Selected papers presented at the 14th Symposium of Magnetic Measurements and Modelling SMMM'2023

Modeling the Anhysteretic Magnetization Curve of Anisotropic Soft Magnetic Materials

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The paper presents the results of validating the model of anhysteretic magnetization curve of anisotropic soft magnetic materials utilizing the Boltzmann distribution of magnetic domain directions. It was confirmed that the editorial mistake in the original paper presenting the concept of anisotropic anhysteretic magnetization curve was reproduced in subsequent publications. Validation presented in the paper covers an anhysteretic magnetization curve model for magnetic materials with axial anisotropy and anisotropic grain-oriented electrical steels. However, the proposed correction of the model of the anisotropic anhysteretic magnetization curve can be extended to other types of anisotropy.

topics: magnetization curve, magnetic materials modeling, magnetic anisotropy, anhysteretic magnetization

1. Introduction

The concept of an anhysteretic magnetization curve [1] is very useful for modeling the magnetic hysteresis loops of soft magnetic materials. It is widely used in developing physical models of the magnetization process [2] and for practical applications, e.g., in gyrator–capacitor models of inductive components implemented with SPICE software [3]. Moreover, the recently presented measuring procedure enables accurate experimental determination of the anhysteretic magnetization curve of cores made of soft magnetic materials [4]. For these reasons, developing efficient and accurate models of the anhysteretic magnetization curve of both isotropic and anisotropic soft magnetic materials is crucial for theoretical analyses and practical applications.

2. Model of the anhysteretic magnetization curve of anisotropic materials

The commonly used model of the anhysteretic magnetization curve of isotropic materials utilizes the concept presented by D.C. Jiles and D. Atherton [1] in 1984. In this model, atomic magnetic moments in the description of paramagnetic materials were substituted by domain magnetic moments to describe the magnetic behavior of ferromagnetic material [2]. In this case, the Boltzmann distribution of domain magnetic moments leads to the model of an anhysteretic magnetization curve described by the Langevin function [2]

$$M_{\mathrm{ah}}(H) = M_s \left[\coth\left(\frac{H_e}{a}\right) - \frac{a}{H_e}\right], \tag{1}$$

where M_s is saturation magnetization, $H_e = H + \alpha M$, H is a magnetizing field, α is quantifying the interdomain coupling, M is the total magnetization of the material, and a is given as

$$a = \frac{N\, k_{\mathrm{B}} T}{\mu_0\, M_s}, \tag{2}$$

where N is the number of domains in unit cubic volume, k_{B} is Boltzmann constant, T is temperature, and μ_0 is magnetic constant.

In successive model development presented by Ramesh et al. in 1997 [5], the anisotropy in Maxwell–Boltzmann distribution was considered, leading to the following equation

$$M_{\mathrm{ah}}(H) = M_s\, \frac{\int_0^{\pi} \mathrm{d}\theta\ \mathrm{e}^{E(1)+E(2)} \sin(\theta)\cos(\theta)}{\int_0^{\pi} \mathrm{d}\theta\ \mathrm{e}^{E(1)+E(2)} \sin(\theta)}, \tag{3}$$

where energies $E(1)$ and $E(2)$ were determined for axial anisotropy as [5]

$$E(1) = \frac{H_e}{a}\cos(\theta) - \frac{K_{\mathrm{an}}}{\mu_0 M_s a}\sin^2(\psi - \theta), \tag{4}$$

$$E(2) = \frac{H_e}{a}\cos(\theta) - \frac{K_{\mathrm{an}}}{\mu_0 M_s a}\sin^2(\psi - \theta). \tag{5}$$

For anisotropic grain-oriented electrical steels, $E(1)$ and $E(2)$ were determined as [6]

$$E\left(1\right)=\frac{H_e}{a}\cos(\theta)-\frac{K_{\mathrm{an}}}{\mu_0 M_s a}\left[\cos^2(\psi-\theta)\sin^2(\psi-\theta)+\frac{\sin^4(\psi-\theta)}{4}\right], \quad (6)$$

$$E\left(2\right)=\frac{H_e}{a}\cos(\theta)-\frac{K_{\mathrm{an}}}{\mu_0 M_s a}\left[\cos^2(\psi+\theta)\sin^2(\psi+\theta)+\frac{\sin^4(\psi+\theta)}{4}\right], \quad (7)$$

where K_{an} is the dominant part of anisotropy energy density. It should be highlighted that the editorial mistake in the original publication presented by Ramesh et al. [5] in 1997 was reproduced in subsequent publications [6]. It can be easily determined that (3) can not be reduced to the Langevin equation for average magnetic anisotropy density equal to zero. Detailed analysis of the original publication indicates that the proper form of (3) should be [7]

$$M_{\mathrm{ah}}\left(H\right)=M_s\,\frac{\int_0^{\pi}\mathrm{d}\theta\ \mathrm{e}^{\frac{E(1)+E(2)}{2}}\sin(\theta)\cos(\theta)}{\int_0^{\pi}\mathrm{d}\theta\ \mathrm{e}^{\frac{(E(1)+E(2))}{2}}\sin(\theta)}. \quad (8)$$

After the above correction, (8) can be used to model the anhysteretic magnetization of materials with axial and grain-oriented types of anisotropy.

3. Validation of the model

The results of experimental measurements, presented previously in the literature [8], were used to validate the model. The influence of axial anisotropy, both parallel and perpendicular to the magnetization axis, was presented by G. Herzer [9]. The magnetic hysteresis loops of FINEMET $Fe_{73.5}Cu_1Nb_3Si_{13.5}B_9$ nanocrystalline alloy were measured after annealing in the magnetic field. As a result, the soft magnetic alloy with parallel anisotropy $K_{||}$ and two alloys with perpendicular anisotropy and values roughly estimated at $K_{1\perp}$ = 6 J/m^3 and $K_{2\perp}$ = 20 J/m^3 respectively, were produced [9]. Magnetic hysteresis of produced ring-shaped samples with axial anisotropy was measured in quasistatic conditions with a hysteresigraph system at room temperature [9].

The influence of grain-oriented anisotropy on the magnetic hysteresis loop of 0.30 mm-thick lamination of high-permeability grain-oriented electrical steel (with 3% silicon content) was presented by F. Fiorillo et al. [10]. Measurements were carried out at the Epstein frame according to the technical standard [11]. The anisotropy of grain-oriented electric steel was estimated at K_{GO} = 100 J/m^3.

TABLE I

The parameters of the anhysteretic magnetization curve of soft magnetic material with parallel and perpendicular axial anisotropy.

Parameter	Parallel	Perpend. 1	Perpend. 2
M_s [A/m]	9.985×10^5		
α	10^{-6}		
K_{an} [J/m^3]	380.7	6.147	9.952
a [A/m]	0.653	0.452	7.754

TABLE II

The parameters of the anhysteretic magnetization curve of soft magnetic material with grain-oriented anisotropy in rolling (RD) and transverse (TD) direction.

Parameter	RD	TD
M_s [A/m]	1.435×10^6	1.077×10^6
α	6.179×10^{-6}	
K_{an} [J/m^3]	22.42	
a [A/m]	22.415	24.056

The identification of the parameters of the anhysteretic loop was carried out during the optimization process. For the modeling, two assumptions were taken:

1. the anhysteretic curve is located inside the magnetic hysteresis loop B–H of electric steel;
2. magnetic hysteresis is relatively small in the case of high-permeability grain-oriented electrical steel (with 3% of silicon content) measured by F. Fiorillo et al. [10].

A target function F for the optimization was the sum of squared differences between the model and experimental results,

$$F=\sum_{i=1}^{n}\Big[B_{\mathrm{an\ model}}(H_i)-B_{\mathrm{meas}}(H_i)\Big]^2, \quad (9)$$

where $B_{\mathrm{an\ model}}(H_i)$ was the result of modeling and $B_{\mathrm{meas}}(H_i)$ was the result of measurements — both for the value of magnetizing field equal to H_i. The values of flux density of the anhysteretic curve $B_{\mathrm{an\ model}}(H_i)$ were calculated for given values of magnetizing field H_i both during the increasing and decreasing of the magnetizing field H.

The differential evolution optimization algorithm [12] was used in the model parameters identification process. The differential evolution algorithm is robust on local minima and enables an efficient identification process. Calculations were performed with the Octave software [13]. For the calculations of integrals in (8), the Gauss–Kronrod quadrature method [14] was utilized.

The results of modeling of anhysteretic magnetization curves for soft magnetic materials with both axial and grain-oriented anisotropy are presented

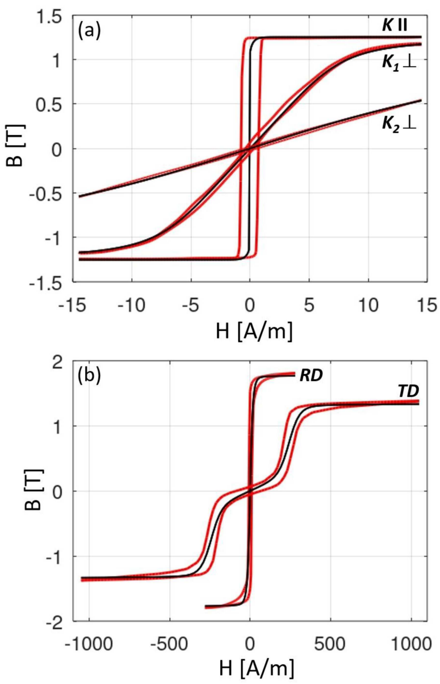

Fig. 1. Results of modeling of anhysteretic magnetization curves of soft magnetic materials: (a) with axial anisotropy, perpendicular $K_{\perp}$ and parallel $K_{||}$ to the magnetizing field H, (b) with grain-oriented anisotropy K_{GO} in the rolling direction (RD) and transverse (TD) direction. Results of modeling — black line, results of $B(H)$ loops measurements — red line.

in Fig. 1, whereas the model parameters of the anhysteretic curves are presented in Tables I and II for the above types of anisotropy, respectively.

The presented results clearly indicated that the model proposed by (4)–(8) very well reproduces the character of the anhysteretic magnetization curve of soft magnetic materials for both axial and grain-oriented anisotropy. This fact is especially important for the grain-oriented electric steel magnetized in the transverse direction, with its sophisticated shape of anhysteretic magnetization curve.

It should also be highlighted that the presented model enables accurate calculation of axial average anisotropy energy density K_{an} for perpendicular axial anisotropy. This good agreement is confirmed by the equation presented by G. Buttino [15]

$$K_{\mathrm{an}} = \frac{B_s^2}{2\mu_0\mu_r}. \tag{10}$$

On the other hand, it was observed that the calculated saturation magnetization M_s for electric steel is different in the rolling direction (RD) than in the transverse direction (TD). This phenomenon can be explained by the fact that in an anhysteretic curve model, saturation magnetization $M_s$should be considered technical saturation, not physical saturation [16].

4. Conclusions

The modeling results confirm that the corrected Maxwell–Boltzmann distribution-based model very well reproduces the character of a hysteretic curve for magnetic materials with axial and grain-oriented anisotropies. This good agreement was confirmed on the basis of experimental results presented previously in the literature.

However, a detailed analysis of achieved model parameters indicates that the physical background of the proposed model of the anhysteretic magnetization curve of anisotropic soft magnetic materials needs development and explanation. This explanation is especially necessary in the area of saturation magnetization of grain-oriented electrical steels with rolling direction and transverse direction anisotropy, as well as in the case of materials with axial anisotropy parallel to the magnetizing field direction.

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