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Advanced Vibration Frequency Control of a Geometrically Non-linear System Resting on a Winkler Foundation Using Electro–Mechanical Coupling Features

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In this paper the study of active/passive vibration frequency control through electro–mechanical coupling features in a slender mechanical system resting on a Winkler foundation is presented. The investigation starts with the formulation of a mathematical model to characterize the dynamic behavior of the composite system created of a core beam and integrated layers of piezoelectric actuators. The key parameters influencing the vibration characteristics, such as foundation stiffness, and geometrical and electro–mechanical properties of the system, are examined and discussed. Based on the obtained numerical results it is shown that the reference system dynamic response may be enhanced through the geometrical and physical parameters of the embedded actuators and can be further improved in significantly greater extent through active piezoelectric actuation. On the other hand, it is observed that together with the increase in the elastic foundation stiffness, the effect of actuation decreases.

topics: piezoelectric (PZT) ceramics, mechanical vibrations, mechanical systems control, elastic structures

1. Introduction

Vibrations are a fundamental phenomenon connected with physics and mechanical engineering, having significant implications for the analysis and design of dynamic systems. In physics, the study of vibrations underpins key concepts such as wave propagation, resonance, and energy dissipation, contributing to a deeper understanding of material behavior and stability in various contexts. In mechanical engineering, vibrations critically affect the performance, durability, and safety of structures and machinery. Uncontrolled vibrations may lead to adverse effects such as increased noise, material fatigue, or — in extreme cases — catastrophic failure.

Krieger-Wojnowsky [1] is considered a pioneer in the study of vibrations. In 1950, he investigated the effect of axial loading on large-amplitude vibrations in bars that were hinged at both ends. It was discovered that introducing an axial displacement into the system causes an internal axial force to oscillate in the bar during vibrations. In 1951, Burgreen [2] presented a solution to the problem of free vibrations in a simply supported beam. A few years later, the same author published a study discussing the impact of the boundary conditions at the bar's end on its natural vibrations [3]. Civalek and Öztürk [4] analyzed the free vibrations of a simply supported Euler–Bernoulli beam with a tapering cross-section resting on a Winkler–Pasternak foundation. The discrete singular convolution (DSC) method was demonstrated to be aplicable in numerical simulations to determine the first three natural frequencies of the system for three different values of foundation stiffness. The influence of the stiffness of different foundation models on the vibration frequency of the beam system can be found, for example, in [5–7].

The investigation of stability and vibrations in slender mechanical systems remains an active area of research to the present day. Contemporary articles highlight the application of various types of "smart materials" to enhance the vibration control capabilities, where particularly prominent in this field are shape memory alloys (SMA) and piezoelectric (PZT) materials. The variation in vibration frequencies and critical forces in a carbon fiber beam with pre-tensioned shape memory wires mounted

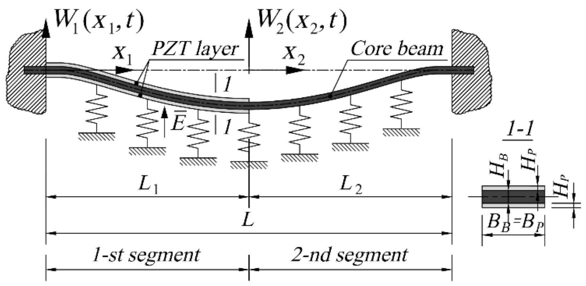

Fig. 1. Clamped–clamped system with two surface-bonded single PZT layers resting on a single-parameter Winkler elastic foundation.

along the neutral axis of the system was examined by Baz et al. [8]. Through the study, it was observed that the appropriate use of reinforcement with pretensioned shape memory wires induces a shift of the fundamental vibration modes of the beams to higher frequency ranges compared to non-reinforced beams. The demonstration that the residual force induced from the PZT element effectively modifies the vibration frequencies, loading capacities, and stability characteristics of the column with the attached PZT rod is shown in [9]. The influence of piezoelectric actuation on lateral oscillation of a beam with varying cross-section is presented in [10].

In this paper, the main goal is to demonstrate the ability of tuning the vibration frequency of a host structure resting on a single-parameter Winkler foundation through a surface-bonded PZT material and its electro–mechanical coupling features.

2. Studied model

For the studied model, a clamped–clamped beam resting on a single-parameter Winkler elastic foundation was chosen, for which the PZT single layers are surface-bonded to the top and bottom surface of the host beam near one of the end supports, as shown in Fig. 1.

It should be highlighted that a beam with both ends restrained against longitudinal displacements gives the most effective control of the critical force and vibration of the system among the cases where one end is free, as stated by de Faria [11] and Zehetner and Irschik [12]. The vibration control of the studied system is achieved by inducing a residual force of equal magnitude and same direction within the PZT layers. In order to exert this force in the system, an electric field needs to be applied to the PZT elements, taking into account that its vector direction explicitly determines the generation of a compressive or tensile force. The derivation of the residual force equation for an n-segment beam has been thoroughly discussed in [13]. It is worth noting that the magnitude of this force strongly depends on the relationship between the axial stiffness of the PZT element and the host beam, as well as on the length, the thickness of the PZT layers itself and the voltage applied.

3. Theory and formulation

Before the system is actuated, it is assumed that the beam is perfectly straight — geometrical imperfections are neglected. Delamination effects as well as material imperfections are ignored in the analysis. The slenderness of the system classifies the problem into the Bernoulli–Euler beam theory. For the piezoelectric elements two different materials are examined: (i) P41 [14], for which the maximum electric field is limited to 2 kV/mm, and (ii) a more recent one, NCE46 [15], with a limit of 5.5 kV/mm, in which the applied electric field–strain relation is linear. In order to make the analysis more general, all quantities are presented in non-dimensional form. In the subsequent sections of this article, the subscript "b" denotes variables related to the beam, while the subscript "p" represent variables associated with the PZT layers.

On the basis of the presented variational formulation using Hamilton's principle in [13] and after some mathematical manipulation, the non-dimensional equations of motion for the system in Fig. 1 (for $i = 1,\ 2$) can be expressed as

$$\frac{\mathrm{d}^4 w_i(\xi_i)}{\mathrm{d}\xi_i^4} \pm \varphi_i f_r^2 \frac{\mathrm{d}^2 w_i(\xi_i)}{\mathrm{d}\xi_i^2} - \left[\mu_i \omega_0^2 - \varphi_i \beta\right] w_i(\xi_i) = 0, \tag{1}$$

where $w_i(\xi_i)$ denotes the transversal displacements of the i-th segment, f_r — residual force exerted in the system, ω_0 — natural frequency, β — elastic foundation stiffness parameter.

The following substitutions are introduced in (1)

$$w_i(\xi_i) = \frac{W_i(x_i)}{L}, \qquad \xi_i = \frac{x_i}{L},$$

$$\varphi_i = (1+r_m)^{-\frac{1}{2}(j^{2i}+1)}, \qquad r_m = \frac{E_p J_p}{E_b J_b},$$

$$f_r^2 = \frac{2 B_p d_{31} E_p V_{app} L^2 A_b L_p}{J_b\big(E_b A_b L + E_p A_p L - E_p A_p L_p\big)},$$

$$\mu_i = \left[\frac{\alpha_1 + (\eta - 1)\,\alpha_2}{\alpha_1\,(1+r_m)}\right]^{\frac{1}{2}(j^{2i}+1)}, \qquad \alpha_1 = \frac{E_p}{E_b},$$

$$\alpha_2 = \frac{\rho_p}{\rho_b}, \qquad \omega_0^2 = \Omega^2 L^4 \frac{\rho_b A_b}{E_b J_b},$$

$$\beta = \frac{k L^4}{E_b J_b}, \qquad j = \sqrt{-1}, \tag{2}$$

where L is the total beam length (assumed as 0.60 m for $f_{r,\max}$ determination), E — Young's modulus of the material, J — moment of inertia of the element cross-section, B — element width, d_{31} — PZT material constant, V_{app} — voltage applied to PZT material, A — cross-section area, ρ — material mass density, Ω — vibration frequency, k — elastic foundation stiffness.

In (1), the plus sign before the residual force f_r^2 denotes the exerted compression in the system, whereas the minus sign denotes the tension. To solve the stated problem, the following boundary and continuity conditions are used

$$w_1(0) = w_1^I(0) = w_2(l_2) = w_2^I(l_2) = 0, \tag{3}$$

$$\begin{aligned} &w_1(l_1) = w_2(0); \\ &w_1^I(l_i) = w_2^I(0); \\ &w_1^{RN}(l_1) = (1 + r_m)\, w_2^{RN}(0) \end{aligned} \tag{4}$$

where $RN = II$ or III and denotes the order of the derivative.

The general solution of (1) is

$$\begin{aligned} w_i(\xi_i) &= A_i \cosh(\gamma_{i1}\xi_i) + B_i \sinh(\gamma_{i1}\xi_i) \\ &\quad + C_i \cosh(\gamma_{i2}\xi_i) + D_i \sinh(\gamma_{i2}\xi_i) \end{aligned} \tag{5}$$

(for $i = 1, 2$), where the coefficients γ_{i1} and γ_{i2} are

$$\gamma_{iz} = \sqrt{\frac{1}{2}\left[\pm\varphi_i f_r^2 + (-1)^z \sqrt{\varphi_i f_r^4 - 4\varphi_i \beta}\right]} \tag{6}$$

(for z=1, 2). By substituting the functions from (5), which describe the transverse displacements of the individual beam segments ($i = 1, 2$), into the boundary and continuity conditions (3) and (4), respectively, a system of twelve homogeneous linear equations with respect to the unknown integration constants A_i, B_i, C_i and D_i is derived. A non-trivial solution to this system exists when the determinant of the coefficient matrix equals zero. Consequently, for the given physical and geometrical parameters of the system and the specified magnitude of piezoelectric actuation, the natural frequency can be determined.

4. Numerical solution

For computation purposes, the geometrical and physical relations are, respectively, as follows

$$b = \frac{B}{L} = 0.05, \qquad h_b = \frac{H_b}{L} = 0.0075, \qquad h_p = \frac{1}{6}h_b, \tag{7}$$

and

$$\begin{aligned} &\frac{E_b}{E_p} = 1.099 \text{ (NCE46)}, \qquad && \frac{E_b}{E_p} = 1.19 \text{ (P-41)}, \\ &\frac{\rho_b}{\rho_p} = 0.353 \text{ (NCE46)}, && \frac{\rho_b}{\rho_p} = 0.365 \text{ (P-41)}, \\ &r_m = 1.5059 \text{ (NCE46)}, && r_m = 1.6313 \text{ (P-41)}, \\ &\beta = \{0, 100\}. \end{aligned} \tag{8}$$

The influence of PZT layers length and the inducted maximum admissible tensile/compressive residual force for two different values of foundation stiffness parameter β on the non-dimensional first natural frequency ω_0 is presented in Fig. 2. In Fig. 3 the absolute value of the relative percentage modification of the first natural frequency for actuated systems with respect to their non-actuated counterparts as a function of the piezosegment length is presented.

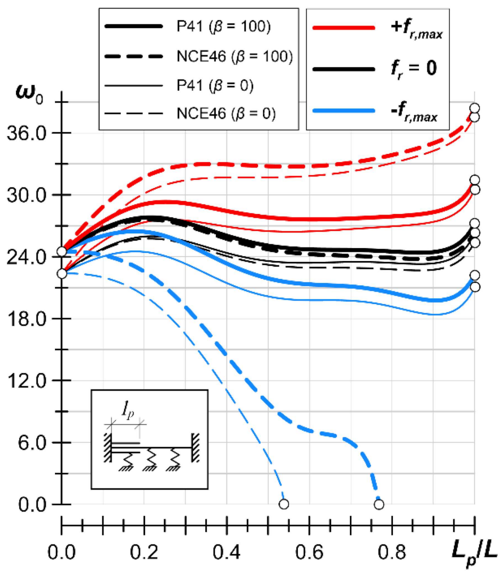

Fig. 2. Influence of piezoelectric segment length, elastic foundation modulus and piezoelectric actuation level on the non-dimensional first natural frequency ω_0 for different PZT materials.

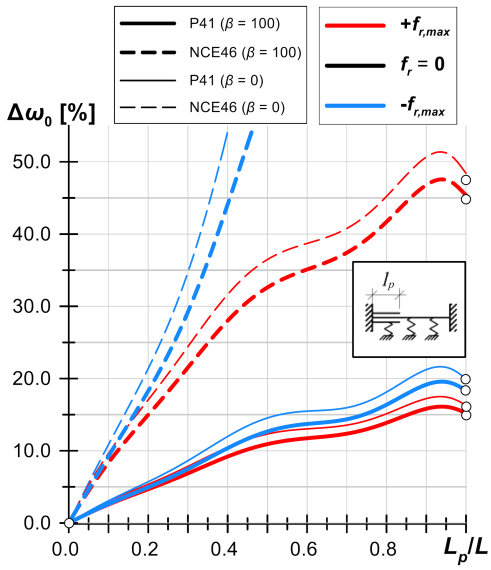

Fig. 3. Absolute relative value of the first natural vibration frequency modification as a function of the piezosegment length for different values of a Winkler foundation stiffness parameter.

Based on the presented results one can state that the NCE46 material gives a greater range in which the first natural frequency ω_0 may be altered compared to the P41 material. The higher the foundation stiffness parameter β, the higher the values of ω_0 are obtained in regard to those with $\beta = 0$.

Regardless of the foundation stiffness, the induction of a compressive residual force in the system lowers the frequency ω_0, whereas the tensile force acts in the opposite way. Exerting the maximum admissible compressive force in a system with the NCE46 material for $\beta = 0$ at $L_p/L \geq 0.5386$, or for $\beta = 100$ at $L_p/L \geq 0.7630$, leads to the loss of system stability. Hence, for long PZT layers, where the PZT material has a high limit of admissible applied voltage, the actuation should be used with special caution.

5. Conclusions

The presented study shows that vibration frequency control using piezoelectric actuators offers a viable and efficient method for enhancing the vibrational performance of dynamic systems. This technique holds significant potential for applications in aerospace, civil and mechanical engineering, where precise vibration control is essential for structural integrity and durability.

References

[1] S. Krieger-Woinowsky, *J. Appl. Mech.* **17**, 35 (1950).

[2] D. Burgreen, *J. Appl. Mech.* **18**, 135 (1951).

[3] D. Burgreen, *J. Eng. Mech.* **84**, 1791-1 (1958).

[4] Ö. Civalek, B. Öztürk, *Geomech. Eng.* **2**, 45 (2010).

[5] M. Gibigaye, C.P. Yabi, G. Degan, *Appl. Math. Model.* **61**, 618 (2018).

[6] S.R. Asemi, A. Farajpour, *Curr. Appl. Phys.* **14**, 814 (2014).

[7] P. Obara, *Arch. Civ. Eng.* **60**, 421 (2014).

[8] A. Baz, S. Poh, J. Ro, J. Gilheany, *J. Sound Vib* **185**, 171 (1995).

[9] K. Sokół, S. Uzny, *AIP Conf. Proc.* **1648**, 850021 (2015).

[10] K. Kuliński, J. Przybylski, *Mech. Res. Commun.* **82**, 43 (2017).

[11] A.R. de Faria, *Comp. Struct.* **65**, 187 (2004).

[12] C. Zehetner, H. Irschik, *Smart Struct. Syst.* **4**, 67 (2008).

[13] J. Przybylski, G. Gąsiorski, *J. Sound Vibr.* **437**, 150 (2018).

[14] Annon Piezo Technology hompage (accessed Nov. 2024).

[15] Noliac homepage (accessed Nov. 2024).