The Influence of Angular Frequencies on Self-Focusing of the Laser Beam in Magnetized Electron–Positron Plasma

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Recent advancements in ultra-short intense pulse technology have made it possible to generate electron–positron plasma in a laboratory. When intense lasers interact with this plasma, various non-linear effects occur, such as self-focusing. This research aimed to investigate how self-focusing is impacted by the primary frequencies of lasers and plasmas. The study developed a specific model that describes the non-linear relationship between self-focusing and angular frequencies of the laser and plasma. Plasma is characterized by the Langmuir angular frequency of all components and the electron cyclotron frequency. The presented results indicate that without an external magnetic field, the beam focusing at different frequencies is similar for both right-handed and left-handed polarizations. However, in the presence of an external magnetic field, a significant difference in the behaviour of right-handed and left-handed polarization is observed. The model also predicts the precise non-linear correlation between the beam width and the angular frequencies of the lasers and plasma.

topics: electron–positron plasma, self-focusing, intense laser, diffraction

1. Introduction

Electron–positron plasma is a large class of paired plasmas of the same mass and charge (with the opposite sign) that exist in the early structure of the universe, pulsars, galactic nuclei, and gamma-ray decay. Experiments have shown that it is possible to create electron–positron plasma in laboratories and fusion devices. Moreover, numerous studies have investigated the creation of electron–positron–ion plasma in laboratories, as well as in gamma-ray generator meteors and solar radiation [1, 2]. The direct production of electron–positron pairs in the collision of two photons is one of the basic processes. Although this process has not been observed in the laboratory due to the lack of intense sources of gamma rays, the emission of synchrotron sources caused by lasers has allowed this process to be observed for the first time. Pair generation during the multi-photon process has been observed in the laboratory at the Stanford Linear Accelerator Center (SLAC), in the collision of a high-energy electron beam with a laser pulse with the power of 10^{12} W/cm^2 [3].

The SLAC experiment was carried out by colliding a beam of electrons with an energy of 46.6 GeV with a high-intensity laser pulse. Interaction of relativistic electrons with heavy nuclei (such as gold) also provides the possibility of producing an electron–positron pairs. Electron–positron plasma is the only type of plasma that makes it possible to study the physics of plasmas with equal masses, and it is currently made in a laboratory. In 2015, the first experimental evidence from the generation of neutral and dense electron–positron plasma was reported in the laboratory [3, 4]. Electromagnetic waves demonstrate notable distinctions in the propagation characteristics when travelling through electron–ion plasma compared to electron–positron–ion plasma. Rizato [5] investigated the propagation of circularly polarized electromagnetic waves in a cold plasma, including electrons and positrons with stationary ions, and concluded that this three-component plasma, which is used to accelerate particles, can amplify electromagnetic waves [5]. Self-focusing is one of the most important nonlinear effects that occur when electromagnetic radiation passes through the plasma.

The self-focusing phenomenon allows the beam to travel several Rayleigh lengths and have a smaller spot diameter compared to its initial size. This phenomenon in electron–ion plasma has been the subject of many researches [6–13]. The amount of focusing and the impact of laser and plasma parameters on the beam-width reduction have also been investigated in many studies [14–19]. The effect of beam polarization on the self-focusing of a laser beam in magnetized electron–ion (e-i) plasma has also been studied by Sepehri-Javan et al. [20]. In another research by Sepehri-Javan et al. [21], the changes in the spot diameter of the ion–electron–positron (i-e-p) plasma were studied using a two-fluid model. The results of these research indicate that for the right-handed polarization, the effect of the external magnetic field and electron temperature on the self-focusing beam is greater than for the left-handed polarization. However, due to the complexity and non-linearity of the dependence of self-focusing on the initial laser and plasma frequencies, the impact of these parameters on the self-focusing has not been studied so far. In this study, the effect of the plasma and laser frequencies on the beam-width reduction is investigated in the presence of an external magnetic field, considering both the right- and left-handed polarization.

2. Focusing parameter

The interaction of a high-intensity laser with a three-component hot plasma, including an electron–positron–ion (e-p-i) component, is considered. It is assumed that the laser beam is propagated in the $e_{\hat{z}}$ direction and the beam has circular polarization. The intended plasma is a combination of the primary electron–ion (e-i) plasma and electron–positron (e-p) plasma. The individual symbols "i", "e_1", "e_2", and "p" refer to ions, electron–positron, electron–ion, and positrons, respectively. Experimental research showed that the percentage of positrons in almost all laboratory generated e-p-i plasma is low — it is below 10% [22–27], and electrons belonging to both e-i and e-p plasmas may have different temperatures.

If the external magnetic field is applied in the z direction as $\boldsymbol{B} = B_0\,\hat{e}_z$, assuming that the propagation of a circularly polarized wave is along the external magnetic field with a slowly changing amplitude and ignoring the translational velocity of heavy ions, the changes in electron density will be as follows [21]

$$n_j = n_{0j} \exp\left(-\frac{1}{2}\left|\boldsymbol{A}\right|^2 Q_j\right); \quad j = \mathrm{e}_1, \mathrm{e}_2, \mathrm{p}, \mathrm{i}. \quad (1)$$

Here, $\boldsymbol{A}$ is the vector potential of wave and Q_{e1}, Q_{e2}, Q_{i}, Q_{p} are defined as follows

$$Q_{\mathrm{e1}} = \frac{\frac{-\zeta_2\beta_{\mathrm{e2}}}{1-\sigma\alpha} + \frac{\zeta_{\mathrm{p}}\beta_{\mathrm{p}}}{1+\sigma\alpha} + \frac{\beta_{\mathrm{e1}}}{1-\sigma\alpha}\left(\frac{\zeta_2}{\delta_2} + \frac{\zeta_{\mathrm{i}}}{\delta_{\mathrm{i}}} + \frac{\zeta_{\mathrm{p}}}{\delta_{\mathrm{p}}}\right)}{1 + \frac{\zeta_2}{\delta_2} + \frac{\zeta_{\mathrm{i}}}{\delta_{\mathrm{i}}} + \frac{\zeta_{\mathrm{p}}}{\delta_{\mathrm{p}}}},$$

$$Q_{\mathrm{e2}} = \frac{-\frac{\beta_{\mathrm{e1}}}{1-\sigma\alpha} + \frac{\zeta_{\mathrm{p}}\beta_{\mathrm{p}}}{1+\sigma\alpha} + \frac{\beta_{\mathrm{e2}}}{1-\sigma\alpha}\left(\delta_2 + \zeta_{\mathrm{p}}\frac{\delta_2}{\delta_{\mathrm{p}}} + \zeta_{\mathrm{i}}\frac{\delta_2}{\delta_{\mathrm{i}}}\right)}{\delta_2 + \zeta_{\mathrm{p}}\frac{\delta_2}{\delta_{\mathrm{p}}} + \zeta_{\mathrm{i}}\frac{\delta_2}{\delta_{\mathrm{i}}} + \zeta_2},$$

$$Q_{\mathrm{i}} = \frac{\frac{\beta_{\mathrm{e1}}}{1-\sigma\alpha} + \frac{\zeta_2\beta_{\mathrm{e2}}}{1-\sigma\alpha} - \frac{\zeta_{\mathrm{p}}\beta_{\mathrm{p}}}{1+\sigma\alpha}}{\delta_{\mathrm{i}} + \zeta_2\frac{\delta_{\mathrm{i}}}{\delta_2} + \zeta_{\mathrm{p}}\frac{\delta_{\mathrm{i}}}{\delta_{\mathrm{p}}} + \zeta_{\mathrm{i}}},$$

$$Q_{\mathrm{p}} = \frac{\frac{\beta_{\mathrm{e1}}}{1-\sigma\alpha} + \frac{\zeta_2\beta_{\mathrm{e2}}}{1-\sigma\alpha} + \frac{\beta_{\mathrm{p}}}{1+\sigma\alpha}\left(\delta_{\mathrm{p}} + \zeta_2\frac{\delta_{\mathrm{p}}}{\delta_2} + \zeta_{\mathrm{i}}\frac{\delta_{\mathrm{p}}}{\delta_{\mathrm{i}}}\right)}{\delta_{\mathrm{p}} + \zeta_2\frac{\delta_{\mathrm{p}}}{\delta_{\mathrm{p}}} + \zeta_{\mathrm{i}}\frac{\delta_{\mathrm{p}}}{\delta_{\mathrm{i}}} + \zeta_{\mathrm{p}}}, \quad (2)$$

and

$$\frac{n_{0\mathrm{i}}}{n_{0\mathrm{e1}}} = \zeta_{\mathrm{i}}, \quad \frac{n_{0\mathrm{p}}}{n_{0\mathrm{e1}}} = \zeta_{\mathrm{p}}, \quad \frac{n_{0\mathrm{e2}}}{n_{0\mathrm{e1}}} = \zeta_2,$$
$$\frac{T_{\mathrm{e2}}}{T_{\mathrm{e1}}} = \delta_2, \quad \frac{T_{\mathrm{p}}}{T_{\mathrm{e1}}} = \delta_{\mathrm{p}}, \quad \frac{T_{\mathrm{i}}}{T_{\mathrm{e1}}} = \delta_{\mathrm{i}}. \quad (3)$$

In fact, β_j and α are also defined as $\beta_j = c^2/(v_{Tj})^2$ and $\alpha = \omega_c/\omega_0$, where $v_{Tj} = \sqrt{k_{\mathrm{B}}T_j/m_0}$ is the thermal velocity of the particle of type j, k_{B} is the Boltzmann constant, m_0 is the electron mass, T_j is the temperature of the particle of type j. Here, $\omega_c = \pm\left|\frac{eB_0}{m_0 c}\right|$ is the cyclotron frequency of electron (positron), where the sign of ω_c is negative for electron and positive for positron. In (2), $\sigma = +1$ indicates a wave with a right-hand polarization, and $\sigma = -1$ with a left-hand polarization. Considering the wave equations and the relativistic fluid model, the equation for the variation of the spot size of laser in the quasi-neutral approximation can be obtained as follows [21]

$$\frac{\partial^2 f}{\partial \xi^2} + \left[\frac{1}{f^3}\left(\frac{\omega_{\mathrm{p}}^2 a_0^2\, r_0^2}{8c^2}N - 1\right)\right] = 0. \quad (4)$$

In (4), a_0 is the amplitude of the dimensionless vector potential, f is the beam-width diameter (the ratio of the diameter of the spot during propagation with respect to the initial diameter of the spot at $z = 0$), $\xi = \frac{Z}{Z_R}$ is the normalized propagation length, Z_R is the Rayleigh length, and N is as follows [21]

$$N = \frac{\omega_{\mathrm{p.e1}}^2}{\omega_{\mathrm{p}}^2}\left(\frac{1}{2(1-\sigma\alpha)^4} + \frac{Q_{\mathrm{e1}}}{2(1-\sigma\alpha)}\right) + \frac{\omega_{\mathrm{p.e2}}^2}{\omega_{\mathrm{p}}^2}\left(\frac{1}{2(1-\sigma\alpha)^4} + \frac{Q_{\mathrm{e2}}}{2(1-\sigma\alpha)}\right) + \frac{\omega_{\mathrm{p.p}}^2}{\omega_{\mathrm{p}}^2}\left(\frac{1}{2(1+\sigma\alpha)^4} + \frac{Q_{\mathrm{p}}}{2(1+\sigma\alpha)}\right). \quad (5)$$

In (5), $\omega_{\mathrm{p}.j} = 4\pi n_{0j}e^2/m_0$ is the Langmuir frequency of j-th component, $\omega_{\mathrm{p}} = (\omega_{\mathrm{p.e1}}^2 + \omega_{\mathrm{p.e2}}^2 + \omega_{\mathrm{p.p}}^2)^{1/2}$ is the total Langmuir frequency.

In order to solve (4), the initial boundary conditions (at $z = 0$) are considered as $\frac{\mathrm{d}f}{\mathrm{d}z} = 0$ and $f = 1$, which are the acceptable conditions for the initial flat wave front. Here, f represents the ratio of the spot size of laser along the propagation to its initial spot size.

The term within the innermost brackets in (4) is due to diffraction and focusing effects, and when the diffraction effect is dominant, this term will be negative. However, if the focusing phenomena are dominant, the term in the bracket becomes positive. If the focusing and diffraction effects are equal, the term becomes zero causing the spot size to remain unchanged during propagation.

Equation (4) is a second order differential equation and has the following form

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \alpha y^n = 0. \quad (6)$$

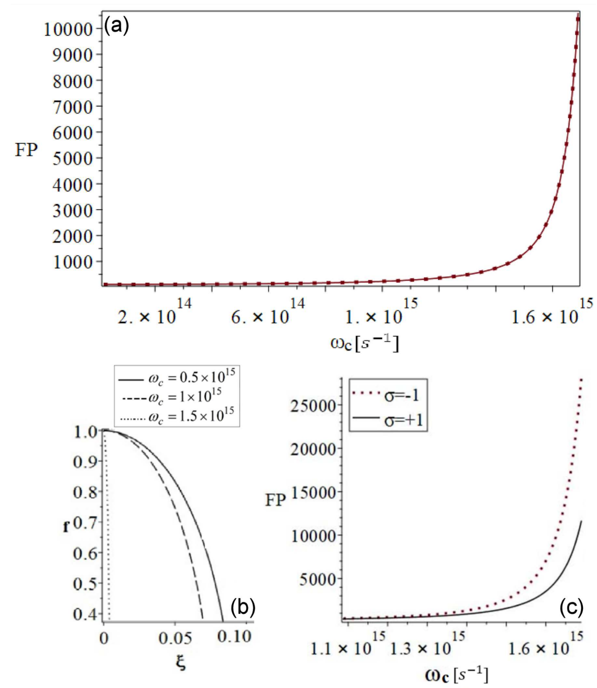

Fig. 1. (a) Plot of FP versus cyclotron frequency. (b) Plot of beam-width parameter versus propagation length for different cyclotron frequencies. (c) FP versus ω_c for $\omega_{\text{p.p}} = 0.25\omega_0$.

In this type of equation with α as a constant and n as any positive or negative number, when the initial condition is $y' = 0$ and $y = y_0$, then: if α is positive, the value of y increases beyond the initial value of y_0, while if α is negative, y will be less than y_0. The amount of reduction or increase of y relative to the initial value is also related to the value of α. For example, if (6) is regarded as $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \alpha y^{-3} = 0$ and the initial condition is taken as $y_0 = 2$, solving the equation for positive α shows that the value of y is less than the initial value of $y_0 = 2$, and for negative α the value of y is larger than the initial value of $y_0 = 2$ for all y. This means that when $\alpha = +5$, the value of y is greater than when $\alpha = +4$ for a given x. Conversely, for negative values of α, there is more reduction in y for $\alpha = -5$ compared to $\alpha = -4$.

Therefore, the term in the bracket in (4) can determine the amount of focusing or diverging of the laser beam. This term is named the focusing parameter (FP)

$$\begin{aligned}\mathrm{FP} = \frac{\omega_{\mathrm{p}}^2 a_0^2 r_0^2}{8c^2} \Bigg[&\frac{\omega_{\mathrm{p.e1}}^2}{\omega_{\mathrm{p}}^2} \left(\frac{1}{2(1-\sigma\alpha)^4} + \frac{Q_{\mathrm{e1}}}{2(1-\sigma\alpha)} \right) \\ &+ \frac{\omega_{\mathrm{p.e2}}^2}{\omega_{\mathrm{p}}^2} \left(\frac{1}{2(1-\sigma\alpha)^4} + \frac{Q_{\mathrm{e2}}}{2(1-\sigma\alpha)} \right) \\ &+ \frac{\omega_{\mathrm{p.p}}^2}{\omega_{\mathrm{p}}^2} \left(\frac{1}{2(1+\sigma\alpha)^4} + \frac{Q_{\mathrm{p}}}{2(1+\sigma\alpha)} \right) \Bigg] - 1. \qquad (7)\end{aligned}$$

FP is a complex function of the laser and plasma characteristics. Moreover, since (4) is a differential equation, the value of f changes over the propagation distance. Therefore, it is not possible to directly find the precise nonlinear dependence of the beam width on the influence parameters such as the angular frequencies of positron, electrons, cyclotron and laser. But based on the FP behaviour, it is possible to find the precise impact of the effective parameters on the self-focusing.

In the following section, the non-linear relationship between the beam width and the angular frequencies of positrons, electrons, laser, and cyclotron is declared based on the FP model.

3. The impact of angular frequencies of laser and plasma on self-focusing

According to the introduced model of focusing parameter (FP), one can analyse the impact of angular frequencies on focusing of laser beam. By utilizing (7), the effect of cyclotron frequency (ω_c), angular positron frequency ($\omega_{\text{p.p}}$), angular frequencies of electrons ($\omega_{\text{p.e1}}$, $\omega_{\text{p.e2}}$), and laser angular frequency (ω_0) on self-focusing is discussed as follows.

3.1. The impact of cyclotron frequency on self-focusing

In order to discuss the impact of the cyclotron frequency ($\omega_c = \frac{eB_0}{m_e c}$), which is related to the external magnetic field (B_0), the parameter FP is plotted against the cyclotron frequency and depicted in Fig. 1a. The initial characteristics of laser and plasma are assumed as: $\omega_0 = 1.88 \times 10^{15}$ s^{-1}, $r_0 = 15 \times 10^{-4}$ cm, $a_0 = 0.271$ ($I \approx 10^{17}$ W/cm^2), $T_{\text{e1}} = T_{\text{e2}} = 10$ keV, $T_{\text{i}} = 0.5$ keV, $T_{\text{p}} = 5$ keV, $\omega_{\text{p.j}} = 0.14\omega_c$, $\omega_{\text{p.e2}} = \omega_{\text{p.e1}} = 0.1\omega_0$.

Based on Fig. 1a, the FP behaviour predicts the following results:

- Increasing the cyclotron frequency leads to an increase in FP, namely stronger focusing occurs in the presence of the external magnetic field.
- The influence of cyclotron frequency is the same for left-handed and right-handed polarization for the above mentioned initial condition. But if the initial values are changed, the effect of ω_c may differ for right-handed or left-handed polarization. For example, in Fig. 1c, the initial value of $\omega_{\text{p.p}}$ is assumed as $\omega_{\text{p.p}} = 0.25\omega_0$. The result indicates that the effect of the cyclotron frequency and the magnetic field will be different for left-handed and right-handed polarization.

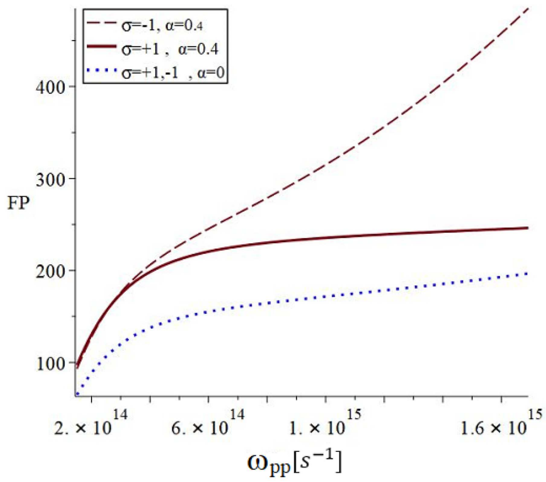

Fig. 2. Focusing parameter versus positron frequency $\omega_{\mathrm{p.p}}$.

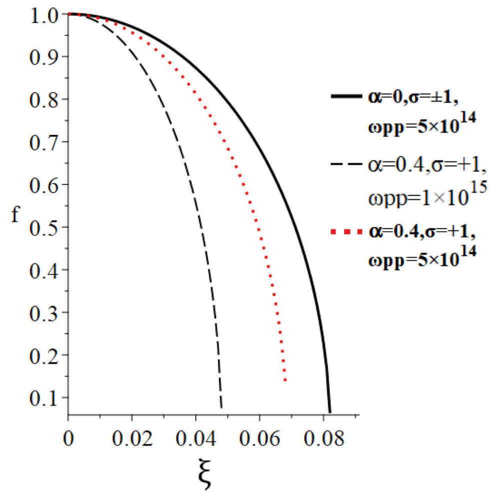

Fig. 3. Beam-width parameter f versus propagation length ξ for left-handed and right-handed polarization.

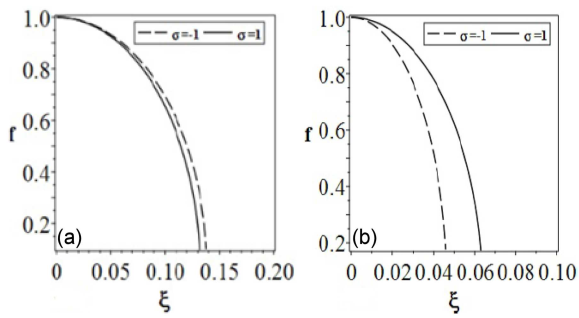

Fig. 4. Beam-width parameter f versus propagation length ξ (a) before the intersection point ($\omega_{\mathrm{p.p}} = 2.65 \times 10^{14}$ s^{-1}) and (b) after the intersection point.

- At high cyclotron frequencies, FP undergoes sudden changes, whereas at low frequencies, FP changes gradually.

To validate the results discussed above, the second-order equation for self-focusing (see (4)) is solved, and the beam-width parameter (f) versus the normalized propagation length (ξ) is plotted. The result is shown in Fig. 1b, and good agreement is seen between the FP predictions and the variation of beam width. For a given propagation length (ξ), the value of f is smaller for larger ω_c. Figure 1b also indicates that more significant changes in the beam width occur at higher ω_c. This means that when a strong external magnetic field is present, even slight alterations in the magnetic field can result in notable changes in the laser's spot size.

3.2. The effect of positron frequency $\omega_{\mathrm{p.p}}$ on self-focusing

In order to investigate the effect of positron frequency on self-focusing, in Fig. 2 the focusing parameter is plotted versus $\omega_{\mathrm{p.p}}$ for right-handed and left-handed polarization beams. The result indicates that increasing the positron frequency $\omega_{\mathrm{p.p}}$ in the plasma can lead to an increase in FP, i.e., further focusing of beam occurs at larger $\omega_{\mathrm{p.p}}$ for both right-handed and left-handed polarizations. The result of solving (4), presented in Fig. 3, shows a good agreement with the above-mentioned result.

Figure 2 also indicates that in the absence of a magnetic field $\alpha = 0$, there is no difference between the self-focusing of the right-handed and left-handed polarizations. But in the presence of an external magnetic field, there is significant difference between the focusing of both mentioned polarizations. As shown in Fig. 2, before the intersection point ($\omega_{\mathrm{p.p}} = 2.65 \times 10^{14}$ s^{-1}), the value of the focusing parameter (FP) is less for the beam with left-handed polarization. However, after the intersection point, the value of FP is greater for the left-handed polarization than for the right-handed polarized beam. In Fig. 5 the beam width is plotted versus the propagation length, and a good agreement with the prediction of the FP model is seen.

In order to validate the above results, in Fig. 3 we have plotted the beam-width parameter, based on (4).

3.3. The effect of angular frequencies of electrons on self-focusing

The graph in Fig. 5 displays the relationship between FP and the electron–positron angular frequency $\omega_{\mathrm{p.e1}}$ and angular frequency electron–ion $\omega_{\mathrm{p.e2}}$. The results indicate that FP acts in a similar manner in $\omega_{\mathrm{p.e1}}$ and $\omega_{\mathrm{p.e2}}$.

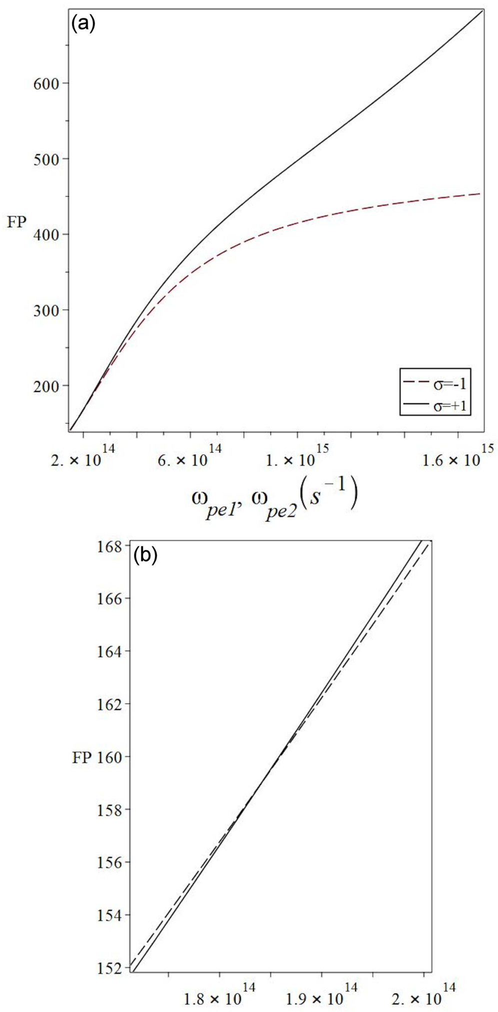

Fig. 5. (a) Plot of FP versus angular frequency of electrons. Inset (b) magnifies the area of intersection of both polarization results.

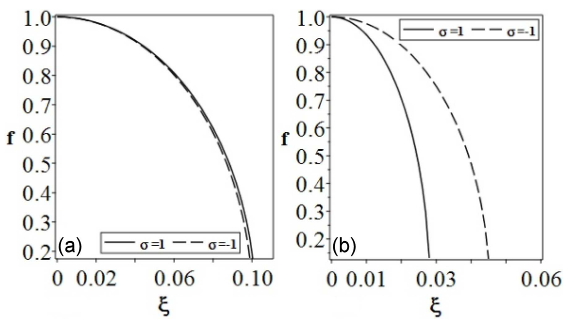

Fig. 6. Beam-width parameter versus propagation length (a) before the intersection point and (b) after the intersection point.

Additionally, Fig. 5 indicates that in contrast to Fig. 2 (plot of FP versus positron frequency $\omega_{\mathrm{p.p}}$), at higher electron frequencies the focusing parameter FP of right-handed polarization is larger compared to the value of FP for left-handed polarization.

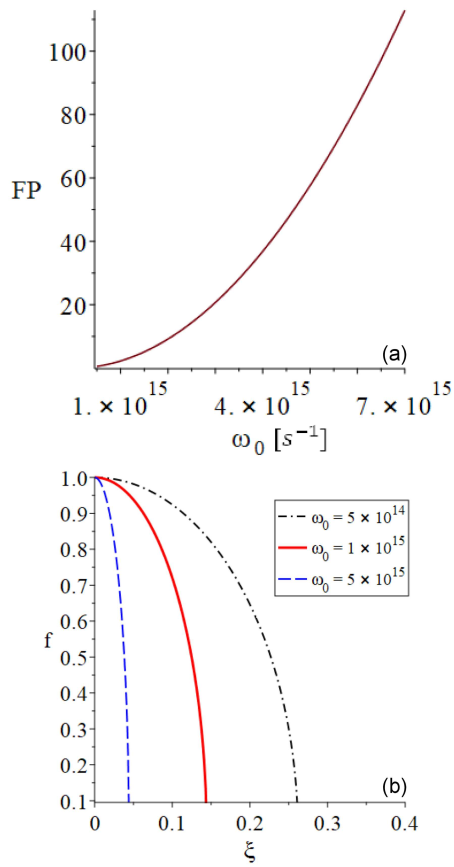

Fig. 7. (a) Plot of FP versus angular frequency of laser. (b) Plot of beam-width parameter.

However, at frequencies lower than the intersection point ($\omega_{\mathrm{p.p}} = 1.85 \times 10^{14}\ \mathrm{s}^{-1}$), the value of FP for right-handed polarization is less than that of left-handed polarization. To confirm this observation, we solved the beam-width equation and plotted the results graphically in Fig. 6. A good agreement is seen.

3.4. The effect of the laser's angular frequency on self-focusing

To study how the angular frequency of a laser affects self-focusing, the dependence of FP versus ω_0 is shown in Fig. 7a. In Fig. 7b, one can see that as the frequency of lasers increases, the laser beam experiences more intense focusing.

4. Conclusions

Based on the introduced model regarding the FP phenomena, the exact relation between the laser and plasma frequencies on self-focusing is established. It is found that increasing the magnetic field leads to stronger self-focusing and that the amount of focusing is the same for both polarizations. In the absence of an external magnetic field, the amount of focusing is similar for both polarizations. But, in the presence of an external magnetic field, there are significant differences between the impact of positron frequency on self-focusing for left-handed and right-handed polarizations.

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