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Numerical Modeling of the Influence of Flat Solar Collector Design on Its Thermal Efficiency with the Use of Fractional Calculus

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The present study is dedicated to the issue of the operation of flat plate solar collectors with different tube cross-sections and different absorbers. Here, the flow of the working medium is analyzed in order to assess the influence of the type of tube cross-section and the type of absorber on the thermal parameters of the installation. Numerical tests were carried out on full-scale simulations with the use of Ansys Fluent software. The analysis showed that the efficiency of the system can be significantly improved by installing an appropriate absorber system. The results also show that changing the thickness and type of material the absorber is made of has a large impact on the efficiency of the collector. In this paper, to describe the phenomenon in question and to verify the obtained data, a mathematical model based on fractional order differential calculus is proposed. After comparing the flow of the working medium through collectors with different tube cross-sections with different types of absorbers, it was shown that the best results were obtained for a collector with a circular cross-section of tubes and an absorber made of copper with a thickness of 0.5 mm.

topics: flat plate solar collector, thermal efficiency, computational fluid dynamics, fractional calculus

1. Introduction

The most popular and most frequently tested and modernized type of solar collectors are flat plate collectors. There are many studies in the literature devoted to investigating the influence of the shape of the pipe on the operation of a flat plate collector. The use of triangular pipes allowed to increase the temperature of the water at the outlet. Authors of [1] compared round and elliptical pipes and found that the round pipe provides the maximum temperature of the water at the outlet with the same heat flux and inlet temperature. Numerical methods based on CFD (computational fluid dynamics) models are used to analyze phenomena occurring in solar collectors. Tests are generally performed using commercial Ansys Fluent software. The paper attempts to verify numerical data with a model based on fractional differential calculus. The growing interest in this method in various fields of engineering and science is mainly due to the possibility of obtaining accurate mathematical models of real objects, with particular emphasis on models with memory properties. Karayer et al. [2] introduced the fractional method for some aspects of quantum mechanics. Jena et al. [3] showed that the damping characteristics when solving the damped beam equation are well defined by the fractional derivative and solved the model of the large membrane vibration equation using Caputo derivatives. This description does not go into structure, but assumes a degree of heterogeneity. Using an ordinary differential equation containing a combination of left-hand and right-hand derivatives of fractional order, we obtain a model without penetration of the structure and without such a number of coefficients. Equations of this type are obtained by modifying the principle of least action and applying the principle of fractional integration by parts. The results obtained on the basis of the proposed model were used to verify the calculations carried out using the Ansys Fluent program. Based on the obtained results, it can be concluded that changes in these parameters significantly affect the thermal efficiency of the systems.

2. Fractional model

2.1. Fractional differential equations

Due to the complex character of the heat and mass flow processes, it becomes necessary for such processes to use the appropriate mathematical models describing the individual processes taking place in the course of the flow working medium. This paper used an ordinary differential equation containing a composition of left- and right-sided fractional order derivatives considered within a limited area [4].

We consider an ordinary fractional differential equation with the composition of left and right Caputo derivatives of order $\alpha \in (0,1)$ in the following form

$$ {}^{C}D^{\alpha}_{a+}\, {}^{C}D^{\alpha}_{b-} T(x) - \lambda T(x) = 0, \qquad x \in\ [a,b], \tag{1}$$

where the fractional derivatives have following forms [5]

$$ {}^{C}D^{\alpha}_{a+} T(x) = \frac{1}{\Gamma(1-\alpha)} \int\limits_{a}^{x} \mathrm{d}\tau\ \frac{T^{'}(\tau)}{(x-\tau)^{\alpha}}, \tag{2}$$

$$ {}^{C}D^{\alpha}_{b-} T(x) = -\frac{1}{\Gamma(1-\alpha)} \int\limits_{x}^{b} \mathrm{d}\tau\ \frac{T^{'}(\tau)}{(\tau-x)^{\alpha}}. \tag{3}$$

Now, (1) is supplemented by the following boundary conditions $T(a) = Ta, T(b) = Tb$.

2.2. Numerical solution

The form of the analytical solution of the considered equation is complicated and cannot be applied to the model. Therefore, the equation is discretized. We introduce homogenous grid $a = x_0 < x_1 < \ldots < x_i < x_{i+1} < \ldots < x_n = b$, where $x_i = i\Delta x$ and $\Delta x = (b-a)/n$. Then we write (1) with the boundary conditions consistent with [5] in the discrete form

$$ T_a = Ta, \qquad\qquad T = Tb,$$

$$ \sum_{k=i}^{n} \left[v(n-i, n-k) \sum_{j=0}^{k} v(k,j) T_j \right] + \lambda T_i = 0, \tag{4}$$

for $i = 1, \ldots, n-1$; where T_i is a value of the function T at the node x_i and coefficients v are given by the formula

$$ v(i,j) = \frac{(\Delta x)^{\alpha}}{\Gamma(2-\alpha)} \times \begin{cases} -i^{1-\alpha} + (i-1)^{1-\alpha}, & \text{for } j = 0; \\ (i-j+1)^{1-\alpha} - 2(i-j)^{1-\alpha} + (i-j-1)^{1-\alpha}, & \\ & \text{for } j = 1, \ldots, i-1; \\ 1, & \text{for } j = 1. \end{cases} \tag{5}$$

3. Numerical results and discussion

In addition, it was checked how the proposed calculation method translates into the accuracy of the results obtained for flow problems, taking into account heat transfer. The test case was selected to reflect the flow phenomena occurring in the pipe in the solar collector, i.e., recirculation zones, stagnation points, and interactions between vortices formed around pipe bends. The simplicity of the selected configurations allowed for the preparation of body-fit meshes that were needed for the calculations performed using the Ansys Fluent code. The calculations performed in the Ansys Fluent program were conducted using the second-order discretization method on tetrahedral meshes fitted to the body, accurately reflecting the shapes of the objects. The results obtained from the Ansys Fluent code with the proposed calculation method and the literature numerical results [6] practically overlap (see Fig. 1), which means that their accuracy is very good.

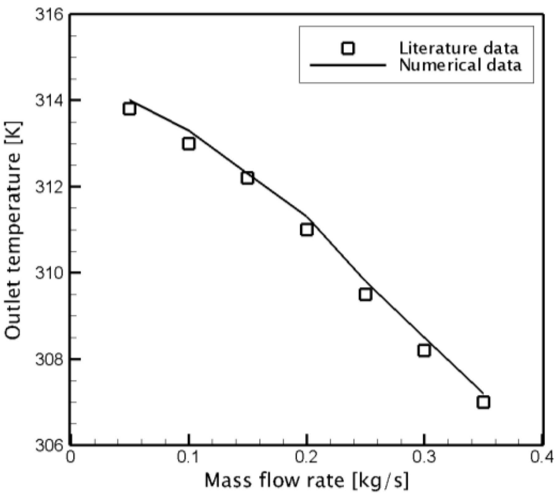

Fig. 1. Comparison of the results obtained using the Ansys Fluent with the literature data [6].

3.1. Influence of tube flattening on the thermal efficiency of collectors

In order to investigate the influence of the tub cross-section shape on heat transfer in flat plate solar collectors, three models with different tube shapes were designed. All configurations were simulated under constant Reynolds number, inlet fluid temperature, and solar radiation.

Figure 2 shows the changes in water temperature along the tube. The collector with a circular cross-section tube had the best performance as it exhibited the lowest velocities, which reduced pressure drop and improved ambient heat transfer to the fluid.

3.2. Comparison of the results of the numerical model and the mathematical model

In this part, a comparison of the proposed model with the results of numerical calculations using the Ansys Fluent program is presented. As shown in Fig. 3, for the circular cross-section of the tube, the results based on the mathematical model differ only slightly from the results of the numerical simulation. We can see that most of the deviations were at a similar level.

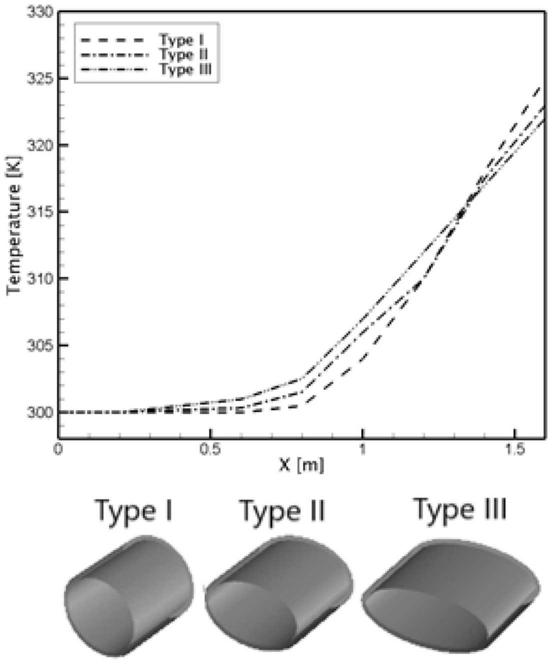

Fig. 2. Temperature profiles in the three tubes under consideration.

Depending on the measuring point, the results are overestimated and underestimated in relation to the numerical data. These fluctuations did not occur regularly, and therefore, their origin could not be identified. Generally, when comparing the numerical data with the results of the fractional model, it can be noticed that the differences are small. The method proposed in the paper allows for the modeling of actual temperature changes inside the collector tube and for the analysis of the temperature distribution inside the tubes.

3.3. Thermal efficiency

The further part of the work focuses on the calculations of the efficiency of collectors with selected pipe cross-sections. The round tube collector (Type I) has proven to be the most efficient collector type. In fact, the efficiency of the collector is reduced by the greater flattening of the cross-section. This is because convective heat transfer is very low for non-circular ducts. The round tube collector is 6.5% and 13% more efficient than Type II and Type III elliptical collectors, respectively. For further research, the collector with the best efficiency (Type I) was selected, and it was analyzed whether and what impact the thickness of the absorber and the material it is made of have on the thermal efficiency. The efficiency of the collector increases with the thickness of the absorber, e.g., when the thickness is increased from 0.1 to 0.6 mm, the efficiency increases by up to 15%, which results from the greater resistance with a greater thickness of the absorber and, thus, lower heat transfer. One of the most important components of a solar collector is the absorber plate, which absorbs the sun's energy and transfers the heat to the pipes and fluid. Absorber plates made of copper, aluminum, and steel with different thermal properties were compared, and the results are presented in Fig. 4. The efficiency of the copper absorber is 3.4% and 35% higher than that of the aluminum absorber and the steel one, respectively.

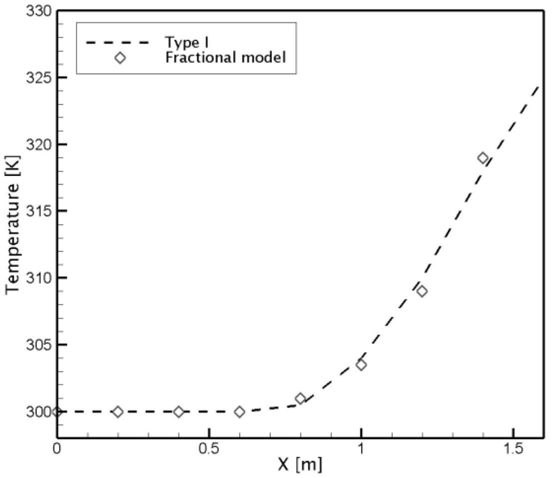

Fig. 3. Comparison of the numerical solution of Ansys Fluent with numerical solution of (1).

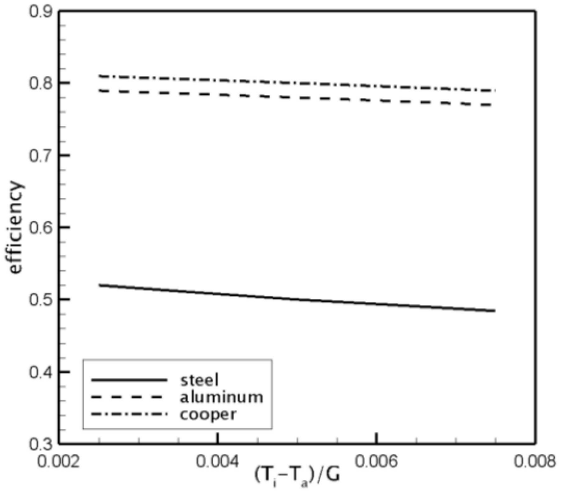

Fig. 4. Collector efficiency of various absorber materials.

4. Conclusions

A three-dimensional numerical simulation was carried out to investigate the efficiency of the flat plate solar collector. Commercial Ansys Fluent software was used to numerically solve the equations of fluid flow and heat transfer. The results showed that the collector with circular ascension tubes is more efficient than the collectors with the elliptical cross-section of the tubes. It was shown that Type I section collector had the best efficiency, where the fluid reached temperatures higher up to 324.8 K at the exit of the tube and achieved an efficiency of 62%, higher than Type II and Type III, which showed an efficiency of 58 and 55%, respectively. Moreover, the thermal efficiency of the collectors significantly changed with the change of the absorber material (the best efficiency for copper and aluminum). The thickness of the absorber also had a great influence, having a significant effect up to a thickness of 0.6 mm. The obtained results of the temperature distribution for the selected case (Type I) correspond to those estimated by the mathematical model. In the obtained distributions, as compared to (1), the differences in temperature values were consistent with the results estimated with the mathematical model. The use of fractional differential equations as a mathematical description of a complex system has the advantage that it does not require detailed information about the analyzed system. The results presented in the paper show that the proposed model describes the temperature profiles well.

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