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Experimental Distributions of the Reflection Amplitude for Networks with Unitary and Symplectic Symmetries

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We present the experimental study of the distributions of the reflection amplitudes $r_i = |S_{ii}|$ of the two-port scattering matrix $\hat{S}$ for networks with unitary and symplectic symmetries for the intermediate absorption strength parameter γ. The experimental results confirm the theoretical predictions obtained within the framework of the Gaussian unitary and symplectic ensembles of the random matrix theory.

topics: random matrix theory, gaussian unitary ensemble, gaussian symplectic ensemble, chaotic scattering

1. Introduction

The theory of quantum chaotic scattering in large complex quantum systems was developed more than seventy years ago [1–3]. However, controllable experimental investigations of such systems due to the effects of decoherence are still extremely difficult. Therefore, many of physical problems from the field of quantum chaos are experimentally undertaken with the help of microwave networks simulating quantum graphs [4–7].

This article shows how microwave networks can be applied to obtain the experimental results on the distributions $P(r)$ of the reflection amplitudes $r_i = |S_{ii}|$ of the two-port scattering matrix

$$\hat{S} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \tag{1}$$

for networks with unitary and symplectic symmetries. The experimental results are compared to the exact random matrix theory (RMT) solutions of this problem [8].

The concept of quantum graphs constructed from a set of vertices connected by one-dimensional quantum wires was introduced more than 80 years ago by Linus Pauling [9]. They are not only basic mathematical objects but are also indispensable in modeling physical networks in the limit where the lengths of the wires are much larger than their widths [4, 10]. Quantum graphs are invaluable tools for studying open quantum systems exhibiting chaotic scattering [11–14]. They have been used to describe a large variety of systems and models, e.g., superconducting quantum circuits [15], quantum circuits in tunnel junctions [16], and the realization of high-dimensional multipartite quantum states [17].

Quantum graphs can be simulated by microwave networks because of the formal equivalence of the Schrödinger equation describing quantum graphs and the telegraph equation of the corresponding microwave networks [4–6]. It was demonstrated that microwave networks can experimentally simulate systems whose fluctuation properties can be described by all three fundamental ensembles in RMT. In the case of the systems characterized by T-invariance, they are the Gaussian orthogonal ensemble (GOE, symmetry index β=1 in RMT) [4, 12, 18–23] and the Gaussian symplectic ensemble (GSE, symmetry index β=4) [7, 14, 24, 25]. For systems for which T-invariance is broken, this is the Gaussian unitary ensemble (GUE, symmetry index β=2) [4–6, 26–28].

It should be emphasized that the other complex quantum systems can be simulated by microwave plane billiards [29–45] and atoms excited in strong microwave fields [46–55].

2. Theoretical outline

The distribution of the amplitude of the diagonal elements S_{ii} of the scattering matrix $\hat{S}$, $P(r)$, where $r = |S_{ii}|$, is an important but very rarely studied characteristic of chaotic systems. The distribution of the reflection amplitude $P(r)$ in the GOE system and in the system with partially violated time reversal invariance was experimentally studied using microwave chaotic cavities [29, 56].

Recently, substantial progress has been made in the investigation of open chaotic systems with violated time reversal invariance (symmetry index $\beta = 2$). The distributions of Wigner's reaction K-matrix in the case of high absorption were for the first time experimentally studied in [6]. Finally, the experimental investigation of distributions of the off-diagonal elements of the two-port scattering and the Wigner's $\hat{K}$ matrices were investigated in [28]. However, the distribution of the reflection amplitude $P(r)$ has not been studied yet.

For open chaotic systems with symplectic symmetry (symmetry index $\beta = 4$), the distribution of the rescaled reflection amplitude $P(\tilde{r})$, where $\tilde{r} = \frac{r}{\langle r \rangle}$ and $\langle r \rangle$ is the average value of r, has been studied in [7].

In this article, we present the first experimental study of the distribution of the reflection amplitude $P(r)$ for an open GUE system. These results are compared with the ones obtained for the GSE system. The openness of the systems will be described by the dimensionless parameter $\gamma = 2\pi\Gamma/\Delta$, characterizing the absorption strength [8, 57], where Γ and Δ are the width of resonances and the mean level spacing, respectively.

The diagonal elements of the scattering matrix $\hat{S}$ can be expressed as $S_{ii} = r_i \mathrm{e}^{\mathrm{i}\theta_i}$, where r_i and θ_i are the reflection amplitude and the phase measured at the i-th port of the network. The relationship between the diagonal elements S_{ii}^{exp} of the two-port scattering matrix $\hat{S}^{\mathrm{exp}}$ measured directly in the experiment and the diagonal entries S_{ii} of the matrix $\hat{S}$ will be discussed later.

In the experimental investigations, quantum graphs with unitary symmetry were modeled by microwave networks with the circulators [5, 6] (see Fig. 1). The GSE microwave networks with symplectic symmetry contained two connected microwave subnetworks with unitary symmetry (see Fig. 2). The time reversal invariance violation was induced by T-shaped circulators of opposite orientation introduced at corresponding vertices. The connections between the subnetworks were realized by two phase shifters in order to maintain the signal phase difference of π (see Fig. 2) and to enforce the appearance of Kramer's doublets specific to the GSE systems [7].

3. Experiment

In this article, the distributions of the reflection amplitudes $P(r)$ are tested for the intermediate values of the absorption parameter $\gamma = 5.1 \pm 0.5$ and $\gamma = 5.6 \pm 0.2$ ($\Gamma \simeq \Delta$) for the networks with unitary

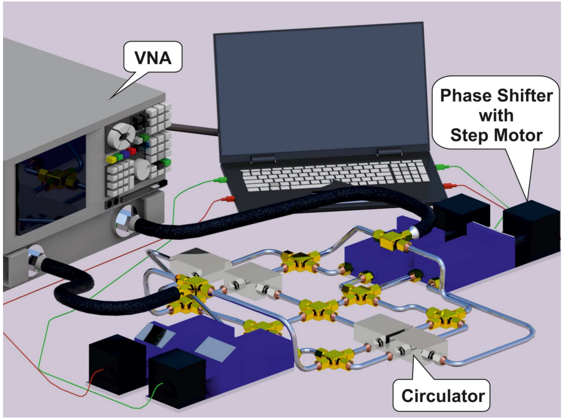

Fig. 1. The scheme of the experimental set-up for measuring the scattering matrix $\hat{S}$ of the 9-vertex microwave networks with violated T-invariance (GUE system) and absorption. The T-violation was induced using four Anritsu PE8403 microwave circulators. Absorption in the networks was caused by the internal absorption of microwave cables, 4 phase shifters, and 4 circulators.

and symplectic symmetries, respectively. In order to achieve such values of the parameter, in addition to the microwave cables, 4 circulators and 4 phase shifters were introduced into the unitary microwave network (see Fig. 1). The microwave network with symplectic symmetry contained altogether 20 1 dB attenuators, 2 circulators, and 4 phase shifters, i.e., 10 1 dB attenuators, 1 circulator, and 1 phase shifter for each connected by 2 phase shifters microwave subnetwork with unitary symmetry (see Fig. 2).

The two-port scattering matrices $\hat{S}^{\mathrm{exp}}$ of the microwave networks with unitary and symplectic symmetries required for the evaluation of the normalized two-port scattering matrix $\hat{S}$ and the distributions $P(r)$, were measured using a vector network analyzer (VNA), Agilent E8364B (see Figs. 1 and 2). The networks were connected to VNA through the leads — HP 85133-616 and HP 85133-617 flexible microwave cables. The T-violation in the unitary network and in the subgraphs of the main GSE network was induced with Anritsu PE8403 and Aerotek microwave circulators with low insertion loss, which operate in the frequency ranges $\nu \in$ (7–14) GHz and $\nu \in$ (3.5–7.5) GHz, respectively. The circulators are non-reciprocal three-port passive devices. A wave that enters the circulator through port 1, 2, or 3 exits through port 2, 3, or 1, respectively.

4. Basic formulas

For systems with GUE and GSE invariance ($\beta = 2$ and $\beta = 4$), the analytic expression for the distribution of the reflection amplitude r can be expressed by the distribution of the reflection

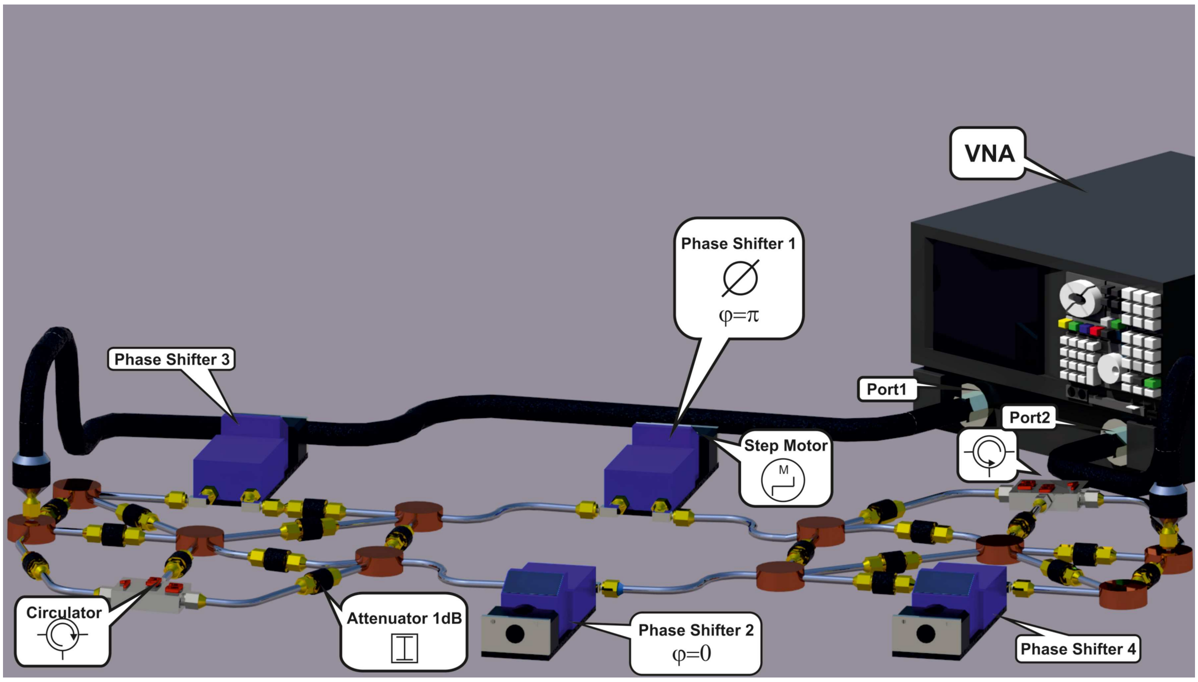

Fig. 2. The scheme of the microwave network with symplectic symmetry. The microwave network is constructed from two GUE subgraphs. Time reversal invariance violation is induced by T-shaped circulators. The subgraphs are connected by two phase shifters (No. 1 and No. 2) that induce a relative phase π. Different realizations of the GSE graph were realized by increasing the lengths of two corresponding bonds with phase shifters (No. 3 and No. 4) by the same amount. The absorption strength parameter γ in the GSE network was controlled by 20 1 dB attenuators.

coefficient $R = r^2$ given by [8]

$$P(r) = \frac{4r}{(1-r^2)^2} P_0\left(\frac{1+r^2}{1-r^2}\right). \tag{2}$$

The probability distribution $P_0(x)$ for GUE systems is given by the expression

$$P_0(x) = \frac{1}{2}\left[A^{\rm GUE}\left(\frac{\alpha}{2}(x+1)\right)^{\beta/2} + B^{\rm GUE}\right] \times \exp\left(-\frac{\alpha}{2}(x+1)\right), \tag{3}$$

where $\alpha = \gamma\beta/2$, $A^{\rm GUE} = {\rm e}^{\alpha} - 1$, and $B^{\rm GUE} = 1 + \alpha - {\rm e}^{\alpha}$.

While, in the case of GSE symmetry, the probability distribution $P_0(x)$ is defined by

$$P_0(x) = \frac{1}{2}\left[A^{\rm GSE}\gamma(x+1) + B^{\rm GSE}\right]{\rm e}^{-\gamma(x+1)} + C(x,\gamma){\rm e}^{-\gamma x}\int_0^{\gamma} {\rm d}t\, \frac{\sinh(t)}{t}, \tag{4}$$

where $A^{\rm GSE} = {\rm e}^{2\gamma} - 1$, $B^{\rm GSE} = 1 + 2\gamma - {\rm e}^{2\gamma}$, and $C(x,\gamma) = \frac{1}{2}\gamma^2(x+1)^2 - \gamma(\gamma+1)(x+1) + \gamma$.

For each realization of a microwave network, the absorption parameter $\gamma = \frac{1}{2}\sum_{i=1}^{2}\gamma_i$ was evaluated by fitting the theoretical mean reflection coefficient

$$\langle r^2\rangle^{\rm th} = \int_0^1 {\rm d}r\, r^2 P(r) \tag{5}$$

to the experimental one, $\langle r_i^2\rangle = \langle S_{ii}S_{ii}^{\dagger}\rangle$, obtained after eliminating the direct processes, where the index $i = 1, 2$ denotes the port 1 or 2. In particular, the diagonal elements S_{ii} of the scattering matrix $\hat{S}$ of a network for the perfect coupling case were obtained by removing the direct processes present in the diagonal elements $S_{ii}^{\rm exp}$ of the scattering matrix $\hat{S}^{\rm exp}$ using the impedance approach [41].

5. Results

In Fig. 3a, the experimentally obtained distribution of the reflection coefficient $P(r) = \frac{1}{2}\sum_{i=1}^{2} P_i(r)$ for the microwave networks with unitary symmetry is shown for the effective absorption strength $\gamma = 5.1 \pm 0.5$ (red dots). The results are obtained by averaging over 700 realizations of the network, which were generated by increasing and decreasing the length of different pairs of network bonds by the same amount, while keeping the total optical length of the network constant at 3.61 m. The corresponding theoretical distribution $P(r)$ calculated from (2) and (3) for the parameter $\gamma = 5.1$ is represented by a red dashed line. A good overall agreement of the experimental distribution $P(r)$ with the theoretical one is observed. For comparison, we also show the theoretical distribution $P(r)$ calculated for GSE systems with the same absorption strength $\gamma = 5.1$ (blue solid line).

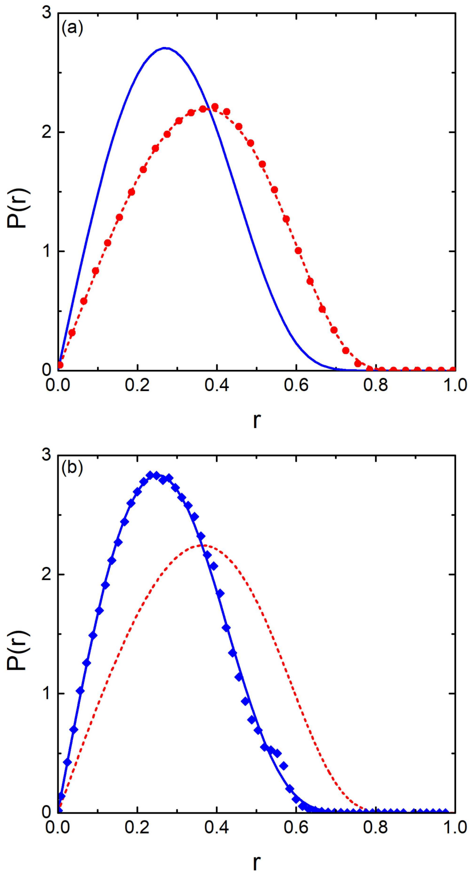

Fig. 3. (a) The experimental distribution of the reflection amplitude $P(r)$ for the microwave network with unitary symmetry for $\gamma = 5.1 \pm 0.5$ (red dots). It is compared with the theoretical ones for GUE (red dashed line) and for GSE (blue full line) calculated for $\gamma = 5.1$. (b) The experimental distribution of the reflection amplitude $P(r)$ for the microwave network with symplectic symmetry for $\gamma = 5.6 \pm 0.2$ (blue diamonds). It is compared with the theoretical ones for GSE (blue full line) and for GUE (red dashed line) for $\gamma = 5.6$.

The experimental distribution of the reflection coefficient $P(r)$ for the microwave networks with symplectic symmetry for the effective absorption strength $\gamma = 5.6 \pm 0.2$ (blue diamonds) is shown in Fig. 3b. The results were obtained by averaging over 30 realizations of the networks. The total optical length of the networks varied from 7.09 to 7.17 m. In Fig. 3b, we also show the corresponding theoretical distribution $P(r)$ (blue full line) calculated from (2) and (4) for the parameters $\gamma = 5.6$. The good overall agreement of the experimental distribution $P(r)$ with the theoretical one confirms that the procedure leading to the determination of the absorption parameter γ using (5) also works very well for the networks with symplectic symmetry. Additionally, the distribution $P(r)$ predicted for GUE systems ($\gamma = 5.6$) is shown with the red dashed line.

6. Conclusions

In conclusion, we have reported on the measurements of the distribution $P(r)$ of the amplitude of the diagonal elements $r = |S_{ii}|$ of the two-port scattering matrix $\hat{S}$ for the unitary and symplectic microwave networks for intermediate loss parameters $\gamma = 5.1 \pm 0.5$ and $\gamma = 5.6 \pm 0.2$, respectively. The experimental results were compared with the theoretical ones [8], showing good overall agreement. Thus, our experimental results validated the theoretical ones.

Acknowledgments

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