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Influence of Meta-Atom Geometry on the Occurrence of Local Resonance Regions in Two-Dimensional Finite Phononic Structures

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In this work, the influence of different cross-sections of meta-atoms and their distribution on the occurrence of local resonance regions in inter-meta-atomic spaces of finite phononic structures was investigated. Software based on the Mathematica package was designed and implemented using the finite difference algorithm in the time domain to simulate mechanical wave propagation in phononic structures. Then, for the recorded time series from the inter-meta-atomic spaces, resonant frequency distributions were determined using Fourier transforms, and an analysis of the differences in frequency distributions depending on the location of the inter-meta-atomic space was carried out.

topics: finite-difference time domain (FDTD), discrete Fourier transform (DFT), local resonant regions, phononic structures

1. Introduction

In the realm of phononic structures, the study of how meta-atom geometry influences acoustic wave filtering properties has emerged as a critical research focus. Researchers are keenly interested in harnessing the controllability of such structures. Much of the attention in this field has traditionally been directed toward understanding phononic band gaps (PhnBG), which represent frequency bands where the passage of acoustic waves through a crystal is prohibited. This area of study has seen significant contributions, as evident in works such as [1–6], which delve into the characteristics and properties of these band gaps. Furthermore, researchers have delved into the geometry and arrangement of scatterers within phononic crystals. By altering the shape and layout of these scatterers, they aim to achieve specific properties within the PhnBG, including its width or blocking of a particular frequency band. For instance, research has examined the influence of scatterer orientation on tuning acoustic bands in two-dimensional phononic crystals [7], used numerical methods to explore tunable acoustic bandgaps [8], and even developed analytical models for tuning rods within phononic crystals to attenuate waves at particular frequencies [9]. The concept of active phononic crystals (APC) has also been a subject of interest, as highlighted in studies [10, 11]. APCs are a specialized class of phononic crystals that incorporate active components or materials to actively control and manipulate the propagation of acoustic or mechanical waves within the structure. Unlike traditional (passive) phononic crystals, which rely solely on the inherent properties of the crystal's structure to control the flow of waves, active phononic crystals use external energy sources or active materials to modify the wave properties in real time. These studies have shown the potential of active phononic crystals in controlling the propagation of waves at specific frequencies, offering the possibility of serving as directional mechanical filters. Another area of exploration has focused on understanding how factors such as the filling factor, lattice constant, and the shape and type of meta-atomic material affect the transmission of mechanical waves in quasi-two-dimensional phononic structures [12]. Such investigations provide crucial insights into designing structures with desired wave properties. In supporting these investigations, numerical methods, particularly the finite-difference time-domain (FDTD) method, have played a pivotal role. FDTD is widely used to simulate wave propagation within periodic acoustic structures, making it a valuable tool for understanding and analyzing the characteristics of phononic crystals [2, 13–16]). In sum, the research surrounding the influence of meta-atom geometry in phononic structures is a multifaceted field, covering band gaps, scatterer properties, active control, and numerical simulations. These efforts are fundamental to advancing our knowledge and capabilities in designing phononic structures with a wide range of practical applications.

In this paper, by performing a series of calculations, we demonstrate how the spatial geometry of meta-atoms influences the resulting resonance regions localized in between them. For this reason, we constructed a special arrangement of meta-atoms, distributed evenly in part of the computational domain, and changed their geometry simultaneously, leaving their spatial location unchanged. The change in geometry was very limited and came down to changing the outer edge of the meta-atom while maintaining the distances between the centers of meta-atoms and their external dimension (radius). This approach allowed us not only to study the frequency response of both kinds of structures, but also made it possible to provide a direct comparison.

2. Computational details

In Fig. 1, we present a schematic representation of the calculation domain. The simulation was naturally divided into two parts: (i) the part in which the meta-atoms were rods made of polylactide (PLA) plastic ($\rho_r = 1240$ kg/m^3 and $V_r = 2220$ m/s) immersed in air ($\rho_a = 1.29$ kg/m^3 and $V_a = 331.45$ m/s) with a square cross-section and (ii) the part in which they were replaced with rods with a circular cross-section. The diameter of each rod corresponds to the side length of the square rod and equals $d = 2.75$ cm. The lattice constant (the distance between the centers of the rods) was set to 4 cm in each case, both for horizontal and vertical spacing. The simulation area is surrounded by PML layers (perfectly matched layers), which prevents the return of the acoustic wave reflected from the end of the computational domain. Through preliminary tests, it was determined that in order to absorb this unwanted reflected acoustic wave, it is necessary to use 16 PML layers with a resulting thickness of 4 cm. The simulation used a soft source located on the left side of the tested structures and marked in the drawing as SRC. This source is a continuous wave type with a sinusoidal envelope and

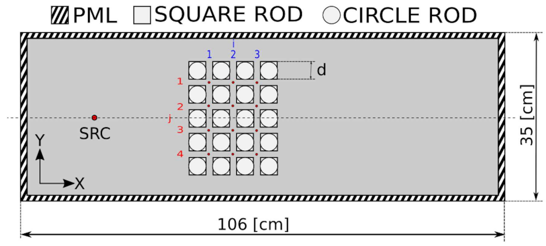

Fig. 1. Schematic diagram of the simulation domain. The square or circle meta-atoms are placed in rectangular array shifted to the left side of computational domain. The small red points indicate measurement locations P_{ij}, while the larger red point is the source of the acoustic wave.

frequency set to 1500 Hz. This frequency of the wave source causes the size of the sound wave (22 cm) to be comparable to the size of meta-atoms, ensuring their interesting interaction.

Data for further frequency analysis were collected from the area marked with red points in Fig. 1 (localised between meta-atoms, exactly in the geometric center between them). Each point in this area has been marked as P_{ij}, where index i stands for a column, while index j denotes a row of the array. According to this designation, point P_{11} is located in the upper left corner of the structure, while point P_{34} is located in the lower right corner of the structure. The overall size of the computational domain was selected in such a way as to limit the possibility of the formation of a standing wave as a result of possible interaction of the source with the boundaries of the box. Also, for this reason, the position of the whole meta-atom array is shifted relative to the center of the volumetric region. The time step in the simulation was set to $d_t = 5.4 \times 10^{-7}$ s, and the total simulation time was more than 32 ms. Data for frequency analysis were collected only after 16 ms of simulation duration, therefore reducing the influence of transient states on results.

3. Results and discussion

First, the frequency behavior of the structure was examined when the meta-atoms had a square cross-section. Figure 2 shows a series of graphs containing frequency analysis (Fourier analysis) for points $P_{11} - P_{34}$ collected during the simulation for all 9 examined spaces between the rods. For the convenience of analysis, the charts have been arranged in such a way that they spatially correspond to the location of the points from which the data for their creation was collected. For example, points P_{11}, P_{12}, P_{13}, and P_{14} are the points lying between the first and second columns of meta-atoms (counting from the source side), analogous to charts that present them. As one can see, each of the presented graphs

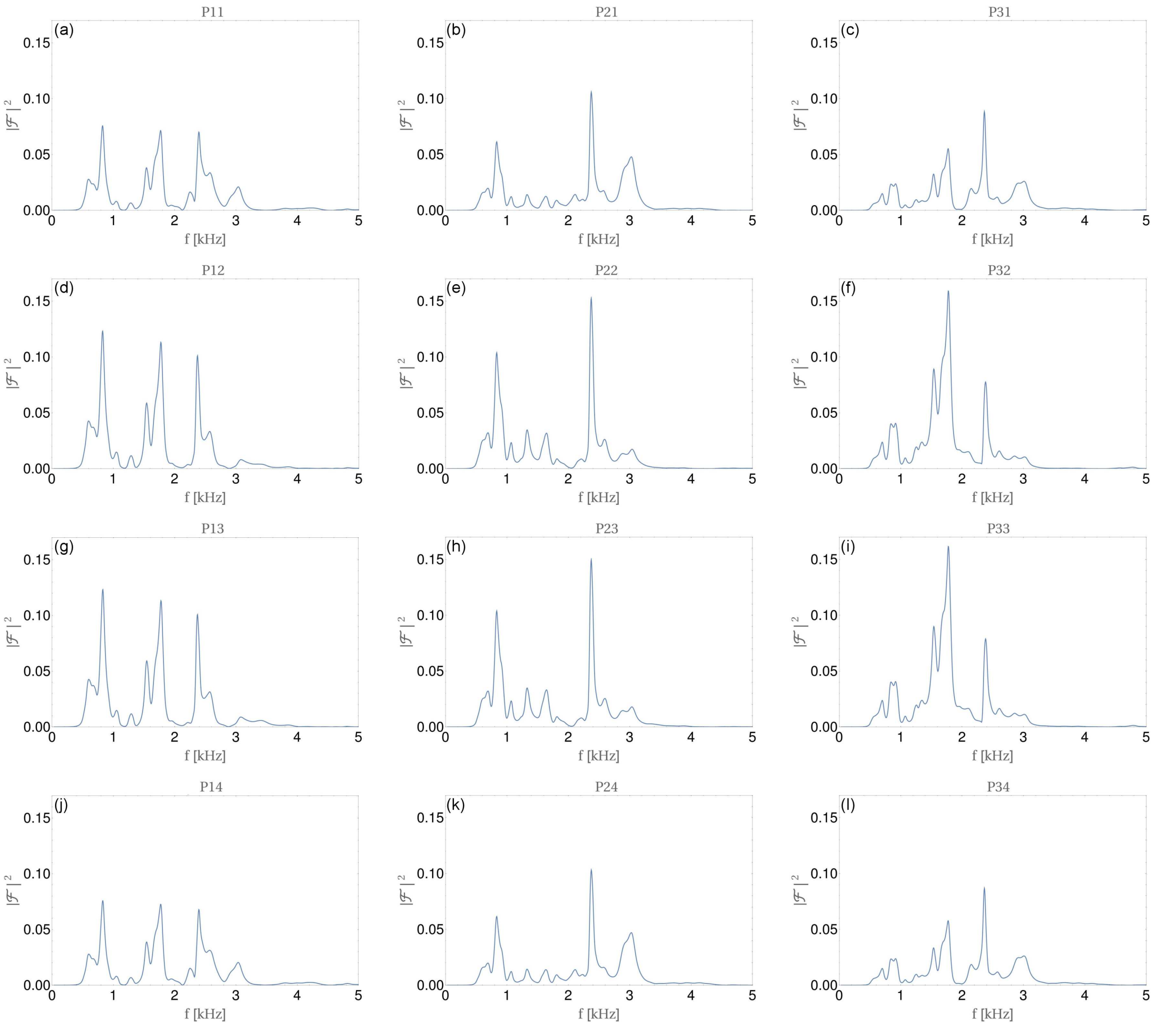

Fig. 2. Frequency analysis $|\mathscr{F}|^2$ recorded at points P_{11} to P_{34} for the structure made of square cross-section PLA rods. The source frequency was set to 1.5 kHz.

shows the presence of more than one frequency unrelated to the source frequency (1.5 kHz); in particular, the presence of the several most intense peaks can be distinguished. At the same time, in the frequency characteristics at each of the tested points, the presence of the main frequency of the sound wave emerging from the source is strongly suppressed. In the case where the measurement points lie in the same layer (column) as the point group PGC1 consisting of P_{11}, P_{12}, P_{13}, and P_{14}, group PGC2 — consisting of P_{21}, P_{22}, P_{23}, P_{24}, and the last group PGC3 (points $P_{31} - P_{34}$), a characteristic feature is the occurrence of the same frequencies within the same group with a simultaneous change in the relative intensity of the maxima. Detailed analysis also reveals that similarities in both the frequency distribution and the relative intensity of peaks occur between groups implementing point rows, e.g., between the group consisting of points P_{11}, P_{21}, P_{31} (PGR1) and the group consisting of points P_{14}, P_{24}, P_{34} (PGR4). These similarities also occur between the groups formed from points P_{12}, P_{22}, P_{32} (PGR2) and P_{13}, P_{23}, P_{33} (PGR3). Such relationships between groups of recording points suggest that the frequency behavior of the system reflects axial symmetry to some extent, assuming that the axis of symmetry is horizontal and passes through the source of the acoustic wave (marked in Fig. 1 as a central dashed line). This means that by moving transversely (along the y-axis) to the direction of acoustic wave propagation, one can easily control which acoustic frequencies are to be included in a given spectrum (and thus, in a sense, modify the frequency characteristics of the system).

In the case where rods with a square cross-section are replaced by rods with a circular cross-section, as presented in Fig. 3, characteristic symmetries are also observed, as in the first case mentioned. The mutual relations in the frequency spectrum between the points forming the columns of points (PGC1, PGC2, and PGC3) covering the positions of frequency peaks are of an identical nature, and within one column of measurement points, the differences occur mainly in the intensity of the peaks,

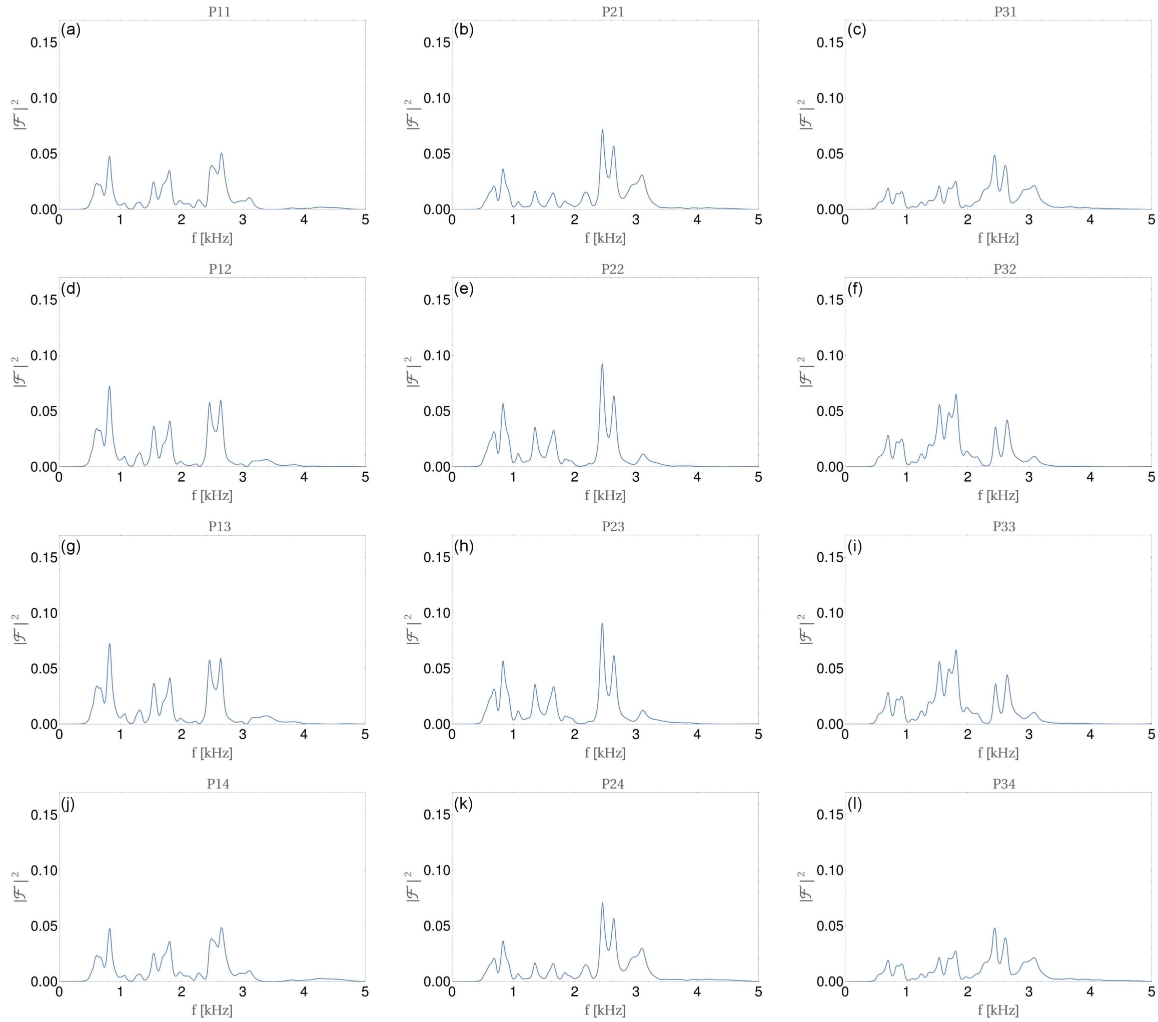

Fig. 3. Frequency analysis $|\mathscr{F}|^2$ recorded at points P_{11} to P_{34} for the structure made of circular cross-section PLA rods. The source frequency was set to 1.5 kHz.

and not in their position or number. Similarly, when considering symmetry along the horizontal axis, in this case it can be seen that the similarity between the groups of points in the rows (PGR1 and PGR4 as well as between PGR2 and PGR3) was preserved despite the change in the geometry of the meta-atoms. This observation confirms that the similarities between groups of points are related to the symmetry of the entire system, and not to the shape of the meta-atoms. As for the differences in spectrograms observed for different shapes of meta-atoms, it can be stated that these differences mainly come down to differences in the intensity of the peaks. The resonance of the acoustic wave between meta-atoms results mainly from the specific structure of the space between them. In the first case, in which the cross-section of the meta-atoms was square, the space between the meta-atoms is essentially parallel along the entire length of both sides of the meta-atom. This creates favorable conditions for the formation of a standing wave in these areas. Unlike the first case, when the meta-atoms have a circular cross-section, although the symmetry of the space between them is preserved, this space is much more complicated in terms of shape, and therefore it is much more difficult to create a standing wave in such a variable area, because standing waves generally require the presence of flat and parallel surfaces. The presence of these local resonance areas has a quite strong and positive effect on the intensity of the observed peaks in the first case examined. Although the nature of the spectra is similar in both cases, an almost twofold reduction in peak intensity is observed in the second case.

4. Conclusions

In this study, we investigated the influence of meta-atom geometry on the occurrence of local resonance regions in two-dimensional finite phononic structures. Our analysis involved two distinct scenarios, namely one where the meta-atoms had a square cross-section and another where the meta-atoms were replaced by rods with a circular cross-section. The results of our investigation offer valuable insights into the role of meta-atom shape in shaping the acoustic response of the system. When considering the case with square cross-section meta-atoms, our findings revealed a complex frequency behavior characterized by the presence of multiple frequencies unrelated to the source frequency. Additionally, the primary frequency of the source wave was significantly suppressed. The analysis also unveiled a certain degree of axial symmetry, particularly concerning the vertical axis passing through the source, allowing for the modification of the system's frequency characteristics by transversely shifting recording points. For the square cross-section meta-atoms, we observed a complex frequency behavior characterized by multiple unrelated frequencies and significant suppression of the source wave's primary frequency. Our analysis revealed axial symmetry, allowing for the modification of frequency characteristics by shifting recording points transversely. In the case of circular cross-section meta-atoms, similar symmetries in frequency spectra were evident, indicating that the system's behavior is primarily determined by overall symmetry rather than meta-atom shape. The differences between the two scenarios mainly concerned the intensity of frequency peaks, with square meta-atoms exhibiting a strong local resonance effect. In conclusion, our study demonstrates that meta-atom geometry significantly influences the frequency behavior of two-dimensional finite phononic structures. The choice of meta-atom shape can enable control over the system's frequency characteristics, making it a key factor to consider when designing phononic structures for specific applications. These findings provide a foundation for further research in the field of acoustic wave manipulation and the design of novel phononic devices.

Acknowledgments

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