ACTA PHYSICA POLONICA A No. 1 Vol. **146** (2024)

Selected papers presented at the 14th Symposium of Magnetic Measurements and Modelling SMMM'2023

The Concept of Heat and the Hysteresis Loop: The Evolution of the Losses Models

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Doi: 10.12693/APhysPolA.146.34

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The origin of iron losses in ferromagnetic materials is commented on, starting with the definition of heat. The different possible dissipative mechanisms inside a hysteresis curve, which originate heat, as well as its relationship to the magnetic Barkhausen noise, are discussed in detail. The loss separation model is better explained by using the concept of heat, especially to understand losses when eddy currents are small (at very low frequencies).

topics: losses, hysteresis, heat, dissipative processes

1. Introduction

Since the time of Fourier [1], heat has been discussed mathematically. The laws derived by Kirchhoff and Fick are analogous to Fourier's law of heat transmission. Nowadays, heat is described essentially as "jumping atoms." In other words, heat is explained as kinetic energy. Increasing temperature manifests itself in an increasing "jumping frequency" of atoms.

Atoms were a controversial subject in the XIX century. It was only in the XX century, after the study of the Brownian movement by Einstein, Smoluchowski, and Perrin, that the concept of atoms was widely accepted. This also influenced the way heat was defined. As the definitions of heat in the XIX century avoided mentioning the controversial atoms, the concept of heat in the XIX century was not very well formulated.

Here, the evolution of loss models over time is discussed, starting with the earlier XIX-century models of Heaviside [2] and J.J. Thomson [3]. As the area of the hysteresis loop is heat, the concept of atoms is important to understand the different dissipative processes that may happen inside the hysteresis cycle, as well as the loss separation. Epstein — the inventor of the Epstein frame — used loss separation as early as 1907 [4]. Anomalous losses were discussed already in the 1930s by Legg, under the name "residual" instead of "anomalous" [5]. The name "residual losses" persists to this day for soft ferrites. Another relevant development is Prigogine's principle of minimum energy production. Thus, domain walls can be understood as "dissipative structures" according to Prigogine's theory [6]. As defined by Prigogine, self-organization is possible without violating the 2nd law of thermodynamics. Domain walls are thus interpreted as structures with self-organization.

In the present study, the different possible dissipative mechanisms inside a hysteresis curve (i.e., heat) are discussed in detail, as well as its relationship with the magnetic Barkhausen noise (MBN), including the mathematical relationship between MBN and hysteresis.

2. The concept of heat

The area of the hysteresis loop is heat. Therefore, a good knowledge of what heat is is relevant for understanding losses. Here, the concept of heat will be briefly reviewed, with special attention paid to historic developments.

The discussion about heat has a long history. It can be traced back to Empedocles of Agrigento, who correctly concluded the existence of air and vacuum [7]. However, the definitions of Empedocles about water, fire, and earth were incorrect. Only after Lavoisier [8], it becomes clear that water is H_2O [9], and that fire is the result of combustion, a reaction involving oxygen. And about the earth? It seems that everything else was described by Empedocles as "earth."

Evidently, in the Empedocles theory, atoms are missing. Atoms were a controversial subject even at the start of the XX century. The concept of atoms by Demokritos was essentially mathematical: When solving an integral $f(x) = \int \mathrm{d}x$, $\mathrm{d}x$ can not be zero. Thus, dx should be "non-divisible" or "a-tom." This was important for the Archimedes exhaustion method, and the modern version of it is named a Riemann sum [10].

One of the most difficult concepts in science is energy [11]. Energy is never absolute, it always needs a reference. Energy always is a variation, ΔE. Thus, energy is not positive or negative. Instead of writing $E = m\,g\,d$ (m is the mass, g is the gravity acceleration, d is the height), it is more accurate to express $\Delta E = mg\,\Delta d$.

The XX century started with two equations [12], i.e.,

$$E = k_{\mathrm{B}}T, \tag{1}$$

$$E = hf, \tag{2}$$

where T is the temperature, k_{B} is the Boltzmann constant, f is the frequency, and h is the Planck constant. The Boltzmann constant first appeared in the paper by Planck [13, 14]. In two simple expressions, i.e., (1) and (2), one makes use of the Planck constant, and another makes use of the Boltzmann constant.

In (1), what is implicit is the concept of atoms. According to (1), the temperature corresponds to the energy. Atoms, however, were a subject of intense discussion in the early XX century [15], and Mach used to ask "did you see one?" Much later, Binnig saw atoms [16].

As aforementioned, atoms were a very controversial subject throughout the XIX century. It was only in the XX century, after the study of the Brownian movement by Einstein, Smoluchowski, and Perrin, that the concept of atoms was widely accepted. This fact turned out to have an impact on the definition of heat. As the definitions of heat in the XIX century avoided mentioning controversial atoms, the concept of heat was not perfectly formulated. For example, Maxwell's 1872 definition of heat [17] is obsolete because Maxwell avoided controversial atoms at that time [18]. On the other hand, (1) translates heat into "jumping atoms" (or oscillating atoms). Maxwell equations were used long before atom theory was accepted, for example by Heaviside to understand losses and the skin effect [19, 20].

From chemistry, the idea of an isomer indirectly suggests the existence of atoms [21]. Maybe the first indirect evidence of atoms is in the Fourier equation for heat. Fourier is cited by Fick [22], which also cites Ohm [23]. Fourier profoundly influenced physicists of the XX century, including Maxwell [24] and Lord Kelvin [25]. Therefore, Ohm applied the Fourier heat equation [23] to the diffusion of electricity in a conductor.

The Fick 1st law is given by

$$J = -D\frac{\partial C}{\partial x} \tag{3}$$

for the one-dimensional case. Here, J is the flux, D is the diffusion coefficient, and C is the concentration; (3) is for the steady state.

The 2nd Fick law given by

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}\left(D\frac{\partial C}{\partial x}\right) \tag{4}$$

is valid when there is a variation of the concentration C with time. If D is independent of concentration, (4) becomes

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2}. \tag{5}$$

The diffusion coefficient D is given by

$$D = D_0 \exp\left(-\frac{Q}{RT}\right), \tag{6}$$

where D_0 is the pre-exponential factor [26], Q is the enthalpy variation, R_{gas} is the gas constant, and T is temperature. There is a relationship between R_{gas} and k_{B}, given by $R_{\mathrm{gas}} = k_{\mathrm{B}}N_{\mathrm{A}}$ (where N_{A} is Avogadro's number). Thus, the energy barrier given by (1) appears in (6). The same mathematics for solving heat problems can be used for solving the problems of atom diffusion in solids [27].

One of the last anti-atomists, Ostwald, surrendered to atom theory in 1908 [28]. Ostwald is famous in materials science due to the "Ostwald Ripening" — the phenomenon responsible for precipitation hardening in aluminum alloys [29], a method still used today for strengthening the wings of airplanes [30]. Ostwald was the 1909 Nobel Prize laureate in chemistry for catalysis [31], even with a lack of perception of the actual origin of this phenomenon.

Possibly the most relevant fact here are the dates on which the papers were published: Einstein, 1905 [32]; Smoluchowski, 1906 [33]; Perrin, 1910 [34]. This means that atoms were still debatable in 1908 [35]. Thus, other areas of science, such as for example electrical engineering, neglected the controversial atoms, especially until 1910.

Existence of atoms means the non-continuity of matter. Matter can not be treated as a continuum if atoms do exist. Besides, a complete revolution will occur in 1914 [36], with the X-ray diffraction and a series of implications. Then, the crystalline structure could be determined, and the atoms could be approximated by a sphere (an ellipsoid, in fact) to determine the crystalline structures. This is named the "rigid sphere model" in which atoms are treated as macroscopic spheres [37].

Then, after 1914, it becomes clear that "jumping atoms" store energy or temperature in the crystalline structure, as given in (1). The higher the jumping frequency, the higher the temperature. Nevertheless, heating as the Joule effect $P = RI^2$ was described much earlier, in 1840 (here P is power, I is current, and R is resistance) [38].

Noise is sound. Sound is vibration. Thus, this discussion is also useful for understanding MBN — the magnetic Barkhausen noise [39]. The noise indicates dissipative processes, namely "jumping atoms" (or oscillating atoms). In the analysis of MBN by

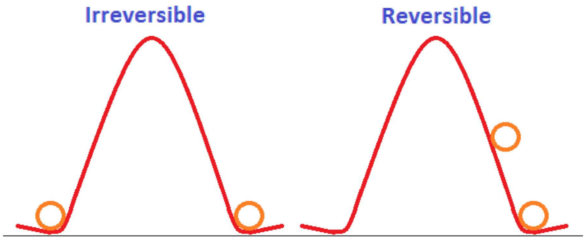

Fig. 1. Ball-hill model for reversible and irreversible processes.

Stoner [40], there is no relationship between the hysteresis curve and MBN [41]. Noise of transformers [42] is an analogous phenomenon, indicating that the loss of energy produces sound. As aforementioned, noise is sound and sound is vibration.

As an example of the similarity between the Fourier heat equation and electromagnetism, Lord Kelvin's theory (1854) for the "telegraph equation" was developed [25] in analogy with the Fourier heat diffusion law. This is shown by

$$\frac{\partial^2 V}{\partial x^2} = R\,C_p \frac{\partial V}{\partial t}. \tag{7}$$

Here, V is the voltage, and C_p is the capacitance. Heaviside introduced the electromagnetic inductance term [43]; then (7) becomes

$$\frac{\partial^2 V}{\partial x^2} = R\,C_p \frac{\partial V}{\partial t} + L\,C_p \frac{\partial^2 V}{\partial t^2}, \tag{8}$$

with L as the inductance. Materials scientists, in contrast with electrical engineers, are typically interested in concentration variation with time, as in (5), which remains in modern use. Thus, equation (7), although very similar to (5), was written differently. Nevertheless, the modern version of (7) is (8), with the Heaviside inductance term.

3. Quasi-static losses

It is important to start by separating between irreversible and reversible processes. There are both types of domain wall movement: reversible and irreversible. Also, there is domain rotation, both reversible and irreversible. Losses happen in irreversible processes.

Figure 1 uses the ball-hill analogy to illustrate the reversible/irreversible process. If the ball goes to the other side of the hill, then the process is irreversible. Figure 2 illustrates reversibility/irreversibility using potential gravitational energy [44]. In Fig. 2, if a block goes from (a) to (b) and then back to (a), then the process is reversible. However, if the block goes from (a) to (b) and then to (c), the process is irreversible. Chen [45] also uses the ball-hill model to describe the irreversible movement of the domain wall.

Among the dissipative processes happening once in each hysteresis cycle, resulting in the hysteresis losses component P_h, the following can be listed [46]:

(i) Irreversible rotation of domains,
(ii) Irreversible domain wall displacement,
(iii) Creation and annihilation of domain walls,
(iv) Elimination of "90° closure domains" associated with magnetostrictive effects.

Microeddy currents surrounding domain walls could generate losses when a domain wall moves between different pinning sites [47]. To avoid this effect, Stewart [48] made the domain wall move very slowly in a very low-frequency experiment. Even so, losses did not become zero. The explanation was given by Shockley in the discussion at the end of the article [48]: "there will be certainly irreversible energy losses due to the fracture of Néel spikes." Therefore, Shockley, in 1951, was already indicating another dissipative mechanism, not only heating by the Joule effect according to the basic formula $P = RI^2$ [49]. Therefore, the suggestion by Becker [50], and especially by Graham [51], that the only cause for losses are microeddys [51] has no experimental basis. Besides, the noise is evidence of the dissipative process. Sound means vibration. Thus, "jumping atoms" (or oscillating atoms) are behind the occurrence of transformer noise [52, 53].

4. History of loss separation

Every theory needs to be experimentally tested. This was Lavoisier's way of ruling out the old theory of phlogiston [54]. Also, calculations by Heaviside appeared in order to explain experimental observations [55]. According to Russell (1904) [56], the eddy current loss problem was first solved by Heaviside (1884) for wires [2] and later by J.J. Thomson for sheets (1892) [3]. In 1904, the classic loss expression

$$P_{cl} = \frac{\pi^2\, f^2\, e^2\, B_{\max}^2}{6\rho} \tag{9}$$

could be found in textbooks such as that of Russell [56]. Here, B is the induction, f is the frequency, e is the thickness, and ρ is the resistivity. Graham [51] could not find the original source of (9). The separation between the eddy losses and the hysteresis losses P_h is mentioned not only in the 1904 book of Russell [56] but also by Morris (1906) [57] and Epstein (1907) [4]. Thus, at the beginning of the XX century, loss separation was commonplace.

The exact origin of the so-called "anomalous loss component" (P_a) is less known. Instead of "anomalous losses," Legg in 1936 [5] uses the term "residual losses" and finds it by the difference $P_t - P_h - P_{cl}$, identical to today's definition of anomalous losses. Therefore,

$$P_a = P_t - P_h - P_{cl}. \tag{10}$$

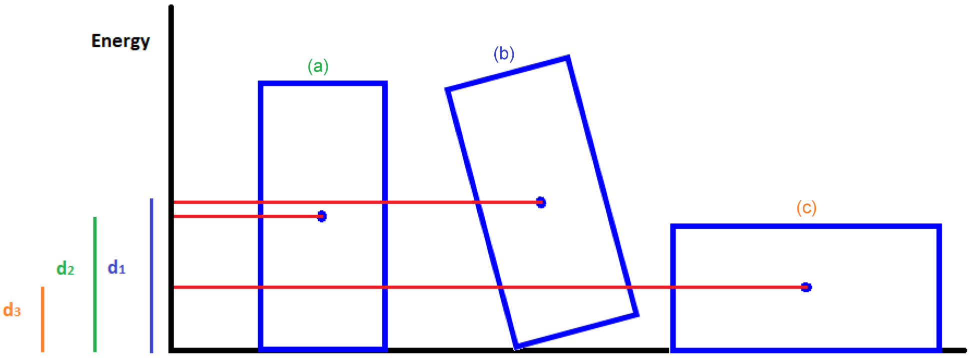

Fig. 2. Illustration of a reversible and irreversible process. The points at the center of the blocks denote the center of gravity. It should be noted that $d_1 > d_2 > d_3$ and that all heights d are defined for the same point of reference. (a) Metastable equilibrium, $E = mg\,d_2$. (b) Unstable equilibrium, $E = mg\,d_1$. (c) Stable equilibrium, $E = mg\,d_3$.

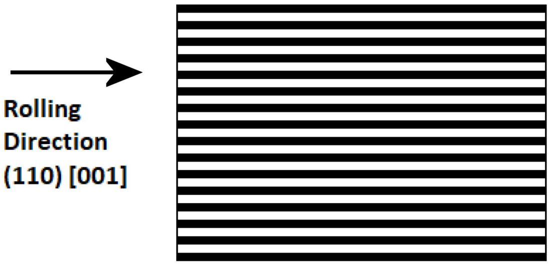

Fig. 3. Typical domain wall structure in GO steels, displaying the 180° domain walls.

The name "residual losses" remains in use for MnZn ferrites [58–62]. In (10), P_t is the total losses, and P_h is the quasi-static losses, given by

$$P_h = f \oint \mathrm{d}H\ B. \qquad (11)$$

Actually, (9) neglects the skin effect, which can be significant at high frequencies or large thicknesses [63]. The "almost undecipherable papers" of Heaviside [55] are among the first to address the skin effect [43]. Equation (9) was deduced for constant permeability, which is an assumption far from reality.

Based on the Pry and Bean model [64], the anomalous losses can be expressed by [65]

$$P_a = \frac{k}{n\,\rho}\sqrt{G_s}\, f^{3/2}\, e^2\, B^2_{\max}, \qquad (12)$$

and so $P_a \propto x/e$, where x is the distance between the domain walls; n is the number of domain walls, $n \propto 1/x$; and $c = k/n$ (n is dimensionless) [65], where k is experimental constant.

The dependence of anomalous losses as a function of frequency as $f^{3/2}$ is explained by Haller–Kramer [66, 67] and Sakaki [68], especially by observing the domain wall structures as a function of frequency [69]. Thus, (12) is similar to (9) except for frequency. The theoretical paper of Haller and Kramer [67] does not mention loss separation anywhere, but, in fact, makes use of loss separation by mixing energies due to eddy-current dissipation and domain nucleation–annihilation dissipation. The dependence of P_a with the square of the thickness has been observed experimentally [70, 71]. Equation (12) was confirmed experimentally for a series of alloys with different resistivities by Hong et al. [72].

In heavily deformed electrical steels, it was found that the anomalous losses were near zero [73], but the reported numbers were slightly negative, as noted in [74]. After that, one of the authors of the 2012 study [73] — F.J.G. Landgraf — examined those steel sheets with a micrometer and discovered that the thickness values used in the calculations of the 2012 paper [73] were slightly overestimated. After this correction was done, the anomalous losses were found to be zero! This result is in remarkable agreement with (12): If $G\,s$ is small then P_a, is near zero; or if n is high, then P_a is also near zero.

Of fundamental relevance to the loss separation model is the experimental observation that magnetic aging only affects the hysteresis losses (P_h) but not the other part, related to P_a. Thus, $P_t = P_p{+}P_a{+}P_h$, but the part $P_p{+}P_a$ is a constant under magnetic aging [49]. In 2006 [49], we were unaware that this experimental observation was reported by Epstein in 1907 [4] and by Beckley and Thompson in 1970 [75]. This result, namely that magnetic aging only affects hysteresis losses P_h, has been confirmed in several other studies [76, 77].

Loss separation in grain-oriented electrical steels has led to complex results [78], especially when considering the transverse direction [79]. For typical non-oriented electrical steels, a slight improvement of texture decreases both the hysteresis losses P_h and the anomalous losses components P_a. The complex results of Pluta [78] can be understood by the analysis of the domain wall structure in the grain-oriented (GO) electrical steels, see Fig. 3, which only displays

TABLE I

Quantitative effect of several variables on three loss components.

Variable	Classical eddy	Hysteresis	Anomalous	References
induction (B)	$B_{\max}^2$	$B_{\max}^{1.6-2.0}$	$B_{\max}^{1.5-2.0}$	still debatable
frequency (f)	f^2	f	$f^{3/2}$	[66], [67], [68]
resistivity (ρ)	$1/\rho$	–	$1/\rho$	[70]
thickness (e)	e^2		e^2	[70], [71]
grain size (G_s)	–	$1/G_s$	$\sqrt{G_s}$	[69]

TABLE II

Qualitative effect of several factors on three loss components.

Increase or improvement	Eddy current	Hysteresis	Anomalous	References
Si or Al content [%]	decreases	decreases	decreases	[72]
better texture	–	decreases	decreases	[78], [79]
number of domain walls (n)	–	increases	decreases	[64]
plastic deformation (rolling)	–	increases	decreases	[73]
applied stress (compression)	–	increases	increases	[80], [82], [83], [84]
number of inclusions	–	increases	(no effect)	[4], [49],[75], [76], [77]

the 180° domain walls. Obviously, nucleation of domains at 90° is required before any of the 90° domain wall movement, as discussed previously [79]. The domain wall displacement at 90° of the rolling direction is very difficult because, according to the Kondorsky law, the H field change or the domain wall displacement is given by $H \sim 1/\cos(\theta)$, and this gives an infinite field for $\theta = 90°$. This entails the need to form domains at 90° by rotation [79], which results in a very strange "stepped" hysteresis shape [80, 81].

The effect of stress — either compressive or tensile — depends on the magnetostrictive characteristics of the material. For grain-oriented electrical steel, compressive stress in general increases losses [80, 82], and this trend is also commonly observed for non-oriented electrical steels [83, 84]. There is a relevant observation: If the number of domain walls increases, then the pinning effect of domain walls at the surface can increase hysteresis losses [85], especially for very thin sheets.

Tables I and II summarize the predictions of the loss separation model given by (9)–(12). Since the model was presented in 2006 [69], it has resisted many experimental tests. However, the induction dependence for each P_h or P_a term is still a subject of debate and deserves to be investigated in future studies.

Electric vehicles have provoked an increasing demand for steels with better properties, high resistivity, and small thickness [86–89], and the loss separation model can be useful for improving these materials. The steel sheet thickness of the Tesla Model 3 is 0.25 mm [90]. It is difficult to improve the mechanical properties of such thin sheets. Then, a possible option for increasing both resistivity and mechanical properties is solid solution strengthening, and high manganese steels provide such a possibility [91].

5. Conclusions

Analogies between Fourier's heat law, Fick's diffusion law, and Ohm's law are discussed, with emphasis on historical developments. The concept of heat became clear after Boltzmann's ideas were accepted, but this only happened in the XX century.

This paper gives an overview of how the concept of heat evolved with time and why it is relevant for understanding the loss separation model. As "jumping atoms" (or oscillating atoms) are associated with heat, it becomes easier to understand the dependence of the hysteresis losses term P_h on the frequency f. The loss separation model was discussed in detail, emphasizing the practical applications.

Acknowledgments

The author thanks Fundação de Amparo à Pesquisa do Estado do Rio de Janeiro) (FAPERJ) and Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq).

References

[1] J.B.F. Fourier, *The Analytical Theory of Heat*, trans. A. Freeman, Cambridge University Press, 2009 (1st ed. 1822).

[2] O. Heaviside, *The Electrician*, 1884, p. 583.

[3] J.J. Thomson, *The Electrician*, 1892, p. 599.

[4] J. Epstein, *J. Inst. Electr. Eng.* **38**, 28 (1907).

[5] V.E. Legg, *The Bell Syst. Tech. J.* **15**, 39 (1936).

[6] G. Nicolis, I. Prigogine, *Self-Organization in Non-Equilibrium System* John Wiley & Sons, 1977.

[7] P. Ball, *The Elements: A Visual History of Their Discovery*, University of Chicago Press, 2021.

[8] H. Guerlac, *Hist. Stud. Phys. Sci.* **7**, 193 (1976).

[9] P. Needham, *Int. Stud. Philos. Sci.* **16**, 205 (2002).

[10] R.B. Guenther, J.W. Lee, *Aspects of Integration: Novel Approaches to the Riemann and Lebesgue Integrals*, CRC Press, Chapman & Hall, 2023.

[11] J. Coopersmith, *Energy, the Subtle Concept: The Discovery of Feynman's Blocks from Leibniz to Einstein*, Oxford University Press, 2015.

[12] M. Planck, *Ann. Phys.* **309**, 553 (1901).

[13] M. Planck, *Eight Lectures on Theoretical Physics Delivered at Columbia University in 1909*, New York Columbia University Press, 1915.

[14] M. Planck, *Nobel Lectures — Physics 1901–1921*, Elsevier Publishing Company, Amsterdam 1967.

[15] C.A. Gearhart, *Boltzmann's Atom: The Great Debate that Launched a Revolution in Physics*, David Lindley Free Press, New York 2001.

[16] G. Binnig, H. Rohrer, *Rev. Mod. Phys.* **59**, 615 (1987).

[17] J.C. Maxwell, *Theory of Heat*, 1872.

[18] C. Charalampous, *Perspect. Sci.* **29**, 189 (2021).

[19] G.S. Smith, *Eur. J. Phys.* **35**, 025002 (2014).

[20] B.J. Hunt, *The Maxwellians*, Cornell University Press, 1994.

[21] S. Esteban, *J. Chem. Educ.* **85**, 1201 (2008).

[22] A. Fick, *Phil. Mag.* **10**, 30 (1855).

[23] G.S. Ohm, The Galvanic Circuit Investigated Mathematically, 1st English trans. from 1st ed., William Francis, New York 1827.

[24] T.N. Narasimhan, *Rev. Geophys.* **37**, 151 (1999).

[25] W. Thomson, *Philos. Mag.* **7**, 502 (1854).

[26] M.F. de Campos, *Mater. Sci. Forum* **727–728**, 163 (2012).

[27] J. Crank, *The Mathematics of Diffusion*, 2nd ed., Oxford University Press, New York 1975.

[28] W. Ostwald, *Outlines of General Chemistry*, trans. T.W. White, Macmillan and co., London 1912.

[29] B.A. Pletcher, K.-G. Wang, M. E. Glicksman, *Int. J. Mater. Res.* **103**, 1289 (2012).

[30] E. Hornbogen, *J. Light Metals* **1**, 127 (2001).

[31] G. Ertl, *Angew. Chem. Int. Ed.* **48**, 6600 (2009).

[32] A. Einstein, *Ann. Phys.* **322**, 549 (1905) (in German).

[33] M. von Smoluchowski, *Ann. Phys.* **326**, 756 (1906) (in German).

[34] J. Perrin, *J. Phys. Theor. Appl.* **9**, 5 (1910).

[35] J. Renn, *Ann. Phys.* **14**, Supplement, 23 (2005).

[36] B.W. Lawrence, *Proc. R. Soc. Lond. A* **89**, 468 (1914).

[37] S. Ono, H. Satomi, J. Yuhara, *Comput. Mater. Sci.* **218**, 111959 (2023).

[38] R. de Andrade Martins, *Notes Rec.* **76**, 117 (2022).

[39] H. Barkhausen, *Phys. Z.* **20**, 401 (1919).

[40] E.C. Stoner, *Rev. Mod. Phys.* **25**, 2 (1953).

[41] L.F.T. Costa, G.J.L. Gerhardt, F.P. Missell, M.F. de Campos, *Acta Phys. Pol. A* **136**, 740 (2019).

[42] C.-H. Hsu, Y.-M. Huang, M.-F. Hsieh, C.-M. Fu, S. Adireddy, D.B. Chrisey, *AIP Adv.* **7**, 056681 (2017).

[43] C. Donaghy-Spargo, *Philos. Trans. R. Soc. A* **376**, 20170457 (2018).

[44] J.W. Cahn, in: *The Selected Works of John W. Cahn*, Eds. W.C. Carter, W.C. Johnson, Wiley 1998.

[45] C.W. Chen, *Magnetism and Metallurgy of Soft Magnetic Materials*, 1977, p. 129.

[46] S.R. Janasi, V.A. Lázaro-Colán, F.J.G. Landgraf, M.F. de Campos, *Mater. Sci. Forum* **775–776**, 404 (2014).

[47] K.H. Stewart, *Proc. Phys. Soc. A* **63**, 761 (1950).

[48] K.H. Stewart, *J. Phys. Radium* **12**, 325 (1951).

[49] M.F. de Campos, M. Emura, F.J.G. Landgraf, *J. Magn. Magn. Mater.* **304**, e593 (2006).

[50] J.J. Becker, *J. Appl. Phys.* **34**, 1327 (1963).

[51] C.D. Graham, *J. Appl. Phys.* **53**, 8276 (1982).

[52] T. Tanzer, H. Pregartner, R. Labinsky, M. Witlatschil, A. Muetze, K. Krischan, in: *2017 IEEE Int. Electric Machines and Drives Conf. (IEMDC)*, 2017.

[53] S. Taguchi, *Trans. ISIJ* **17**, 604 (1977).

[54] J.R. Partington, D. McKie, *Ann. Sci.* **4**, 113 (1939).

[55] I. Yavetz, *From Obscurity to Enigma: The Work of Oliver Heaviside 1872–1889*, 1995.

[56] A. Russell, *A Treatise on the Theory of Alternating Currents*, Vol. 1, 1st Ed. Cambridge University Press, 1904.

[57] D.K. Morris, G.A. Lister, *J. Inst. Electr. Eng.* **37**, 264 (1906).

[58] S.H. Chen, S.C. Chang, C.Y. Tsay, K.S. Liu, I.N. Lin, *J. Eur. Ceram. Soc.* **21**, 1931 (2001).

[59] O. Inoue, N. Matsutani, K. Kugimiya, *IEEE Trans. Magn.* **29**, 3532 (1993).

[60] H. Kobiki, A. Fujita, S. Gotoh, *J. Phys. IV Proc.* **7**, C1-103 (1997).

[61] W.H. Jeong, Y.H. Han, B.M. Song, *J. Appl. Phys.* **91**, 7619 (2002).

[62] A. Fujita, H. Kobiki, S. Gotoh, *J. Magn. Soc. Jpn.* **22**, Supplement, S1 (1998).

[63] P. Jabłoński, M. Najgebauer, M. Bereźnicki, *Energies* **15**, 2869 (2022).

[64] R.H. Pry, C.P. Bean, *J. Appl. Phys.* **29**, 532 (1958).

[65] M.F. de Campos, *Acta Phys. Pol. A* **136**, 705 (2019).

[66] T.R. Haller, J.J. Kramer, *J. Appl. Phys.* **41**, 1034 (1970).

[67] T.R. Haller, J.J. Kramer, *J. Appl. Phys.* **41**, 1036 (1970).

[68] Y. Sakaki, *IEEE Trans. Magn.* **16**, 569 (1980).

[69] M.F. de Campos, J.C Teixeira, F.J.G. Landgraf, *J. Magn. Magn. Mater.* **301**, 94 (2006).

[70] E.T. Stephenson, *J. Appl. Phys.* **57**, 4226 (1985).

[71] M.A. Trindade, M.F. de Campos, F.J.G. Landgraf, N.B. de Lima, A. Almeida, *Mater. Sci. Forum* **930**, 466 (2018).

[72] J. Hong, H. Choi, S. Lee, J.K. Kim, Y.M. Koo, *J. Magn. Magn. Mater.* **439**, 343 (2017).

[73] D.L. Rodrigues, J.R.F. Silveira, G.J.L. Gerhardt, F.P. Missell, F.J.G. Landgraf, R. Machado, M.F. de Campos, *IEEE Trans. Magn.* **48**, 1425 (2012).

[74] S.E. Zirka, Y.I. Moroz, S. Steentjes, K. Hameyer, K. Chwastek, S. Zurek, *J. Magn. Magn. Mater.* **394**, 229 (2015).

[75] P. Beckley, J.E. Thompson, *Proc. IEEE* **117**, 2194 (1970).

[76] K.M. Marra, F.J.G. Landgraf, V.T. Buono, *J. Magn. Magn. Mater.* **320**, e631 (2008).

[77] A.A. de Almeida, F.J.G. Landgraf, *Mater. Res.* **22**, e20180506 (2019).

[78] W.A. Pluta, *J. Magn. Magn. Mater.* **499**, 166270 (2020).

[79] M.F. de Campos, M.A. Campos, F.J.G. Landgraf, L.R. Padovese, *J. Phys. Conf. Ser.* **303**, 012020 (2011).

[80] F. Brailsford, Z.H.M. Abu-Eid, *Proc. Inst. Electr. Eng.* **110**, 751 (1963).

[81] F. Brailsford, *Physical Principles of Magnetism*, Van Nostrand, London 1966.

[82] D. Brown, C. Holt, J.E. Thompson, *Proc. Inst. Electr. Eng.* **112**, 183 (1965).

[83] K. Ali, K. Atallah, D. Howe, in: *Int. Workshop on Rare-Earth Magnets and Their Applications 14*, São Paulo 1996, World Scientific, Singapore 1996, p. 632.

[84] A. Baghel, J. Blumenfeld, L. Santandrea, G. Krebs, L. Daniel, *Electr. Eng.* **101**, 845 (2019).

[85] N. Morito, M. Komatsubara, Y. Shimizu, *History and Recent Development of Grain Oriented Electrical Steel. at Kawasaki Steel*, Kawasaki Steel Technical Report No. 39, 1998, p. 3.

[86] M. Najgebauer, J. Szczyglowski, A. Kaplon, in: *2015 Selected Problems of Electrical Engineering and Electronics (WZEE), Kielce (Poland)*, IEEE, 2015.

[87] G. Ouyang, X. Chen, Y. Liang, C. Macziewski, J. Cui, *J. Magn. Magn. Mater.* **481**, 234 (2019).

[88] N. Leuning, M. Jaeger, B. Schauerte et al., *Materials* **14**, 6588 (2021).

[89] Y. Du, R. O'Malley, M.F. Buchely, *Appl. Sci.* **13**, 6097 (2023).

[90] C.-L. Lin, H.-M. Dai, C.-H. Chao, S. Wei, C.-F. Yang, *Sensors Mater.* **35**, 4131 (2023).

[91] M.F de Campos, *Przegląd Elektrotechniczny* **2019**, 7 (2019).