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The Influence of Eccentricity and Pyramidal Error on Rotation Angle Measurement Within a 360-Degree Range

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In this study, the dual-autocollimator method was used to investigate the accuracy of the metrological calibration of multi-face polygons, focusing on the effects of pyramidal error and eccentricity. Measurements were conducted on a 12-face polygon, where mirrors positioned 90° apart were observed using two autocollimators — one as a reference and the other for measuring the yaw angle deviations as the polygon was rotated by 30°. A reading of the results for 12 angles with four repetitions was performed. Different levels of variability in the readings were observed for the mirrors. This article presents the procedure for assessing the sources and values of uncertainty in the measurement. The errors of misalignment and eccentricity of the rotary table axis and the angle standard were considered. Significant impacts of pyramidal error and eccentricity were identified.

topics: multi-face polygon, autocollimators, pyramidal error, eccentricity

1. Introduction

Measurements of rotation angle can be carried out using angular encoders. These are standard and precise devices commonly used in the industry, e.g., in systems requiring high-fidelity feedback such as *computerized numerical control* (CNC) machines, surgical robots, and other automated systems [1–6]. Regular calibration is essential to maintain the high accuracy of these devices. One method for calibrating encoders involves using a multi-face polygon with one or two autocollimators. The accuracy of this method hinges on the precise calibration of the polygon mirror, which is typically manufactured from materials like steel or quartz glass and features flat mirror faces with well-defined angles. These polygons are manufactured with 12, 20, 24, 36, or even up to 72 faces [7]. Calibration is typically performed by measuring the angles between the mirror faces using two autocollimators. This method can achieve a standard deviation of 0.3" and uncertainty of ±0.5" [7]. A variation of this technique, known as the shift-angle method [8], uses two autocollimators and a rotary table with an encoder as the reference, achieving uncertainties as low as ±0.28". While these methods are effective, they are time-consuming due to the need to reposition the autocollimators for each mirror face, which can also introduce additional errors. Some publications like [9] have demonstrated that pyramidal errors and surface flatness significantly influence the precision of angular measurements. The extent of these deviations is contingent upon the surface characteristics and the specific measurement configuration, with reported errors ranging from 0.02" to 0.5" under various experimental conditions.

Further studies have proved the relationship between measurement deviations and factors such as mirror topography, polygon eccentricity, and pyramidal error [10, 11]. Experimental analyses involving different mirrors revealed that variations in mirror geometry affect the accuracy of angular measurement. A theoretical model was subsequently developed to account for these deviations in autocollimator measurements. Despite the presence of such errors, their overall impact on measurement precision was determined to be negligible.

This paper presents a study investigating the effect of pyramidal and eccentricity errors on 360° angular measurements, especially when using a multi-face polygon with a dual autocollimator. The study employs a rigid Taylor Hobson Talyrond 100 roundness table with angular and linear adjustments to apply the two types of errors. The experiment effectively demonstrates how these errors directly impact the measurements, using a 12-face polygon under conditions identical to the actual calibration process.

2. Material and method

2.1. Calibration polygon using the two autocollimator method

Angle measurements were performed using the setup shown in Fig. 1. The setup consists of a polygon (no. 1) placed on a rotary table of a form measuring machine (no. 2) with a stable frame (no. 3). The rotary table allows for both angular (nos. 4, 5) and linear (nos. 6, 7) adjustments. Measurements were taken from two polygon mirrors positioned 90° apart, using two autocollimators (nos. 8 and 9), also set by 90° apart.

Autocollimator I (no. 8) was initially set to 0°, while Autocollimator II (no. 9) was precisely aligned to 90°, perpendicular by the polygon. As the polygon rotated in 30° increments, Autocollimator II measured the yaw angle deviation (about the z-axis) in arcseconds. Four repetitions were performed for each rotation, with the polygon completing a full 360° turn in 30° steps. Autocollimator I remained fixed at 0° at each step, while Autocollimator II recorded the deviation at 90°.

Table I and Fig. 2 present the results of the four repetitions of conducting a 12-face polygon using two autocollimators.

The result of the *yaw deviation angle* (YDA) for each position could be written as

$$\mathrm{YDA} = \frac{\sum\limits_{j=1}^{n} (o_1 - o_2)_j}{n}, \tag{1}$$

where o_1 is an indication of Autocollimator I (no. 8), $o_2 = 0$ is an indication of Autocollimator II (no. 9), and $n = 4$ is the number of repetitions.

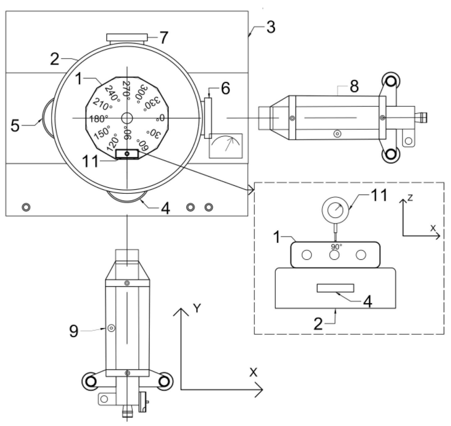

Fig. 1. The set-up for measurement of yaw deviation angle of polygon's mirrors set by 90° apart, where 1 — a 12 face polygon; 2 — rotary table (Taylor Hobson form measuring machine FMM); 3 — frame of the FMM; 4, 5 — angular adjustments; 6, 7 — linear x, y adjustments; 8, 9 — TA51-2 autocollimators; 11 — digital indicator 0.01 mm).

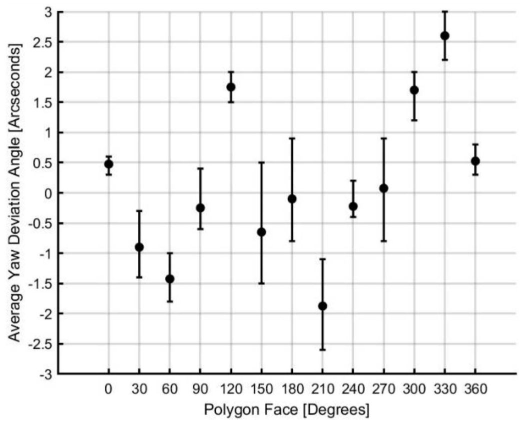

Fig. 2. Results obtained using two autocollimators and a 12-face polygon.

TABLE I

Results of the four repetitions for performing the 12-face polygon using two autocollimators.

Polygon face [degree]	Four repetitions (Ri) for measuring yaw deviation angle [arc seconds]			
	R1	R2	R3	R4
0	0.6	0.6	0.4	0.3
30	−1.4	−0.9	−0.3	−1.0
60	−1.5	−1.8	−1.0	−1.4
90	−0.3	−0.5	0.4	−0.6
120	2.0	1.5	2.0	1.5
150	0.5	−1.5	−0.4	−1.2
180	0.9	−0.4	−0.1	−0.8
210	−1.1	−2.6	−1.8	−2.0
240	0.2	−0.3	−0.4	−0.4
270	0.9	−0.2	0.4	−0.8
300	1.2	1.6	2.0	2.0
330	2.6	3.0	2.6	2.2
360	0.6	0.4	0.3	0.8

After processing the experimental data from four repetitions of rotating the polygon over 360°, we obtained results from Autocollimator II, with Autocollimator I as the reference for each 30° increment. The maximum variation observed across the four repetitions was 2 arcseconds.

To further assess the variability and stability of the setup, an experiment involving 100 repetitions was conducted with a single 90° mirror of the polygon. With the setup using a dual autocollimator system and a 12-face polygon on a rotary table, it was possible to evaluate the method's reliability and determine the maximum standard deviation as a measure of uncertainty. In this experiment, Autocollimator I (no. 8) was set to 0° as the reference mirror, while Autocollimator II (no. 9) was precisely aligned to 90° as the tested mirror, perpendicular to the polygon. As the polygon

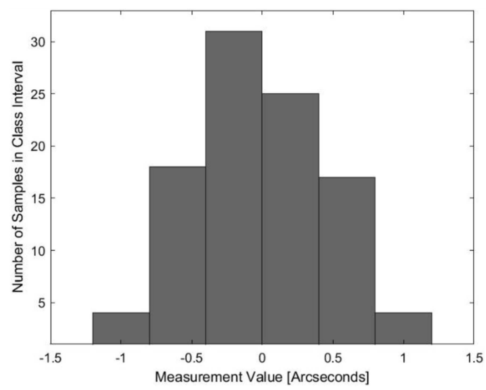

Fig. 3. Histogram of 100 repetitions on the 90° mirror of the polygon using two autocollimators method.

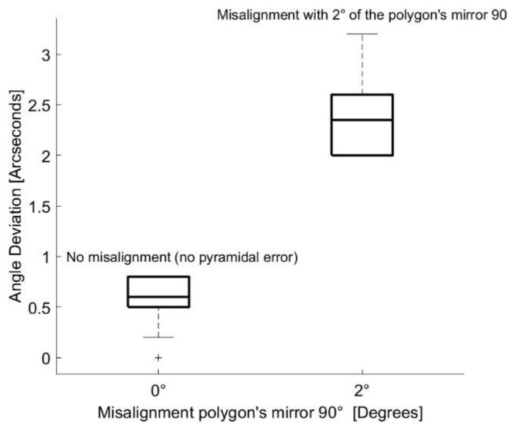

Fig. 4. Pyramidal errors of the polygon mirror at 90° measured using two autocollimators. (Box plot comparison of angular deviation in arcseconds between 0° and 2° misalignment of the polygon's mirror 90° conditions. The boxes represent the interquartile range (IQR), and the whiskers indicate the range of the data, excluding outliers.).

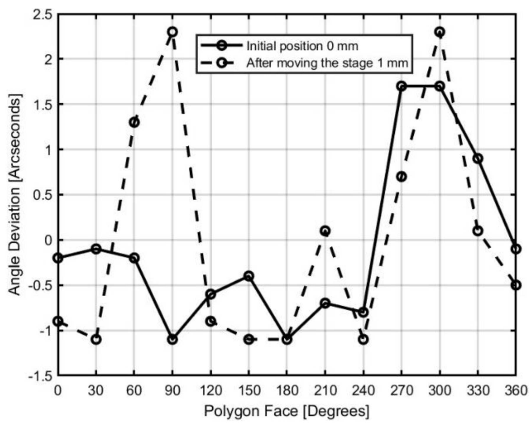

Fig. 5. Results obtained using the two autocollimators and 12-face polygon. Solid line — no linear movement (no eccentricity). Dashed line — after moving the rotary table of 1 mm in linear movement.

rotated by 360° and returned to 0° with respect to Autocollimator I, Autocollimator II measured the yaw angle deviation of the 90° mirror in arcseconds. The data in Fig. 3 are consistent with normal distribution, confirmed by the Shapiro–Wilk test, with a p-value of 0.065 being above the 0.05 significance level. This indicates with 95% confidence that the normal distribution can model the data.

The standard deviation of the measurements was calculated to be 0.45 arcseconds, and the confidence interval for the mean of these measurements is ± 0.2 arcseconds. These results are consistent with the manufacturer's specifications for the autocollimator. Thus, it can be concluded that the setup (including the rotary table, standard clamping, and ambient conditions) did not significantly affect the measurements. Additionally, increasing the number of repetitions could further reduce random factors affecting the results.

2.2. Pyramidal errors

Pyramidal error during the rotation of the polygon mirror about its axis can lead to deviations from ideal angular measurements due to misalignment, manufacturing tolerances, or mechanical imperfections [11]. Additionally, the lack of flatness and pyramidal errors of the polygon's faces, along with misalignment issues and errors introduced by the autocollimator during calibration, further complicate the process [12]. Understanding the impact of these errors is crucial for the accurate calibration of polygon mirrors, as they significantly affect measurement precision. As shown in Fig. 1, a digital gauge indicator (no. 11) was used to evaluate the effect of pyramidal error on polygon calibration and to understand how this source of error influences the measurement process. This sensor measured changes in the Pitch angle (about the x-axis), indicating a pyramidal error in this study.

Figure 1 also presents the setup to assess pyramidal error on a polygon mirror at 90° (dashed rectangle). Two autocollimators were positioned: Autocollimator I (no. 8) was fixed at 0° as a reference, while Autocollimator II (no. 9) measured the deviation. Ten repetitions of measurements were initially taken, with the polygon rotating 360° for each repetition and then returning to the same mirror for re-recording. These repetitions were performed with the rotary table leveled to ensure a consistent Pitch angle. Afterwards, a deliberate 2° misalignment was introduced using the adjustment mechanism (no. 4) shown in Fig. 1, followed by another ten readings using Autocollimator II to observe the impact of this induced misalignment.

Figure 4 presents the data obtained from the digital indicator (no. 11 in Fig. 1) alongside the measurements recorded by Autocollimator II (no. 9

in Fig. 1). These results highlight the influence of pyramidal error on the Pitch angle and underscore the importance of precise alignment in minimizing this error during polygon calibration. The impact is statistically significant, and the deviation value exceeds the accuracy requirements for the polygon.

The angular alignment of the polygon axis and the table rotation axis was achieved by adjusting the tilt of the surface on which the polygon is mounted. The goal was to minimize the readings of indicator (no. 11) by fine-tuning adjustments (nos. 4 and 5). This adjustment process involved aligning the table surface at positions (nos. 4 or 5) opposite the digital indicator. The angular adjustment was made with an accuracy of 1 arcminute based on the resolution of both the digital indicator and the adjustment mechanisms. At this level of precision, the effect of pyramidality is negligible and can be estimated by examining the variation in the readings.

2.3. Eccentricity measurements

The eccentricity error is the radial distance between the grating's axis of rotation and the center of the rotary grating, which must be accounted for during calibration [12]. The influence of eccentricity is closely related to the accuracy of the standard surface [9]. Our experiment was designed to test the effects of eccentricity up to 1 mm on the calibration of a multi-face polygon. The setup shown in Fig. 1 makes it possible to compare the measurement data obtained by Autocollimator II (no. 9) based on the position indication of Autocollimator I (no. 1). The experiment involved one complete 360° rotation in 30° steps without eccentricity, followed by a 1 mm linear displacement of the rotary table using the linear adjustment (no. 6). Figure 5 illustrates the data observed from this experiment.

Figure 5 shows that the maximum deviation error caused by a 1 mm change in the polygon position is 3.4” mm. This is a significant value, indicating a large error, mainly when this method is used to calibrate highly accurate encoders.

3. Conclusions

The measurements of the polygon angles over 360° typically show a range of less than 1 arcsecond for most angles. However, some angles exhibit variability up to twice as much. We identified the primary sources of error such as axis misalignment (pyramidal error) and mirror surface flatness issues, which are exacerbated by eccentricity. To achieve precise measurements, the parallelism of the rotation axis and the polygon axis must be aligned within one arcminute. At an axis angle deviation of 2°, the error range matches the largest observed deviations in the setup. Therefore, the adjustment process should ensure a coarse alignment within 2° and a precise alignment within one arcminute.

Additionally, we tested the effect of eccentricity by measuring the polygon mirrors using the autocollimator within a full 360° rotation with 30° increments. We then repeated the measurements after shifting the table by 1 mm linearly. This test revealed a maximum deviation of 3.4 arcseconds between the two sets of readings. This significant deviation indicates that eccentricity can introduce notable errors, particularly in high-precision rotary encoders. Such errors are assigned to specific mirrors and are systematic.

Acknowledgments

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