

# The Influence of Additional Mass Elements on the Acoustic Spectrum of the Bell

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The paper presents a proposal to introduce additional mass elements in order to change the resonance frequencies of the bell. This innovative approach is intended to eliminate the need of fine tuning this type of instruments in a traditional method, i.e., through mechanical working, but only by adding appropriately selected elements of specific weight, quantity, and location. Another advantage of this method is that it does not interfere with the external appearance of the bell, because the components are assembled inside the object. The parameters of the added elements can be selected using heuristic algorithms that allow for precise removal and shifting of sound components. Preliminary results of numerical and experimental studies indicate that the presented approach should find practical application. It can be developed by replacing mass elements with mechanical resonators or elements made of metamaterials.

topics: bell, finite element method (FEM), tuning, sound partial

## 1. Introduction

Acoustic properties are one of the most important parameters determining the quality of bells. The sound emitted by a bell consists of components whose frequencies are multiples of the first lowest tone, creating a series with the numerical ratio of 1 : 2 : 2.4 : 3 : 4 [1–4]. This dependence defines the relationships between the first five partial tones and the fundamental tone, which in the order indicated by the series are called: hum, fundamental, tierce, quint, and nominal (for our purpose we designate them as ‘H’, ‘F’, ‘T’, ‘Q’, and ‘N’, respectively). In reality, it is not possible to achieve the ideal proportion defined by the series, but the deviations are strived for as small as possible. If the difference between the expected and received sound is too large, the bell should be tuned. This process consists in machining and is performed on the inner surface of the object, because the outer face most often has decorations and ornaments.

In order to tune the bell to the desired frequencies, it is necessary to know how mass reduction will affect the sound components, especially since it is an irreversible process, and in extreme cases, it may cause a permanent increase in sound defects. There are known works by the authors [2] indicating in a

rather general way the places where the mechanical working (material removal) will cause a decrease or increase in the value of individual vibration frequencies. However, these publications do not signify how much mass reduction in a given area will change the frequency of the partial tones compared to the original, i.e., how the values of individual frequencies will change.

In the paper, an innovative solution was proposed, in which instead of the turning process, additional mass elements are introduced into the system. The advantage of this method is the ability to change the size, number, and location of the added elements without irreversibly interfering with the shape of the bell.

## 2. Numerical research

The proposed solution was first analyzed numerically using the finite element method (FEM) [5]. The analyzed bell model has a mass of 24.6 kg and a diameter of 335 mm. It was assumed that with the defined rib and size, the expected fundamental tone would be  $d/3$  (European tonality), which corresponds to a frequency of 1174.66 Hz. In this case, the theoretical values of the sound components have the values listed in the  $\omega_{opt}$  column in

TABLE I

Sound components of an untuned bell: optimal ( $\omega_{opt}$ ), numerical ( $\omega_{num}$ ), experimental ( $\omega_{exp}$ ). Here,  $\delta = \left| \frac{\omega_{opt} - \omega_{exp}}{\omega_{exp}} \right| \times 100\%$ .

Sound component (SC)	$\omega_{opt}$	$\omega_{num}$	$\omega_{exp}$	$\delta$
	[Hz]			[%]
Hum (H)	587.33	589.97	570	3.04
Fund. (F)	1174.66	1185.6	1158.5	1.39
Tierce (T)	1409.59	1413.6	1365.5	3.23
Quint (Q)	1761.99	1773.4	1718	2.56
Nominal (N)	2349.32	2375.8	2281	3.00

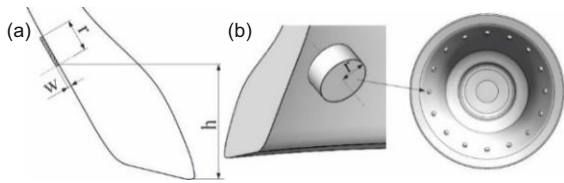


Fig. 1. (a) Location and (b) shape of additional mass elements introduced into the system.

Table I. Higher vibration frequencies (see  $\omega_{num}$  in Table I) were obtained from simulation tests, because the numerical model considers solidification shrinkage and other allowances, taking into account the process of forming and casting the bell. In order to verify whether the assumed surpluses are sufficient, experimental studies were carried out [6], and Table I sets up the sound components of the analyzed real object ( $\omega_{exp}$ ).

The acoustic properties of a real bell deviate from the desired ones, therefore the system must be tuned to increase all values of the vibration frequencies. The largest discrepancy concerns the tierce and exceeds it by 44.09 Hz (3.2%), while the smallest inconsistency is by 16.16 Hz (about 1.4%) and characterizes the impact sound (fundamental).

In the case of traditional tuning, material is removed from the inside of the bell, causing an increase or decrease of the frequency values, and the power of the mass reduction effect changes as a height function of the bell machining. According to the literature [2], it is not possible to simultaneously increase all the sound components by removing material. Milling at the bottom of the bell causes a growth in the vibration frequency for the H, F, T, and N components, without the Q component. The quint can only be reduced regardless of the machining location. Hence, in the examined case, the traditional approach makes it impossible to achieve the intended sound of the bell. The sole solution would be to re-cast the bell, in which all the components are above the desired tones and to tune it “downwards”.

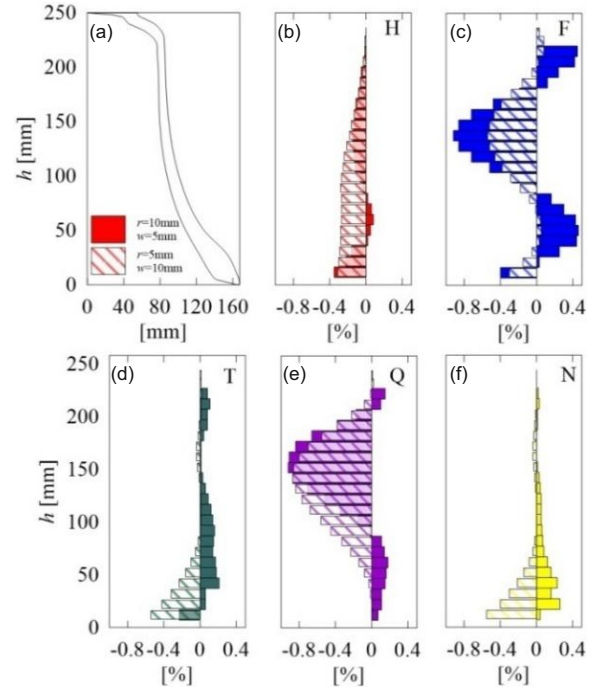


Fig. 2. (a-f) The influence of additional mass elements on the components of the bell sound.

Therefore, a new approach to tuning bells was proposed, namely by adding mass elements to the system. A series of computer simulations were conducted, involving 16 evenly and symmetrically arranged cylinders (Fig. 1). The position of the center ( $h$ ) of the additional masses was changed and on this basis the influence of these elements on the sound components was determined.

Figure 2 shows the percentage influence of additional elements on vibration frequencies. The changes were calculated according to the formula

$$\delta = \frac{(\omega_{mod} - \omega_{num})}{\omega_{num}} \times 100, \quad (1)$$

where  $\omega_{num}$  is the numerical frequency of the bell before tuning,  $\omega_{mod}$  — the numerical frequency of the bell taking into account additional mass elements.

The obtained results show that the position and size of the cylinders have a significant influence on the values of the sound components. These mentioned factors make it possible to both lower and increase the frequency of the sound component vibrations. The resulting impact on the bell’s sound is small in percentage terms (maximum 1%), because the mass of the added elements (110.53 g for 16 cylinders with  $r = 5$  mm,  $w = 10$  mm; 220.19 g for 16 cylinders with  $r = 10$  mm,  $w = 5$  mm) in relation to the bell’s mass constitutes only about 0.45% and 0.9%, respectively. This is not a problem, because the mass of the cylinders can be numerically adapted to obtain the target sound of the bell.

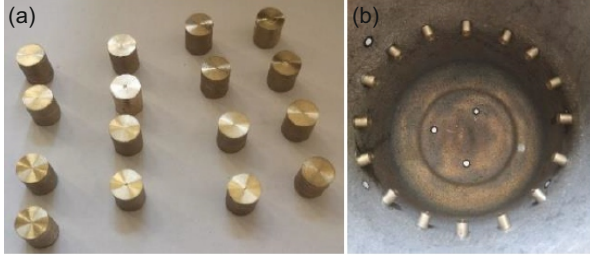


Fig. 3. Additional elements before assembly (a) and the bell tuned according to the proposed method (b).

### 3. Experimental research

In order to verify the proposed method, preliminary experimental studies were performed to document the changes in the vibration frequency in the modified system. Here, 16 cylinders, each with a radius of 5 mm and a height of 10 mm, made of the same material as the bell, were attached to the object (Fig. 3a). The tests were carried out in two stages, by mounting (Fig. 3b) the elements at two heights ( $h$ ), namely 42 mm and 142 mm, measured from the bottom edge of the bell, i.e., at the heights shown in the numerical tests as ensuring a meaningful influence of the change in the mass of the vibrating body on the values of the frequencies of the sound component.

Table II summarizes the obtained experimental values of the sound components without ( $\omega_{exp}$ ) and with the participation of additional mass elements ( $\omega_{m\_exp}$ ). Moreover, Table III provides analogous numerical values ( $\omega_{num}$ ,  $\omega_{m\_num}$ ) for the same cases.

Analyzing the obtained results (Table II) with the tendency of frequency changes shown in Fig. 2, it is surprising that there is no change in the value of the F component ( $\delta_{exp} = 0\%$ ) for the height of placing additional 16 mass elements at  $h = 42$  mm.

The explanation is the result of the vibration frequency (Table III) from the performed numerical analysis and the overall analysis of the obtained results. For this height, the comparison of the numerical and experimental results indicates differences with an error ranging from  $-0.09\%$  for the Q component to  $+0.03\%$  for F. Whereas, the comparative analysis of the vibration frequency results for the height  $h = 142$  mm is burdened with an error from  $-0.03$  for T to  $0.02$  for F.

It should be noted that the indicated heights are characterized by dissimilarity in the thickness of the bell cross-section and, consequently, a significant difference in the local mass. In addition, it should be expected that the cast material with a larger cross-section may contain internal defects in the form of unevenly distributed porosity, especially

TABLE II

Experimental values of bell sound components taking into account additional elements. Here,  $\delta_{exp} = \frac{(\omega_{m\_exp} - \omega_{exp})}{\omega_{exp}} \times 100\%$ .

SC	$\omega_{exp}$ [Hz]	$\omega_{m\_exp}$ [Hz]	$\delta_{exp}$ [%]
$h = 42$ mm / $h = 142$ mm			
H	570/570	568/569	-0.35/-0.18
F	1158.5/1158.5	1158.5/1152.5	0.00/-0.52
T	1365.5/1365.5	1362.5/1365	-0.22/-0.04
Q	1718/1718	1716/1703	-0.12/-0.87
N	2281/2281	2277/2281.5	-0.18/0.02

TABLE III

Numerical values of bell sound components taking into account additional elements. Here,  $\delta_{num} = \frac{(\omega_{m\_num} - \omega_{num})}{\omega_{num}} \times 100\%$ .

SC	$\omega_{num}$ [Hz]	$\omega_{m\_num}$ [Hz]	$\delta_{num}$ [%]
$h = 42$ mm / $h = 142$ mm			
H	589.97/589.97	588.30/588.88	-0.28/-0.18
F	1185.6/1185.6	1185.9/1179.2	0.03/-0.54
T	1413.6/1413.6	1410.3/1413.4	-0.23/-0.01
Q	1773.4/1773.4	1772.8/1757.9	-0.03/-0.87
N	589.97/2375.8	2370.8/2375.3	-0.21/-0.02

in the case of “bell” castings made in the traditional way. In view of the above, it should be stated that growth of the mass of the bell by installing additional elements inside of the bell in order to increase or decrease the frequencies of the H, F, T, Q, and N components is possible, and the final effect is predictable and depends on the mass of the additives used.

Comparing the changes that taken place in the case of numerical and experimental studies, it can be seen that the considered approach is innovative and correct, because the additional elements affect the bell tone almost in the same way in the numerical case as in the case of the real object. Naturally, there are minor deviations that can have many causes, ranging from internal casting defects to the quality of assembly of additional mass elements.

### 4. Conclusions

The paper proposes an innovative approach to bell tuning, which involves introducing additional mass elements into the system. This solution has several advantages. First, it does not directly interfere with the structure of the bell by removing material, as is the case with traditional tuning

techniques. Second, if an element is added by mistake, its position and size can be changed without damaging the bell. Third, the presented method can also be used for bells that are already incorrectly tuned or as a supplement to loss tuning in order to achieve the best possible acoustic effect.

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