work, they proposed that the theory of relativity and quantum mechanics are deeply connected. They argued that the violations of causality associated with tachyons explain the probabilistic nature of quantum effects. According to them, the presence of tachyons in the theory does not lead to causal paradoxes as in [2], but rather changes our notion of causality. Their novel ideas triggered a heated debate in the scientific community [5–11].

In traditional relativity, where only subluminal reference frames are considered, a tachyon is a particle that is never at rest. Thereby, various authors considered the concept of superluminal reference frames. Such studies date back at least to the work of Parker [12], where the Lorentz transformation has the following form

$$|u|<1: \begin{cases} t' = \gamma(u)\big(t - ux\big), \\ x' = \gamma(u)\big(x - ut\big), \end{cases}$$
$$|u|>1: \begin{cases} t' = -\operatorname{sgn}(u)\,\gamma(u)\big(t - ux\big), \\ x' = -\operatorname{sgn}(u)\,\gamma(u)\big(x - ut\big), \end{cases} \tag{1}$$

and the $-\operatorname{sgn}(u) \to \operatorname{sgn}(u)$ version of (1) is said to lead to the same physics ($\operatorname{sgn}(x) = 0, \pm 1$ is the sign function). According to these equations, the primed reference frame moves with the dimensionless velocity u relative to the unprimed one (the velocity is measured in the units of the speed of light) and $\gamma(u) = 1/|1-u^2|^{1/2}$. We say that the velocity u is subluminal for $|u| < 1$ and superluminal for $|u| > 1$.

Besides (1), there are other versions of the Lorentz transformation present in the literature, such as

$$u \in \mathbb{R} \setminus \{1,-1\}: \begin{cases} t' = \gamma(u)\big(t - ux\big), \\ x' = \gamma(u)\big(x - ut\big), \end{cases} \tag{2}$$

or its $\gamma(u) \to \operatorname{sgn}(1-u^2)\gamma(u)$ version (see, for example, [13, 14] or references cited in [15]). However, it can be shown that none of them is actually

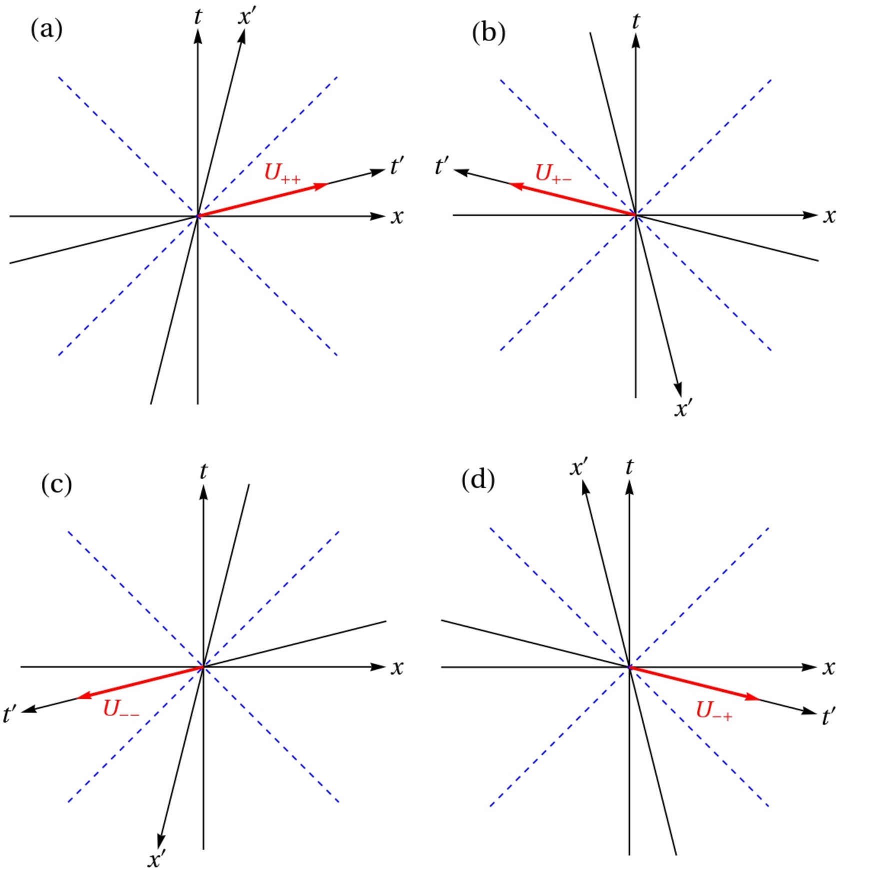

Fig. 1. (a–d) The schematic illustration of four possibilities for the orientation of the axes of the superluminal reference frames. The red arrows display 2-velocities $U_{\alpha\beta} = \big(\alpha\gamma(u), \beta\gamma(u)|u|\big)$, where $\alpha, \beta = \pm$ and $|u| > 1$. The light cones are plotted with blue dashed lines.

acceptable [15], which illustrates the fact that extending the standard Lorentz transformation to the superluminal regime involves some subtle issues.

In this work, we will introduce the 2-velocity of a reference frame and the clockwork postulate in Sect. 2, which will allow us to present superluminal Lorentz transformations from a different angle. We will examine in Sects. 3 and 4 the new formalism in the context of basic special relativity effects. The discussion of (1) and (2) from the perspective of our formalism will be presented in Sect. 5. The summary of our work will be given in Sect. 6.

2. The 2-velocity of the reference frame

When one thinks about the Lorentz transformation in $1+1$ dimensional spacetime, one considers two reference frames moving relative to each other with some velocity, say u. Then, one uses such a velocity to parametrize the coefficients in the transformation relating the spacetime coordinates in the two reference frames. Such coefficients take the form

$$\pm\gamma(u), \qquad \pm\gamma(u)|u|, \tag{3}$$

where the signs in both expressions are chosen independently. While in principle there is nothing wrong with such a procedure, we find it hardly satisfactory for the following reason. Namely, the special theory of relativity is about physics happening in spacetime, where time-like and space-like features appear on *equal footing* in different contexts. Thereby, it is our opinion that it would be more natural to use the 2-velocity to characterize the relation between the spacetime coordinates in the two reference frames. Such an observation also naturally follows from (3), which suggests the consideration of the 2-vector

$$U = \big(\pm\gamma(u), \pm\gamma(u)\,|u|\big), \tag{4}$$

which is reminiscent of the 2-velocity of a relativistic particle. The basic property of (4) is that

$$U{\cdot}U \equiv (U^0)^2 - (U^1)^2 = \operatorname{sgn}(1-u^2) = \begin{cases} +1 & \text{for } u \text{ subluminal,} \\ -1 & \text{for } u \text{ superluminal,} \end{cases} \tag{5}$$

regardless of the choice of signs in (4) (all dot products are defined in this work with the metric tensor $\operatorname{diag}(1,-1)$; the signs in (4) are chosen independently so that there are four expressions encoded in such an equation).

We propose the following parametrization of the Lorentz transformation

$$t' = U{\cdot}U\left(U^0 t - U^1 x\right),$$
$$x' = U{\cdot}U\left(U^0 x - U^1 t\right), \tag{6}$$

which leads to the following inverse transformation after the employment of (5)

$$t = U^0 t' + U^1 x',$$
$$x = U^0 x' + U^1 t'. \tag{7}$$

To give physical meaning to U, we remark that the orientation of the t' axis on the spacetime diagram (t, x) is given by U. Such an observation follows from the $x' = 0$ version of (7). The four superluminal options resulting from different sign choices in (4) are illustrated in Fig. 1.

Clockwork postulate. In order to add meaning to the four possible orientations of the t' axis, we introduce the *clockwork postulate* according to which *the proper time of an inertial observer always increases.* In other words, an observer always becomes older (never younger) in his own rest frame, which nicely fits the basic assumption of special relativity that all inertial observers and/or reference frames are equivalent. This postulate implies that the stationary observer in the primed reference frame always moves in the positive direction of the t' axis. The above-stated clockwork postulate should not be confused with the clock postulate (also known as the clock hypothesis), which is based on the assumption that the rate of operation of the ideal clock in motion depends only on its instantaneous velocity (see [16] for a recent critical discussion of the clock postulate).

The remark formulated below (7), along with the clockwork postulate, leads to the conclusion that U is the relativistic 2-velocity of the primed reference frame relative to the unprimed one. Thereby, the four options displayed in Fig. 1 come from the fact that the primed reference frame can move either forwards ($U^0 > 0$) or backwards ($U^0 < 0$) in time t and either in the positive ($U^1 > 0$) or in the negative ($U^1 < 0$) direction of the x axis. In other words, all spacetime options for the motion of the reference frame are encoded in U. Finally, we note that reference frames moving either forwards or backwards in time and in different directions in space were considered in 1+1 (1+3) dimensional spacetime in the paper of Viera [17] (Sutherland and Shepanski [18]). However, the above-introduced U-parametrization of these transformations was not explored in [17, 18].

3. Properties of the U-parametrized transformation

We begin the discussion here with the derivation of the addition law for 2-velocities. We consider three inertial reference frames O, O', and O''. We assume that O' is moving with the 2-velocity V relative to O, O'' is moving with the 2-velocity U relative to O and U' relative to O'. To determine U', we note that such a 2-velocity is defined via

$$\begin{pmatrix} t' \\ x' \end{pmatrix} = \begin{pmatrix} U'^0 & U'^1 \\ U'^1 & U'^0 \end{pmatrix} \begin{pmatrix} t'' \\ x'' \end{pmatrix} \tag{8}$$

that can be compared to

$$\begin{pmatrix} t' \\ x' \end{pmatrix} = V{\cdot}V \begin{pmatrix} V^0U^0 - V^1U^1 & V^0U^1 - V^1U^0 \\ V^0U^1 - V^1U^0 & V^0U^0 - V^1U^1 \end{pmatrix} \begin{pmatrix} t'' \\ x'' \end{pmatrix}, \tag{9}$$

which is obtained via the transformation $O' \to O$ followed by the $O \to O''$ transformation.[†1] This leads to the identification

$$U' = V{\cdot}V\left(V{\cdot}U, \varepsilon_{\mu\nu} V^\mu U^\nu\right), \tag{10}$$

where $\varepsilon_{\mu\nu} = -\varepsilon_{\nu\mu}$, $\varepsilon_{01} = +1$.

Alternatively, one may obtain (10) by the Lorentz transformation of the 2-velocity U. This is achieved by the following replacements imposed on (6): $U \to V$ followed by $(t, x) \to (U^0, U^1)$ and $(t', x') \to (U'^0, U'^1)$. Moving on, we note that (10) results in

$$U'{\cdot}U' = (V{\cdot}V)(U{\cdot}U) = \begin{cases} +1 & \text{for } U \text{ and } V \text{ subluminal}, \\ +1 & \text{for } U \text{ and } V \text{ superluminal}, \\ -1 & \text{otherwise}. \end{cases} \tag{11}$$

Thereby, the relative velocity of two reference frames is always superluminal when, relative to some other reference frame, one of them is superluminal, the other one is subluminal. Otherwise, the discussed relative velocity is subluminal, which

[†1] By the transformation $O' \to O$, we understand here the transformation (6) subjected to the replacement $U \to V$.

we find interesting (obvious) when both reference frames are superluminal (subluminal) with respect to some other reference frame.

Equation (10) can be cast into the following, more familiar form. With every 2-vector U, we associate the velocity u via

$$u = \frac{U^1}{U^0}. \tag{12}$$

Such a definition follows from the $x' = 0$ version of (7) and leads to the following U-parametrization $U = \big(U^0, U^0 u\big) = \pm\big(\gamma(u), \gamma(u)u\big)$. Combining (10) and (12), we arrive at

$$u' = \frac{V^0U^1 - V^1U^0}{V^0U^0 - V^1U^1}, \tag{13}$$

which is antisymmetric with respect to the $U \leftrightarrow V$ transformation in accordance with standard expectations. We find it curious that (10) does not exhibit such a symmetry. We note that whenever $V \sim (1, v)$ and $U \sim (1, u)$, (13) leads to the standard formula (see, e.g., [19, 20])

$$u' = \frac{u - v}{1 - uv}. \tag{14}$$

Keeping in mind that $V^0 = \operatorname{sgn}(V^0)\gamma(v)$, $V^1 = \operatorname{sgn}(V^0)\gamma(v)v$, etc., we arrive at another representation of (10)

$$V{\cdot}U \neq 0: \quad U' = V{\cdot}V \operatorname{sgn}(V{\cdot}U)\big(\gamma(u'), \gamma(u')u'\big), \tag{15}$$

$$V{\cdot}U = 0: \quad U' = \big(0, \operatorname{sgn}(V^0U^1)\big). \tag{16}$$

Two remarks are in order now.

Firstly, we note that $V{\cdot}U = 0$ is satisfied by $V = \pm(U^1, U^0)$ or equivalently $U = \pm(V^1, V^0)$, which can be easily visualized on spacetime diagrams such as the ones depicted in Fig. 1. In the traditional nomenclature, $V{\cdot}U = 0$ amounts to the well-known condition $uv = 1$ for $|u'| = \infty$ (see, e.g., [19, 20]), where u and v are defined as in (12). We would like to stress that the condition $V{\cdot}U = 0$, unlike $uv = 1$, has a clear geometrical meaning in Minkowski spacetime. We also note that the right-hand side of (16) implies $|u'| = \infty$.

Secondly, (15) can be used to argue that reference frames propagating backwards in time inevitably appear in our formalism when superluminal velocities are considered, which is counterintuitive. Indeed, taking $V{=}\big(\gamma(v), \gamma(v)v\big)$ and $U{=}\big(\gamma(u), \gamma(u)u\big)$, where $V{\cdot}U \neq 0$ and both 2-velocities describe reference frames moving forwards in time t, we see from (15) that U' describes propagation backwards in time t' when[†2]

$$\operatorname{sgn}(1{-}v^2)\operatorname{sgn}(1{-}uv) = -1, \tag{17}$$

which can be satisfied when at least one of the velocities is superluminal.

[†2] Strictly speaking, from the perspective of the reference frame O', the reference frame O'' propagates backwards in time t' when (17) holds.

Finally, we mention that the transformation

$$(t, x) \;\leftrightarrow\; (t', x') \tag{18}$$

is induced by

$$(U^0, U^1) \to U{\cdot}U\,\big(U^0, -U^1\big), \tag{19}$$

which can be seen as velocity reciprocity in our formalism (see [21] for the recent comprehensive discussion of velocity reciprocity in relativity theories). For subluminal U, (19) reduces to $\big(U^0, U^1\big) \to \big(U^0, -U^1\big)$; then, the spatial component of the 2-velocity gets flipped. For superluminal U, (19) reads $\big(U^0, U^1\big) \to \big(-U^0, U^1\big)$; then, the temporal component of the 2-velocity gets flipped. We mention in passing that (18) is enforced by the flip of the velocity, i.e., $u \to -u$, in subluminal and superluminal transformations (1). This is not the case in our formalism in the superluminal case.

4. Length "contraction" and time "dilation"

We discuss here further properties of transformation (6) and its inverse (7), i.e., we again assume that the primed reference frame moves with the 2-velocity U with respect to the unprimed reference frame.

Length "contraction". Measuring the length of a rod requires an observer, in his own reference frame, to simultaneously determine the spatial positions of the rod's endpoints. We consider the rod whose endpoints are located at $A = (t'_A, x'_A)$ and $B = (t'_B, x'_B)$ in the primed reference frame and at $C = (t_C, x_C)$ and $D = (t_D, x_D)$ in the unprimed reference frame, respectively. We assume that the rod is at rest in the primed reference frame and so its proper length ℓ' is given by $|x'_A - x'_B|$ even when the events A and B are not simultaneous in the primed reference frame. The rod's length in the unprimed reference frame is $\ell = |x_C - x_D|$ as long as $t_C = t_D$. It follows from (6) that the spatial coordinates of the events C and D, as observed in the primed reference frame, are

$$x'_C = U{\cdot}U\,\big(U^0x_C - U^1t_C\big),$$
$$x'_D = U{\cdot}U\,\big(U^0x_D - U^1t_D\big). \tag{20}$$

Subtracting these two equations from each other, and keeping in mind that $x'_C = x'_A$, $x'_D = x'_B$, and $t_C = t_D$, we obtain

$$\ell = \frac{\ell'}{\big|(U{\cdot}U)U^0\big|} = \frac{\ell'}{\gamma(u)} \tag{21}$$

via (4). These considerations are illustrated in Fig. 2, where we set $t'_A = t'_B$ and consider the primed reference frame moving either forwards or backwards in time with a superluminal velocity.

Time "dilation". We consider a clock at rest in the primed reference frame. This clock measures the time interval $\Delta t'_{AB} = t'_B - t'_A$ between events $A = (t'_A, x'_A)$ and $B = (t'_B, x'_B)$ occurring at $x'_A = x'_B$. As measured by the clock resting in the unprimed

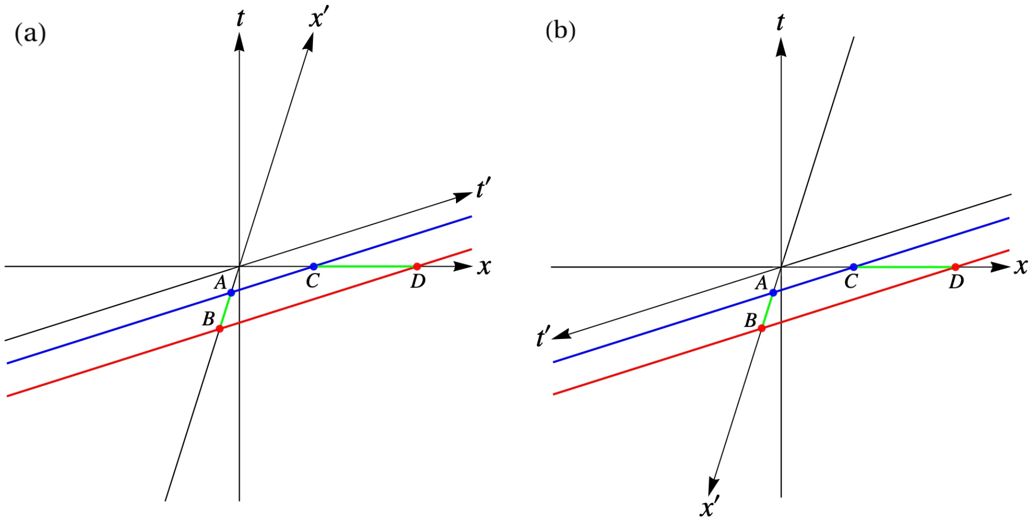

Fig. 2. The schematic plot illustrating the discussion of length "contraction" (see Sect. 4 for the definition of the events A, B, C, and D). The rod is depicted via the thick green line. Its endpoints are marked by blue and red dots (worldlines of the rod's ends are shown in the same colors). While panels (a) and (b) are prepared for the same $U^1/U^0 = u > 1$, U^0 is larger (smaller) than zero in panel (a) (panel (b)). The rod is simultaneously at rest in both primed reference frames. Its appearance in the unprimed reference frame does not depend on whether the primed reference frame moves forwards or backwards in time.

reference frame, the time interval separating these events is

$$\Delta t_{AB} = t_B - t_A = U^0 \Delta t'_{AB} = \mathrm{sgn}\,(U^0)\gamma(u)\Delta t'_{AB} \tag{22}$$

according to (7) and (4). It is clear from (22) that the sequence of events A and B is swapped when $\mathrm{sgn}\,(U^0) = -1$, i.e., when the primed reference frame is moving backwards in time t. Such a conclusion can be easily visualized with the help of the diagrams in Fig. 1 (one may assume for simplicity that the events in the primed reference frame take place on the t' axis).

Finally, we note that for $|u|>\sqrt{2}$, $\ell>\ell'$ (contraction is not always seen) and $|\Delta t_{AB}|<|\Delta t'_{AB}|$ (dilation is not always seen). These remarks explain our use of quotation marks around the words contraction and dilation. We remark that the special role of $u=\sqrt{2}$ in the context of length "contraction" and time "dilation" was noted in [22].

5. Restricted transformations

We discuss here transformations restricted to only two (out of four possible) spacetime orientations of the 2-velocity U. On the one hand, this will give us another opportunity to illustrate how our U-parametrization works in practice. On the other hand, it will allow us to discuss from a different angle the superluminal Lorentz transformations that were studied in the literature.

We begin the discussion of such transformations from (2), which looks like a natural extension of the standard Lorentz transformation to superluminal velocities. In our U-parametrization, (2) reads

$$u \in \mathbb{R} \setminus \{1,-1\}: \; U = \mathrm{sgn}\,(1-u^2)\,\big(\gamma(u), \gamma(u)u\big). \tag{23}$$

Transformations having such a structure, however, do not form a group, which can be shown with the help of (15). Namely, we choose u and v such that $u\,v \neq 1$, which allows us to put (23) and $V = \mathrm{sgn}\,(1-v^2)\,\big(\gamma(v),\gamma(v)v\big)$ into (15). This leads to

$$U' = \mathrm{sgn}\,(1-u^2)\,\mathrm{sgn}\,(1-uv)\,\big(\gamma(u'),\gamma(u')u'\big) = \mathrm{sgn}\,(1-v^2)\,\mathrm{sgn}\,(1-uv)\,\mathrm{sgn}\,(1-u'^2)\,\big(\gamma(u'),\gamma(u')u'\big), \tag{24}$$

where the last equality follows from

$$\mathrm{sgn}\,(1-u'^2) = \mathrm{sgn}\,(1-v^2)\,\mathrm{sgn}\,(1-u^2) \tag{25}$$

obtained from (14). The result (24) does not agree with the $u \to u'$ version of (23) when (17) holds. If both u and v are subluminal, then (17) cannot be satisfied, which is expected because we deal in such a case with the standard Lorentz transformation. However, when at least one of these velocities is superluminal, then the other can always be chosen so as to satisfy (17). The very same problem appears when one uses the $\gamma(u) \to \mathrm{sgn}\,(1-u^2)\gamma(u)$ version of (2), where

$$u \in \mathbb{R} \setminus \{1,-1\}: \quad U = \big(\gamma(u), \gamma(u)u\big). \tag{26}$$

Thereby, (2) and its $\gamma(u) \to \mathrm{sgn}\,(1-u^2)\gamma(u)$ version break the group property that any Lorentz transformation should satisfy, which was noted in [15]. In our formalism, the clockwork principle leads to the conclusion that the superluminal (23) [(26)] describes reference frames moving only backwards [forwards] in time t and either in the positive or negative direction of the x axis; this is evident from the fact that the superluminal U representing (23) [(26)] is depicted in Fig. 1c and d [Fig. 1a and b)]. This observation shows that the restriction to reference frames, which according to our formalism propagate in a fixed direction in time, is insufficient when superluminal velocities are considered.

Then, we take a close look at the transformation (1) employed in [4, 12, 15, 22]. Transformations having such a structure form a group (see, e.g., [4] for a recent insight into this issue, as well as [15]). This is seen in our formalism as follows. The U-parametrization of (1) is

$$|u|<1: \quad U = \big(\gamma(u), \gamma(u)u\big),$$
$$|u|>1: \quad U = \big(\mathrm{sgn}\,(u)\gamma(u), \gamma(u)|u|\big), \tag{27}$$

which can be equivalently written as

$$u \in \mathbb{R}\setminus\{1,-1\}: \quad U = \mathrm{sgn}\,(1{+}u)\,\big(\gamma(u), \gamma(u)u\big) \tag{28}$$

thanks to the compact representation of (1) proposed in [6]. We choose u and v such that $uv \neq 1$, and substitute (28) and $V = \mathrm{sgn}\,(1{+}v)\,\big(\gamma(v), \gamma(v)v\big)$ into (15), getting

$$U' = \mathrm{sgn}\,(1{-}v)\,\mathrm{sgn}\,(1{+}u)\,\mathrm{sgn}\,(1{-}uv)$$
$$\times\big(\gamma(u'), \gamma(u')u'\big) = \mathrm{sgn}\,(1{+}u')\,\big(\gamma(u'), \gamma(u')u'\big), \tag{29}$$

where the last equality is obtained from (14). The result (29) proves that U' is given by the $u \to u'$ version of (28) when $uv \neq 1$. For $uv = 1$, one may easily arrive at the same conclusion by using (16) instead of (15). Moreover, following the same line of reasoning, one may show that the $-\mathrm{sgn}\,(u) \to \mathrm{sgn}\,(u)$ version of (1), which is U-parametrized as

$$u \in \mathbb{R}\setminus\{1,-1\}: \quad U = \mathrm{sgn}\,(1{-}u)\,\big(\gamma(u), \gamma(u)u\big) =$$
$$\begin{cases} |u|<1: \; U = \big(\gamma(u), \gamma(u)u\big), \\ |u|>1: \; U = -\big(\mathrm{sgn}\,(u)\gamma(u), \gamma(u)|u|\big), \end{cases} \tag{30}$$

also satisfies the group property. Proceeding as above, we note that the clockwork principle leads to the observation that the superluminal (28) [(30)] describes reference frames moving in the positive [negative] direction of the x axis and either forwards or backwards in time t; U corresponding to the superluminal (28) [(30)] is shown in Fig. 1a and d [Fig. 1b and c].

To fix such problematic one-directional movement in space, one is forced to use the reinterpretation principle, which states that motion backwards in time t in the positive (negative) direction of the x axis represents motion forwards in time t in the negative (positive) direction of the x axis [23]. Therefore, the reinterpretation principle allows one to work with only two orientations of spacetime axes, as in Fig. 1a and d or Fig. 1b and c (instead of four, as in Figs. 1a–d). However, one should be aware of the fact that by choosing (1) as in [4, 12, 15, 22], one deals with the situation, where the superluminal observer moving in the positive (negative) direction of the x axis uses a clock with a normal (inverted) mechanism — the hands of the clock are moving clockwise (counterclockwise) in these two cases, so to speak. Similarly, by using (1) subjected to the $-\mathrm{sgn}\,(u) \to \mathrm{sgn}\,(u)$ replacement, one assumes that the superluminal observer uses a clock with a normal (inverted) mechanism when it moves in the negative (positive) direction of the x axis. We see such dependence of the clock's mechanism on the direction of motion as a counterintuitive feature.

6. Conclusions

In the spirit of the theory of relativity, we have considered reference frames moving in all possible directions in space and time. In particular, this implies that we have taken into account reference frames moving both forwards and backwards in time. While the exploration of both options does not seem to be necessary in the subluminal context, the situation is far less clear in the superluminal context, where one is bound to encounter counterintuitive features and a lack of experimental data leaves various possibilities open. We hope that our work will stimulate discussion on this issue.

In our formalism, we have parametrized the Lorentz transformations via the 2-velocity of a reference frame and introduced the clockwork principle to give physical meaning to this quantity. We have then studied the group property of such transformations and re-examined basic effects such as length “contraction” and time “dilation”. The new formalism has then been compared to the standard approach discussed in [4, 12, 15, 22]. In the course of these studies, we have identified counterintuitive features both in our formalism and in the above-mentioned standard approach (see the discussion around (17) and at the end of Sect. 5). In fact, we believe that any approach to superluminal systems is going to encounter some conceptual difficulties that require detailed discussion. Such a remark partly motivates the research pursuits discussed in this work, which gives a non-standard perspective on superluminal physics. Finally, we note that it is of interest to extend the formalism presented in this work to higher-dimensional spacetimes, particularly in light of the recent developments presented in [24].

References

[1] G. Feinberg, *Phys. Rev.* **159**, 1089 (1967).

[2] R.C. Tolman, *The Theory of Relativity of Motion*, University of California Press, 1917.

[3] R. Ehrlich, *Symmetry* **14**, 1198 (2022).

[4] A. Dragan, A. Ekert, *New J. Phys.* **22**, 033038 (2020).

[5] A. Grudka, A. Wójcik, *New J. Phys.* **24**, 098001 (2022).

[6] A. Grudka, J. Stempin, J. Wójcik, A. Wójcik, *Phys. Lett. A* **487**, 129127 (2023).

[7] R. Horodecki, *New J. Phys.* **25**, 128001 (2023).

[8] F.D. Santo, S. Horvat, *New J. Phys.* **24**, 128001 (2022).

[9] A. Dragan, A. Ekert, *New J. Phys.* **24**, 098002 (2022).

[10] A. Dragan, A. Ekert, *New J. Phys.* **25**, 128002 (2023).

[11] A. Dragan, A. Ekert, *New J. Phys.* **25**, 088002 (2023).

[12] L. Parker, *Phys. Rev.* **188**, 2287 (1969).

[13] R. Goldoni, *Nuovo Cim. A* **14**, 501 (1973).

[14] R.N. Henriksen, *How might tachyons appear?*, arXiv:2004.04045, 2020.

[15] L. Marchildon, A.F. Antippa, A.E. Everett, *Can. J. Phys.* **61**, 256 (1983).

[16] K. Lorek, J. Louko, A. Dragan, *Class. Quantum Gravity* **32**, 175003 (2015).

[17] R.S. Vieira, *Rev. Bras. Ensino Fís.* **34**, 3306 (2012).

[18] R.I. Sutherland, J.R. Shepanski, *Phys. Rev. D* **33**, 2896 (1986).

[19] A. Dragan, *Unusually Special Relativity*, World Scientific, 2022.

[20] L.M. Sokołowski, *Elementy szczególnej teorii względności*, Wydawnictwo Uniwersytetu Warszawskiego, 2023.

[21] P. Moylan, *Am. J. Phys.* **90**, 126 (2022).

[22] A. Dragan, *Why devil plays dice?*, arXiv:0806.4875, 2008.

[23] O.M.P. Bilaniuk, V.K. Deshpande, E.C.G. Sudarshan, *Am. J. Phys.* **30**, 718 (1962).

[24] A. Dragan, K. Dębski, S. Charzyński, K. Turzyński, A. Ekert, *Class. Quantum Grav.* **40**, 025013 (2022).