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# Fine-Tuning of the Oscillation Frequency in Slender Mechanical Beam-Systems Through the Use of Smart Materials Features

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In the study of physics and engineering, natural vibrations are a fundamental characteristic of all systems covering microscopic particles up to large-scale structures. In this work, the significance of active structures technology application and its features for altering the vibration frequency far from the excitation band in slender geometrically non-linear beam-systems is emphasized. The subject of interest covers a hosts beam members with surface-bonded active/passive piezoelectric layers. By precisely adjusting the voltage across these layers based on their poling direction and electric field vector, it is possible to create non-zero stress along the beam, adjusting its eigenfrequency. The provided study is based on the mathematical formulation and derived motion equations using Hamilton's principle, taking into account the linear stress–strain relationship with electro–mechanical coupling. The influence of physical parameters and piezoelectric actuation on the fine-tuning of vibration frequency are investigated. The obtained numerical results clearly indicate that the vibration frequency not only changes non-linearly with the piezo-layers location and their structural parameters, but also the piezoelectric actuation can be efficiently used to additionally alter the systems' dynamic response.

topics: dynamic response modification, vibration frequency, piezoelectric actuation, active structures

## 1. Introduction

In physics/engineering, all systems — including molecular systems and particles — tend to vibrate at a natural frequency depending upon their structure. When an oscillating force is applied at the resonant frequency of the system, the amplitude of oscillation tends to infinitely increase, which may result in the system malfunction or, even worse, its collapse. In recent studies, smart materials, including piezoelectric (PZT) materials, shape memory alloys (SMA), and magnetorheological fluids, play a significant role in mechanical vibration investigations by adaptively responding to external stimuli such as stress, temperature, or magnetic fields. These materials offer advanced solutions for vibration control, damping, and sensing applications, significantly enhancing the performance and functionality of mechanical systems.

The modification of the frequency and critical forces in a carbon fiber beam with pre-tensioned SMA wires mounted along the neutral axis of the system was investigated by Baz et al. [1]. Based on the works [2, 3] presenting the application of PZT-elements, it is stated that the most effective control of both the critical force and the system vibrations is achieved when the boundary conditions have zero longitudinal displacements at both ends of the considered rod. Taking this into account the influence of residual forces on the natural frequency versus the amplitude and proposing criteria to observe the frequency changes under applied external loads was demonstrated in [4]. The improvement of the static and dynamic response of beams, columns, trusses, plates etc., through the application of PZT materials, can be found *inter alia* in [5–7].

In this study, the primary goal is to show how the vibration frequency of a clamped–clamped and pinned–pinned slender host beam may be fine-tuned through the surface-bonding of smart material and especially through the use of its electro–mechanical coupling feature.

## 2. Physical model

The physical model of the studied system consists of a slender beam with a rectangular cross-section, with both end supports preventing longitudinal displacements and perfectly bonded to the top and bottom host beam surface of single PZT layers. The scheme of the clamped–clamped (C-C) piezo-beam system is presented in Fig. 1. Additionally, a pinned–pinned (P-P) system is studied. For better clarification of further constant descriptions used across text, the "B" subscript denotes the core beam element, and "P" — the PZT element. Two beam lengths $L = 0.25$ m and $L = 0.50$ m, as well as two locations of the piezo elements, are investigated, respectively: symmetrically located between supports (configuration "A") and located near one of the supports (configuration "B").

The core beam is made of aluminum material (Young moduli $E_B$=70 GPa, density $\rho_B = 2720$ kg/m$^3$), whereas for PZT elements, the NCE-46 material is adopted according to the manufacturer [8] ($E_P = 76.923$ GPa, $\rho_B = 7700$ kg/m$^3$, PZT material constant $d_{31} = 1.30\times10^{-10}$ C/N). The PZT layers have the same width as beam ($B_P = B_B = 20$ mm) and the heights are $H_P = B_B/40 = 0.5$ mm and $H_B = B_B/10 = 2.0$ mm. The voltage applied is assumed as $V_{app} = 160$ V for each PZT layer providing linear relationship between induced axial piezoelectric force and applied voltage. When piezo elements are symmetrically bonded to the core beam, the first and third segments are identical in terms of physical and mechanical properties. Geometrical and physical imperfections of the system as well as delamination of the piezo layers are neglected. The electric field is considered to be uniformly distributed throughout the piezo segment. In the studied system, transverse vibrations around its straight equilibrium configuration are considered.

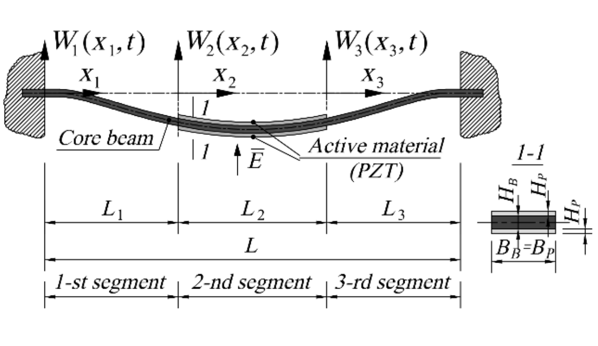


Fig. 1. Clamped–clamped host beam scheme with symmetrically bonded active material (PZT) layers.

## 3. Mathematical formulation

In order to address the problem of transverse vibrations in the studied systems, Hamilton's principle is employed taking into account that systems' geometrical dimensions allow us to classify the problem under the classical Euler–Bernoulli beam theory. The process of deriving the equations of motion for the $n$-segmented beam was presented and discussed in [9]. Hence, for the studied problem, the derived and simplified differential equations describing the motion of $i$-th segment ($i = 1, 2, 3$) are

$$E_i J_i \frac{\partial^4 W_i(x,t)}{\partial x_i^4} \pm F_r \frac{\partial^2 W_i(x,t)}{\partial x_i^2} + \rho_i A_i \frac{\partial^2 W_i(x,t)}{\partial t^2} = 0, \tag{1}$$

where $J_i$ is the moment of inertia [m$^4$], $A_i$ — cross-section area [m$^2$], $F_r$ — axial residual force induced by PZT actuation [N], and $W(x,t)$ is transversal displacement [m].

The axial residual force appearing in (1) is defined as

$$F_r = \frac{2B_B\, d_{31}\, E_P\, V_{app}}{1 + \frac{E_B A_B + E_P A_P}{E_B A_B}\left(\frac{L}{L_P} - 1\right)}, \tag{2}$$

where $V_{app}$ denotes the voltage applied to the piezo-layers [V], $L$ corresponds to the total beam length, and $L_P$ stands as the PZT-layer length.

Depending on the electric field direction one can exert a compressive (plus sign) or tensile residual (minus sign) force in the system, thus in (1) the $F_r$ force member is preceded by the plus and minus sign.

The following geometrical, natural boundary and continuity conditions are used:

- in the case of C-C system,

$$W_1(0,t) = W_3(L_3,t) = \frac{\partial W_1(0,t)}{\partial x} = \frac{\partial W_3(L_3,t)}{\partial x} = 0, \tag{3}$$

- in the case of P-P system,

$$W_1(0,t) = W_3(L_3,t) = \frac{\partial^2 W_1(0,t)}{\partial x^2} = \frac{\partial^2 W_3(L_3,t)}{\partial x^2} = 0, \tag{4}$$

- continuity conditions between segments ($i = 1, 2, 3$ and $j = 2, 3$),

$$W_i(L_i,t) = W_{i+1}(0,t), \tag{5}$$

$$\frac{\partial W_i(L_i,t)}{\partial x_i} = \frac{\partial W_{i+1}(0,t)}{\partial x_{i+1}}, \tag{6}$$

$$E_i J_i \frac{\partial^j W_i(L_i,t)}{\partial x_i^j} = E_{i+1} J_{i+1} \frac{\partial^j W_{i+1}(0,t)}{\partial x_{i+1}^j}. \tag{7}$$

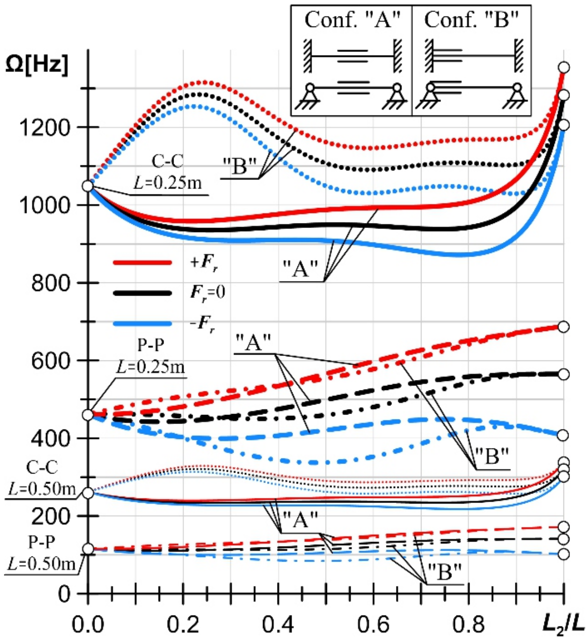


Fig. 2. The influence of piezo-layers location, length and piezoelectric actuation on the natural frequency of C-C and P-P beams for $L = 0.25$ m and 0.50 m, respectively.

In order to obtain approximate results, due to the occurrence of geometric non-linearity in the studied systems, the Lindstedt–Poincaré perturbation method (comp. [7, 10]) was employed.

## 4. The influence of geometrical and physical parameters on the natural frequency

The influence of the location of the piezo-layers, their length and piezoelectric actuation on the natural vibration frequency in C-C and P-P systems is presented in Fig. 2. Small icons included visualize the location of the PZT-layers. Moreover, for actuators located near the left support, dash–dotted and dotted lines are used, whereas for the symmetrical bonding continuous lines. By black lines, the non-actuated systems are depicted, the red lines show the induced compressive force, and the blue lines show the tensile residual force. The thicker lines correspond to the total beam length $L = 0.25$ m, whereas the thinner ones to $L = 0.50$ m.

On the basis of the presented results, one can notice that in C-C beams, regardless of their length, within the same type of piezo actuation or its absence, the attachment of PZT elements near one of the supports leads to an increased natural frequency ($\Omega$) in the whole range of $L_2/L$. For the P-P beams, the way of attaching the active material makes it possible to find the ranges in which the obtained frequency is higher in relation to the second way of bonding. For the non-actuated P-P system with $L = 0.50$ m this is observed when $L_2/L < 0.28$, but this range changes together with the electric field applied to the actuators, i.e., $L_2/L < 0.42$ for $+F_r$ and $L_2/L < 0.19$ for $-F_r$. For non-actuated beams at $L = 0.50$ m, the highest shift in natural frequency between the locations of the piezo-layers is $\approx 348$ Hz at $L_2/L = 0.24$ for C-C system and $\approx 44$ Hz at $L_2/L = 0.58$ for P-P beam.

By sensing the PZT actuators, regardless of their location, it is possible to fine-tune the natural frequency with respect to the non-actuated system. Generally, the longer the piezo-layers with respect to the total length in both P-P and C-C systems, the greater the influence of piezoelectric actuation. Moreover, the shorter the beam, the greater the quantitative range over which the frequency may be tweaked. In a sandwich P-P system ($L_2/L = 1.00$) for $L = 0.25$ m the frequency $\Omega$ may be altered by $\approx 281$ Hz, where the compressive residual force alters $\Omega$ by a maximum of 123 Hz and the tensile force by 158 Hz. In the P-P system for $L = 0.50$ m the piezoelectric control is possible in a significantly lower range of $\sim 71$ Hz. As soon as the C-C system is taken into consideration for $L = 0.25$ m comparing the lines referring to $-F_r$ and $+F_r$ at $L_2/L = 1.00$ it is possible to tune the natural frequency $\Omega$ by 170 Hz, whereas for $L = 0.50$ m by 37 Hz. Despite quantitative differences comparing the same supporting systems, regardless of total beam length, the qualitative differences are almost identical.

## 5. Conclusions

In this study, the physical problem of dynamic response adjustment via smart materials application and its features in slender non-linear beam systems is investigated. The presented analysis shows that making a sandwich/stepped beam through bonding PZT-layers to the top and bottom host beam surface changes the natural frequency with respect to the reference system. This makes the system geometrically non-linear, and the transversal vibration frequency strongly depends on the geometrical and physical parameters of the attached layers as well as their location. Moreover, it is revealed that through the electro–mechanical material features, once an electric field is applied to the system, piezoelectric actuation may be used as an efficient tool for alteraction of the dynamic response of the system far from the excitation band. The crucial influence on the range of possible adjustments are: system supporting, system geometrical dimensions, location of the piezoelectric material, its length, thickness, and applied voltage. Regardless of the case studied, the employment of electro–mechanical coupling feature allows to tweak the frequency in a more convenient and precise way, as well as in a significantly wider extent in comparison to standard methods.

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