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Squeezed States Generation in a Three-Mode System of Nonlinear Quantum Oscillators

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Three identical, interacting with each other nonlinear oscillators are considered. In addition, they are also externally driven by a coherent field of constant amplitude. The possibility of generating two-mode squeezed states is analyzed in this system. The two-mode principal squeeze variance is used to study the properties of the squeezed states of the system. The time evolution of this variance is analyzed, as well as the effect of the strength of the interaction between the oscillators and the damping strength on the generation of squeezed states.

topics: nonlinear oscillator, three-qubit system, squeezed states

1. Introduction

The nonclassical properties of the states can manifest themselves through various phenomena such as entanglement or squeezing. These two phenomena can often be observed in the same quantum systems [1–3]. Therefore, the squeezed states have found applications in various quantum technologies. These states are a resource in such quantum branches as quantum teleportation [4], quantum metrology [5], or quantum information processing [6, 7].

This paper will study a model of three mutually interacting nonlinear oscillators externally driven by a coherent field. The model discussed here is a source of the strongly entangled states [8]. Therefore, we expect this system also to be a source of two-mode squeezed states.

2. The model

We consider the model of three nonlinear Kerr-like oscillators (subsystems) characterized by the nonlinearity constant χ. The oscillators are mutually coupled by linear interaction in such a way that they form a chain. In addition, the boundary oscillators (the first and the last) are externally driven by a coherent field (see Fig. 1). This system can be described by the following Hamiltonian

$$\hat{H} = \sum_{j=1}^{3} \frac{\chi}{2} \left(\hat{a}_j^{\dagger}\right)^2 \hat{a}_j^2 + \sum_{j=1}^{2} \varepsilon \left(\hat{a}_j^{\dagger}\hat{a}_{j+1} + \hat{a}_{j+1}^{\dagger}\hat{a}_j\right) + \sum_{j=1,3} \alpha \left(\hat{a}_j + \hat{a}_j^{\dagger}\right), \tag{1}$$

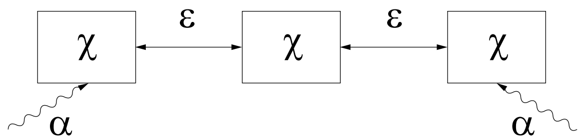

Fig. 1. The model of three interacting nonlinear quantum oscillators driven by an external coherent field.

where the operators $\hat{a}_j^{\dagger}$ and $\hat{a}_j$ are the bosonic creation and annihilation operators for the mode j ($j = 1, 2, 3$), respectively. The parameter ε describes the linear interaction between the subsystems. For simplicity, we also assume that the external excitations in modes 1 and 3 have the same strength and that two internal interactions ($1 \leftrightarrow 2$ and $2 \leftrightarrow 3$) are equal to each other. Importantly, under some conditions ($\alpha, \varepsilon \ll \chi$), our system behaves as the nonlinear quantum scissors [9–11], and the system's evolution is closed within eight three-mode states (see [8] for details).

In our studies, we assume that the system is initially in a vacuum state. Furthermore, we analyze two cases. In the first case, all damping processes are neglected, and the system's evolution is described by the unitary evolution operator $\hat{U} = \mathrm{e}^{-\mathrm{i}\hat{H}t}$ (we use units of $\hbar = 1$). Then, the wave function describing the state of the system is obtained as follows

$$|\psi(t)\rangle = \hat{U}|\psi(t=0)\rangle. \tag{2}$$

In the second case, the evolution includes a damping process. We assume that the system is damped in all three modes. Therefore, to describe the evolution of our system, we apply the master equation approach.

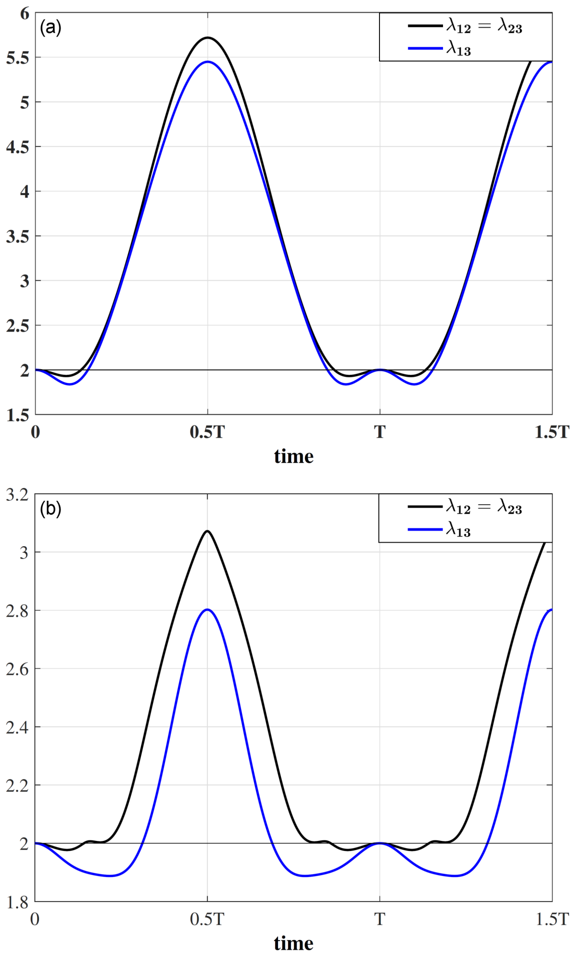

Fig. 2. Time-evolution of the parameter λ_{ij} for $\alpha = 0.001\chi$ and (a) $\varepsilon = \frac{12\alpha}{10+\sqrt{28}}$, (b) $\varepsilon = \frac{12\alpha}{10-\sqrt{28}}$. Time is measured in units of $\frac{1}{\chi}$.

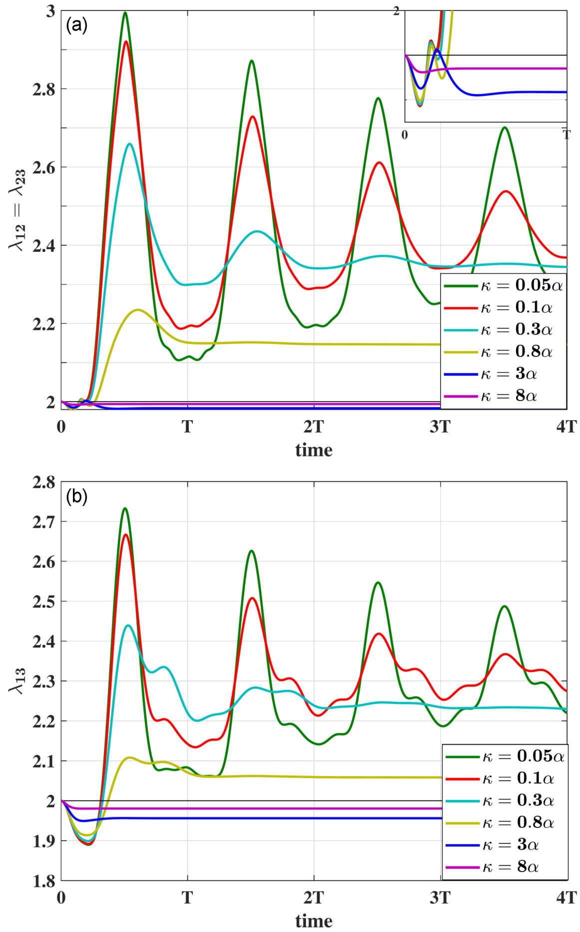

Fig. 3. Time-evolution of the parameter λ_{ij} for $\alpha = 0.001\chi$, $\varepsilon = \frac{12\alpha}{10-\sqrt{28}}$, and for various values of damping parameter κ.

Thus, the time evolution of the density operator $\hat{\rho}$ describing our model is given by

$$\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}t} = -\frac{1}{\mathrm{i}}\left(\hat{\rho}\hat{H} - \hat{H}\hat{\rho}\right) + \sum_{j=1}^{3}\left[\hat{C}_j\hat{\rho}\hat{C}_j^{\dagger} - \frac{1}{2}\left(\hat{C}_j^{\dagger}\hat{C}_j\hat{\rho} + \hat{\rho}\hat{C}_j^{\dagger}\hat{C}_j\right)\right]. \quad (3)$$

The operators $\hat{C}_j$ $(j = 1, 2, 3)$ describe the damping in modes 1, 2, and 3, respectively. These operators are defined as $\hat{C}_j = \sqrt{2\kappa}\,\hat{a}_j$. The parameter κ is the damping constant characterizing the interaction with a zero-temperature bath. We assume here that the damping constants corresponding to all modes are identical ($\kappa = \kappa_1 = \kappa_2 = \kappa_3$).

3. The results and discussion

We focus here on the possibility of generating two-mode squeezed states. To analyze the squeezing phenomena, we use the two-mode principal squeeze variance [12–14]

$$\lambda_{ij} = 2\Big[1+\langle\Delta\hat{a}_i^{\dagger}\Delta\hat{a}_i\rangle+\langle\Delta\hat{a}_j^{\dagger}\Delta\hat{a}_j\rangle+2\mathrm{Re}\langle\Delta\hat{a}_i^{\dagger}\Delta\hat{a}_j\rangle - \left|\langle(\Delta\hat{a}_i)^2\rangle+\langle(\Delta\hat{a}_j)^2\rangle+2\langle\Delta\hat{a}_i\Delta\hat{a}_j\rangle\right|\Big], \quad (4)$$

where $\langle\Delta\hat{a}_i\Delta\hat{a}_j\rangle = \langle\hat{a}_i\hat{a}_j\rangle - \langle\hat{a}_i\rangle\langle\hat{a}_j\rangle$, and i, j denotes the modes. The two-mode squeezed states are produced if the parameter λ_{ij} does not exceed two.

Figure 2 shows the time evolution of the squeezing parameter λ_{ij} for two values of the internal interaction strength corresponding to the periodic solution (see [8] for details). We see here that for all pairs of subsystems, we can observe the two-mode squeezing. Due to the geometry of the system, the variances of λ_{12} and λ_{23} are equal to each other. The degree of two-mode squeezing is weaker for pairs of modes 1–2 and 2–3 than for modes 1–3. The character of the time evolution of the two-mode principal squeezing variances depends on the values of the coupling ε between the oscillators. By changing the strength of the coupling, we can influence the time over which two-mode squeezing is generated.

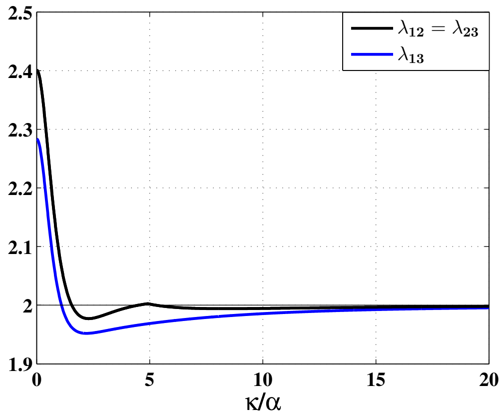

Fig. 4. Steady-state solutions for the two-mode principal squeeze variance vs the value of the damping parameter κ, where $\varepsilon = \frac{12\alpha}{10-\sqrt{28}}$, and $\alpha = 0.001\chi$.

For example, by comparing Fig. 2a and Fig. 2b, we can see that for stronger coupling, the squeezing for modes 1–3 appears for longer periods of time.

In the next step, we will analyze the damped system. In Fig. 3, we can see the time evolution of the parameter λ_{ij} for some values of the damping parameter κ. Since the results for the damped case corresponding to the two previously considered values of the excitation strength ε are very similar, in Fig. 3, we present results only for the stronger internal interaction case, where $\varepsilon = \frac{12\alpha}{10-\sqrt{28}}$. We can see that, as in the case of the system without damping and for the system with damping, we also observe the two-mode squeezing for all pairs of subsystems. For weak damping, the values of the parameters λ_{ij} oscillate. As the parameter κ increases, the oscillatory character of the time-evolution of λ_{ij} disappears.

Importantly, for weak damping, the squeezed states appear only for the initial period of the evolution. For the strongly damped system, the squeezed states can be produced during the whole evolution of the system. Moreover, it can be seen that for the strongly damped systems, we obtain stable squeezed states. Unfortunately, as the damping strength increases, we observe that the degree of stable squeezing decreases. This relationship is better illustrated in Fig. 4, which shows the steady-state solutions for various κ values. This figure also shows that the final value of the two-mode squeezing parameter λ_{ij} depends on the strength of the damping. Furthermore, the final squeezing is the strongest for subsystems 1–3.

4. Conclusions

In this paper, the system containing three nonlinear oscillators characterized by Kerr-type nonlinearity was discussed. The oscillators were coupled with each other in such a way that the system formed a chain, and the first and the last subsystems were coherently excited.

For such a system, the possibility of generating the two-mode squeezed states has been investigated. We have analyzed the time evolution of the two-mode principal squeeze variances for the undamped and damped systems. We have shown that in these two cases (damped and undamped), the analyzed system can be a source of two-mode squeezed states and that the strength of damping influences the produced squeezing. In addition, it has been shown that stable two-mode squeezed states are generated when the system is strongly damped.

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