# Modeling of $Cr^{3+}$ -Doped $Cs_2CdCl_4$ Single Crystal

R.  $KRIPAL^*$ 

EPR Laboratory, Department of Physics, University of Allahabad, Allahabad-211002, India

Received: 31.01.2023 & Accepted: 30.06.2023

Doi: 10.12693/APhysPolA.144.76

\*e-mail: ram\_kripal2001@rediffmail.com

Crystal field parameters of the  $Cr^{3+}$ -doped  $Cs_2CdCl_4$  single crystal are determined by the superposition model. The zero-field splitting parameter D is then evaluated employing microscopic spin Hamiltonian theory. The theoretical zero-field splitting parameter for  $Cr^{3+}$  in  $Cs_2CdCl_4$  single crystal at the axial symmetry site is in good agreement with the experimental value. The local distortion is considered to find the crystal field parameters.

topics: inorganic compounds, crystal fields, optical properties, electron paramagnetic resonance

#### 1. Introduction

Electron paramagnetic resonance (EPR) is useful in obtaining information about the local site symmetry and zero-field splitting (ZFS) parameters of transition ions doped in different crystals [1, 2]. It also identifies the defects responsible for the charge compensation in the system.  $Cr^{3+}$  is one of the most investigated transition metal ions with  $3d^3$  electronic configurations and  $^4A_2$  ground state for the electron paramagnetic resonance [3].

Due to the combined effect of the crystal field and the spin-orbit coupling, the principal mechanism for the zero-field splitting of the ground state of  $3d^3$  ions is the spin-orbit interaction from an admixture of higher multiplets [4]. The superposition model (SPM) has been useful in finding the crystal field parameters and spin Hamiltonian parameters for several  $3d^n$  ions [5, 6].

Inorganic metal halide perovskites like Cs<sub>2</sub>CdCl<sub>4</sub> are important due to their intriguing properties, such as tunable band gaps [7] and long carrier diffusion lengths [8]. These inorganic metal halide perovskites are investigated as promising materials for various applications, including solid-state lighting [9], photocatalysis [10], photovoltaics [11], and thermoelectric [12]. These materials, when doped with transition metal ions, are of particular interest due to their extensive applications in light-emitting display technologies, optoelectronics, lasers, solar cells, and biomedical imaging [13]. The optical and scintillation properties of undoped,  $Sb^{3+}$ -doped, and  $Tl^+$ -doped  $Cs_2CdCl_4$  single crystals were investigated. The light yield of undoped  $Cs_2CdCl_4$  is enhanced by doping with  $Sb^{3+}$  and  $Tl^+$ . The thermally stimulated luminescence characteristics are also achieved by doping with  $Sb^{3+}$  [14].

The EPR study of  $Cr^{3+}$  ion doped in various single crystals has been done due to the zero-field splitting of this ion being quite sensitive to small distortion in the lattice. EPR study of  $Cr^{3+}$  ion in  $Cs_2CdCl_4$  crystal has been reported [15]. It is important to find the site of the paramagnetic impurity  $Cr^{3+}$ , whether substitutional or interstitial, when incorporated in  $Cs_2CdCl_4$ . EPR study [15] indicated that  $Cr^{3+}$  ion substitutes at the  $Cd^{2+}$ ion site with charge compensation and uncompensation. The EPR spectra of Cs<sub>2</sub>CdCl<sub>4</sub> crystals doped with trivalent chromium suggest three distinct  $Cr^{3+}$ centers. Two of the centers have axial symmetry about the crystallographic c-axis while the third exhibits a rhombic one. The rhombic  $Cr^{3+}$  centers occupy four orientationally distinct sites in the Cs<sub>2</sub>CdCl<sub>4</sub> crystals. Two structurally distinct types of  $Cs^+$  ions surround a  $Cr^{3+}$  ion occupying a cadmium site. There are eight equivalent  $Cs^+$  (l) ions and two equivalent  $Cs^+$  (2) ions. A deficiency of one Cs<sup>+</sup> ion from either group will balance the charge discrepancy. If one of the  $Cs^+$  (1) ions is missing, the symmetry at the  $Cr^{3+}$  ion is reduced. The zero-field splitting tensor for a  $Cr^{3+}-Cs^+$  (1) center should be rhombic [15]. As any of the eight  $Cs^+$  (1) ions could be lost, there are eight possible orientations for the  $Cr^{3+}-Cs^+$  (l) center in the  $Cs_2CdCl_4$  lattice. Four orientations are distinguishable by EPR [15]. These orientations become equivalent when the magnetic field is along the crystallographic *a*-, *b*-, or *c*-axes. If one of the two  $Cs^+$  (2) ions is missing, the symmetry at the  $Cr^{3+}$  ion is only reduced to  $C_{4v}$ , and the center is axial. Only one magnetically distinguishable orientation for the  $Cr^{3+}-Cs^+$  (2) center is obtained [15]. The one axial center results from an uncompensated  $Cr^{3+}$  ion and is designated as  $Cr^{3+}(0)$  (Center I). The other axial centers can be attributed to  $Cr^{3+}$  ions which are associated with  $Cs^+$  (2) deficiencies,  $Cr^{3+}-Cs^+$  (2) (Center II) [15]. The EPR parameters, namely zero-field splitting D, the g factors  $g_{\parallel}$  and  $g_{\perp}$ , and hyperfine structure constants  $A_{\parallel}$  and  $A_{\perp}$  of Ti<sup>2+</sup> and V<sup>3+</sup> (3d<sup>2</sup>) ions in CdS crystals with trigonal symmetry have been determined by applying the high-order perturbation formulae based on the cluster approach [2]. The local lattice distortion of a Cr<sup>3+</sup>–(0) center (Center I) in a Cs<sub>2</sub>CdCl<sub>4</sub> crystal has been studied by fitting the calculated EPR parameters D,  $g_{\parallel}$ , and  $g_{\perp}$  to the experimental values with semi-SCF *d*-orbit wave functions model and perturbation loop method in a tetragonal strong field scheme [16].

In the present investigation, the ZFS parameter D for the Cr<sup>3+</sup> ion in Cs<sub>2</sub>CdCl<sub>4</sub> (Center I, II) is evaluated using crystal field parameters from the superposition model and perturbation equation. The thus obtained ZFS parameter D matches well with the experimental value. The present method gives a better match of calculated D with the experimental value compared to earlier studies [2, 16]. In addition, crystal field (CF) energy values are also computed and compared with the experimental values in the present investigation.

# 2. Crystal structure

The Cs<sub>2</sub>CdCl<sub>4</sub> crystal is tetragonal with a = 5.26and c = 16.88 Å, Z = 2 [17]. The space group is I4/mmn, and the crystal is isostructural with K<sub>2</sub>NiF<sub>4</sub> [18]. Atoms are in the following positions: 4Cs in (e), z = 0.357, 2Cd in (a), 4Cl I in (c), and 4Cl II in (e), with z = 0.149. The structure is presented in Fig. 1. The local site symmetry at the Cr<sup>3+</sup> ion is tetragonal or axial.

The crystallographic axis system CAS (a, b, c) is taken as shown in Fig. 1. A common axis system  $(a \parallel x, b \parallel y, c \parallel z)$  is taken to simplify the calculations.

## 3. Theoretical aspect

The ZFS parameter of the  $Cr^{3+}$  ion is obtained using the microscopic spin Hamiltonian (MSH) theory [19]. Taking the ZFS and Zeeman terms, the effective spin Hamiltonian for  $Cr^{3+}$  ions at axial symmetry is given by [5]

$$\mathcal{H} = D \left[ S_z^2 - \frac{1}{3} S(S+1) \right] + \mu_{\mathrm{B}} g_{\parallel} B_z S_z$$
$$+ \mu_{\mathrm{B}} g_{\perp} \left( B_x S_x + B_y S_y \right), \tag{1}$$

where  $g_{\parallel}$  and  $g_{\perp}$  are the spectroscopic splitting factors,  $\mu_{\rm B}$  is the Bohr magnetron, S is the electronic spin,  $\mathbf{B} = (B_x, B_y, B_z)$  is the external magnetic field, and D is the second rank axial ZFS parameter.

For a doped crystal, the crystal field is given as

$$\mathcal{H}_c = \sum B_{kq} C_q^{(k)},\tag{2}$$



Fig. 1. Ligands around  $Cr^{3+}$  in  $Cs_2CdCl_4$  with symmetry adopted axis system (SAAS).

where  $B_{kq}$  are the crystal field parameters in Wybourne notation and  $C_q^{(k)}$  are the Wybourne spherical tensor operators.

In  $Cs_2CdCl_4$  single crystal, the local symmetry around  $Cr^{3+}$  ions is axial. In axial symmetry, the ZFS parameter *D* is given by [5]

$$D = \frac{1}{2} \bigg[ \varepsilon \left( \left| E' \left( {}^{4}F \downarrow {}^{4}A_{2g} \downarrow {}^{4}B_{1} \right) \right\rangle \right) - \varepsilon \left( \left| E'' \left( {}^{4}F \downarrow {}^{4}A_{2g} \downarrow {}^{4}B_{1} \right) \right\rangle \right) \bigg],$$
(3)

where the free-ion  ${}^{4}F$  ground term of  $3d^{3}$  (Cr<sup>3+</sup>) ions splits in octahedral symmetry into  ${}^{4}A_{2g}$ ,  ${}^{4}T_{2g}$ , and  ${}^{4}T_{1g}$  states,  ${}^{4}A_{2g}$  being the ground state, and not split in the axial field, but its irreducible representation changed into  ${}^{4}B_{1}$ . Due to the combined action of the axial field and magnetic interactions,  ${}^{4}B_{1}$  will split further, and this is expressed by group theory as  $B_1 \otimes D^{(3/2)} \to B_1 \otimes (E' \oplus E'') \to E' \oplus E'';$ the value of splitting is 2D, where D is ZFS parameter of the ground state. Diagonalization of full Hamiltonian matrices yields the energy levels and eigen-vectors in terms of  $B_{kq}$ , B, C, and  $\xi$ , where  $B_{kq}$  are defined above; B and C are Racah parameters giving electron–electron repulsion, and  $\xi$ is the spin-orbit coupling parameter. In terms of average covalency parameter N, the Racah parameters [20–22] and the spin–orbit coupling parameter can be written, respectively, as

$$B = N^4 B_0, \quad C = N^4 C_0, \quad \text{and} \quad \xi = N^2 \xi_0,$$
(4)

where  $B_0$  and  $C_0$  are Racah parameters for free ion and  $\xi_0$  is free ion spin orbit coupling parameter. For the Cr<sup>3+</sup> ion,  $B_0 = 918 \text{ cm}^{-1}$ ,  $C_0 = 3850 \text{ cm}^{-1}$ , and  $\xi_0 = 276 \text{ cm}^{-1}$  are taken [21, 22].

Taking the values of Racah parameters ( $B = 516 \text{ cm}^{-1}$ ,  $C = 3797 \text{ cm}^{-1}$ ) found in the optical study of  $\text{Cr}^{3+}$ -doped crystal having halide (fluorine) ligands [23], the average covalency parameter N = 0.9568 is obtained employing

$$N = \frac{1}{2} \left( \sqrt{\frac{B}{B_0}} + \sqrt{\frac{C}{C_0}} \right).$$
 (5)

Fractiona	l position of	f Cr <sup>3+</sup>	ion along v	with spl	herical	coordinate	s ( $R$	$, \theta, q$	6) of	ligand	s in	$Cs_2C$	$dCl_4$	single	crystal	

Position of $Cr^{3+}$			<i>y</i> [Å]		Spherical coordinates			
(Fractional)	Ligands	x [Å]		z [Å]	of ligands			
(Practional)					$R \; [nm]$	$\theta$ [°]	$\phi$ [°]	
	$\operatorname{Cl} \operatorname{I}(1)$	0	0.5	0	0.2630	90	90	
Site: Substitutional	Cl I(2)	0.5	0	0	0.2630	90	0	
Cd1 (0, 0, 0)	Cl II(1)	0	0	0.149	0.2515	0	45	
	Cl II(2)	0	0	-0.149	0.2515	180	45	

### 4. Results and discussion

The SPM yields the crystal field parameters [24, 25] as

$$B_{kq} = \sum_{j} \overline{A_k}(R_j) K_{kq}(\theta_j, \varphi_j), \qquad (6)$$

where the coordination factor  $K_{kq}(\theta_j, \varphi_j)$  is an explicit function of the angular position of the ligand. The intrinsic parameter  $\overline{A_{\kappa}}(R_j)$  is given by

$$\overline{A_k}(R_j) = \overline{A_k}(R_0) \left(\frac{R_0}{R_j}\right)^{t_k},\tag{7}$$

where  $R_j$  is the distance of the  $d^n$  ion from the ligand,  $\overline{A_k}(R_0)$  is the intrinsic parameter of the reference crystal,  $t_k$  gives the power law exponent, and  $R_0$  is reference distance. Using the superposition model, the relation between the local structure parameters and crystal field parameters is written as [5]

$$B_{20} = -2\overline{A_2} \left( \frac{R_0}{R_{10} + \Delta R_1} \right)^{t_2} - 4\overline{A_2} \left( \frac{R_0}{R_{20} + \Delta R_2} \right)^{t_2},$$
(8)
$$B_{40} = 16\overline{A_4} \left( \frac{R_0}{R_{10} + \Delta R_1} \right)^{t_4} + 12\overline{A_4} \left( \frac{R_0}{R_{20} + \Delta R_2} \right)^{t_4},$$
(9)

$$B_{44} = 2\sqrt{70}\overline{A_4} \left(\frac{R_0}{R_{20} + \Delta R_2}\right)^{t_4}.$$
 (10)

Here, the reference distance  $R_0$  is taken as 0.255 nm, which is slightly larger than the sum of ionic radii of the Cr<sup>3+</sup> ion (0.0615 nm) and Cl<sup>-</sup> ion (0.181 nm) [22]. Due to the incorporation of the impurity ion in the crystal, the local environment of the paramagnetic ion may be different from that of the host ion, and the bond lengths will change. The bond lengths may be expressed in terms of the distortion parameters  $\Delta R_1$  and  $\Delta R_2$ , i.e.,  $R_1 =$  $R_{10} + \Delta R_1$ ,  $R_2 = R_{20} + \Delta R_2$ . Note that  $\overline{A_k}$  and  $t_k$  are already defined. For octahedral coordination,  $\overline{A_4}$  is represented as [26]

$$\overline{A_4}(R_0) = \frac{3}{4}Dq.$$
(11)

Using optical study [23],  $Dq = 1494 \text{ cm}^{-1}$  and so the value of  $\overline{A_4}(R_0) = 1120.5 \text{ cm}^{-1}$ . It should be noted that the ratio of  $\overline{A_2}(R_0)$  and  $\overline{A_4}(R_0)$ 

lies between 8 and 12 [21, 27, 28]. In the present investigation,  $\frac{\overline{A_2}}{\overline{A_4}} = 10$  is taken, which gives  $\overline{A_2} = 11205 \text{ cm}^{-1}$ . The crystal field parameters  $B_{kq}$ of  $Cr^{3+}$  ion in  $Cs_2CdCl_4$  are calculated by taking the parameters  $\overline{A_2}$  and  $\overline{A_4}$ , as well as the arrangement of chlorine atoms around the  $Cr^{3+}$  ion having matched experimental ZFS parameter. For the  $Cr^{3+}$ ion in  $Cs_2CdCl_4$ , the parameters  $t_2 = 4$  and  $t_4 = 6$ have been used. The position of the transition ion  $Cr^{3+}$  and the spherical coordinates of ligands when the  $Cr^{3+}$  ion substitutes the  $Cd^{2+}$  ion are shown in Table I. The ZFS parameter D of the  $Cr^{3+}$  ion in  $Cs_2CdCl_4$  crystal is calculated from (3) [5]. The distortion parameters are taken as  $\Delta R_1 = 0.1290$  nm and  $\Delta R_2 = 0.1289$  nm with  $R_{10} = 0.2572$  nm and  $R_{20} = 0.2572$  nm for Center I. The dis-tance  $R_{10}$  is the average of the Cr<sup>3+</sup>–Cl<sup>-</sup> I(1) and  $Cr^{3+}-Cl^-$  II(1) bond lengths, and  $R_{20}$  is the average of the  $Cr^{3+}-Cl^{-}I(2)$  and  $Cr^{3+}-Cl^{-}II(2)$ bond lengths. The distortion parameters for Center II are  $\Delta R_1 = 0.0810$  nm,  $\Delta R_2 = 0.0816$  nm with  $R_{10} = 0.2572$  nm and  $R_{20} = 0.2572$  nm. The calculated crystal field parameters and ZFS parameter, along with reference distance with distortion, are presented in Table II. The above parameters without distortion with the same reference distance also are given in Table II. The ZFS parameter calculated using crystal field parameters from the superposition model for both Centers I and II without distortion is larger than the experimental value [15]. The ZFS parameter calculated employing crystal field parameters from the superposition model for both Centers I and II with distortion is in good agreement with the experimental value [15], as can be seen in Table II. With the help of  $B_{kq}$ parameters and the CFA program [29–30], the optical spectra of the  $Cr^{3+}$ -doped  $Cs_2CdCl_4$  crystal are calculated. The energy levels of the impurity ion are computed by diagonalizing the complete Hamiltonian, which consists of the Coulomb interaction (in terms of B and C parameters), Trees correction, the spin-orbit interaction, the crystal field Hamiltonian, the spin-spin interaction, and the spin-other-orbit interaction. The computed energy values are shown in Table III (input parameters are given below), along with the experimental values [15] for comparison.

## TABLE II

Crystal field (CF) parameters and zero-field splitting (ZFS) parameter D of  $Cr^{3+}$ -doped  $Cs_2CdCl_4$  single crystal with and without distortion.

	$\Delta R_1 \text{ [nm]}$ $\Delta R_2 \text{ [nm]}$		$R_0$ [nm]	pa	Crystal field cameters [cm <sup>-</sup>	Zero-field splitting parameter $ D $	
				B <sub>20</sub>	B40	$B_{44}$	$(\times 10^{-4}) \ [\mathrm{cm}^{-1}]$
	0.1290	0.1289	0.255	-12779.8	2599.255	1554.731	228.1
Ι	0.0000	0.0000	0.255	-64902.6	29758.97	17784.39	4855.0
							$228.0^{*}$
	0.0810	0.0816	0.255	-21611.8	5732.773	3405.186	595.4
II	0.0000	0.0000	0.255	-64902.6	29758.97	17784.39	4855.0
							$595.0^{*}$

\*Experimental [15]

TABLE III

Calculated energy band positions of  $Cr^{3+}$ -doped  $Cs_2CdCl_4$  single crystal together with the experimental values for comparison.

Transition from ${}^{4}A_{2}(F)$	Observed energy bands $[cm^{-1}]$ (Bef [23])	Calculated energy bands $[cm^{-1}]$ with distortion					
$\operatorname{HOIII} A_2(T)$	bands [cm ] (iter [25])	Ι	II				
$4T_{\rm c}$ (E)	14049	13855, 14009, 14178,	13747, 13940, 14098,				
$I_{2g}(I')$	14940	14820, 14832, 14941	14920, 15026, 16603				
$^{2}E_{g}(G)$	15263	15014, 15359	16825, 16975				
$4T_{\rm c}$ (P)	20406	18818, 18995, 19163,	19697, 19967, 20033,				
$I_{1g}(I)$	20400	19881,19946,20872	20183, 20485, 20760				

It is concluded from Table III that there is a reasonable match with the experimental energy values [23].

Input parameters are as follows: numbers of free ion parameters = 5; number of d shell electrons = 3; number of fold for rotational site symmetry = 1; Racah parameters in A, B, and C, spin-orbit coupling constant, and Trees correction are 0, 516, 3797, 240, and 70 cm<sup>-1</sup>, respectively; number of crystal field parameters = 3;  $B_{20}$ ,  $B_{40}$ ,  $B_{44}$  are taken from Table II; spin-spin interaction parameter, M0 = 0.0000; spin-spin interaction parameter, M00 = 0.0000; spin-other-orbit interaction parameter, M22 = 0.0000; magnetic field, B = 0.0 Gs; angle between magnetic field **B** and z-axis = 0.00 degree.

It is observed from Table II that considering a small lattice distortion gives good agreement between theoretical and experimental results. This suggests that the distortion model employed here is quite reasonable. The parameters  $\Delta R_1$  and  $\Delta R_2$  are positive, which shows that the bond length of Cr–Cl is larger than Cd–Cl. When distortion in the crystal is neglected, a larger value of D is obtained. Taking a small distortion, the calculated ZFS parameter D is very close to the experimental value. This indicates that local distortion plays an important role in the contribution to the ZFS parameter and therefore must be taken into account in the calculation.

## 5. Conclusions

 $Cr^{3+}$  ions substitute at  $Cd^{2+}$  sites in  $Cs_2CdCl_4$ crystal. The differences of charge and ionic radii between  $Cr^{3+}$  and  $Cd^{2+}$  ions provide local distortion around  $Cr^{3+}$  impurity. The theoretical investigation of the ZFS parameter has been done using the perturbation formula and crystal field parameters found in the superposition model. Taking local distortion, the ZFS parameter values of  $Cr^{3+}$  ion in  $Cs_2CdCl_4$  crystal for both Centers I and II are in good agreement with the experimental values. It has also been found that there is a reasonable agreement between the computed and experimental optical energy values. Thus the theoretical result supports the conclusion of the experimental study.

## Acknowledgments

The author is thankful to the Head of the Department of Physics for providing the departmental facilities and to Prof. C. Rudowicz, Faculty of Chemistry, Adam Mickiewicz University, Poznan, Poland for the CFA program.

#### References

- J.A. Weil, J.R. Bolton, J.E. Wertz, Electron Paramagnetic Resonance: Elementary Theory and Practical Applications, Wiley, New York 1994.
- [2] Z. Wen-Chen, W. Shao-Yi, D. Hui-Ning,
   Z. Jian, *Spectrochim. Acta A* 58, 537 (2002).
- [3] Y.Y. Yeung, D.J. Newman, *Phys. Rev. B* 34, 2258 (1986).
- [4] C. Rudowicz, P. Gnutek, M. Açıkgöz, Appl. Spectrosc. Rev. 54, 673 (2019).
- [5] Q. Wei, Acta Phys. Pol. A **118**, 670 (2010).
- [6] Y.Y. Yeung in: Optical Properties of 3d-Ions in Crystals: Spectroscopy and Crystal Field Analysis, Ch. 3, Eds. N.M. Avram, M.G. Brik, Springer, 2013, p. 95.
- [7] L. Protesescu, S. Yakunin, M. I. Bodnarchuk, F. Krieg, R. Caputo, C.H. Hendon, R.X. Yang, A. Walsh, M.V. Kovalenko, *Nano Lett.* **15**, 3692 (2015).
- [8] B. Li, Y. Zhang, L. Fu, T. Yu, S. Zhou, L. Zhang, L. Yin, *Nat. Commun.* 9, 1 (2018).
- [9] M. Jiang, Z. Hu, Z. Liu, Z. Wu, L.K. Ono, Y. Qi, ACS Energy Lett. 4, 2731 (2019).
- [10] T. Wang, D. Yue, X. Li, Y. Zhao, Appl. Catal. B 268, 118399 (2020).
- [11] Y. Wang, D.M. Ibrahim, L.K. Ono et al., *Science* 365, 591 (2019).
- [12] H. Xie, S. Hao, J. Bao, T.J. Slade, G.J. Snyder, C. Wolverton, M.G. Kanatzidis, J. Am. Chem. Soc. 142, 9553 (2020).
- [13] M.P. Lewis, N. Keerthana, N.D. Hebbar, K.S. Choudhari, S.D. Kulkarni, *Curr. Appl. Phys.* **32**, 71 (2021).
- [14] T. Sakai, M. Koshimizu, Y. Fujimoto, D. Nakauchi, T. Yanagida, K. Asai, *Sens. Mater.* **30**, 1564 (2018).

- [15] D. Kay, G.L. McPherson, J. Phys. C Solid State Phys. 14, 3247 (1981).
- [16] T.X. Zeng, J.J. Chen, T.H. Chen, Z.X. Liu, Y. Huang, *Rad. Eff. Defects Solids* 164, 187 (2009).
- [17] S. Siegel, E. Gebert, Acta Cryst. 17, 790 (1964).
- [18] D. Balz, Naturwissenschaften 40, 241 (1953).
- [19] C. Rudowicz, Magn. Reson. Rev. 13, 1 (1987).
- [20] W.L. Yu, M.G. Zhao, J. Phys. C Solid State Phys. 18, L525 (1984).
- [21] A. Abragam, B. Bleaney, Electron Paramagnetic Resonance of Transition Ions, Clarendon Press, Oxford 1970.
- [22] C.K. Jorgensen, Modern Aspects of Ligand Field Theory, North-Holland, Amsterdam, 1971, p. 305.
- [23] R. Nistora, L. Andreici, N.M. Avram, Acta Phys. Pol. A 116, 538 (2009).
- [24] W.L. Yu, M.G. Zhao, *Phys. Rev. B* 37, 9254 (1988).
- [25] D.J. Newman, B. Ng, *Rep. Prog. Phys.* 52, 699 (1989).
- [26] D.J. Newman, B. Ng, Crystal Field Handbook, Cambridge University Press, Cambridge, 2000.
- [27] D.J. Newman, D.C. Pryce, W.A. Runciman, Am. Miner. 63, 1278 (1978).
- [28] T.H. Yeom, S.H. Choh, M.L. Du, J. Phys. Condens. Matter 5, 2017 (1993).
- [29] Y.Y. Yeung, C. Rudowicz, J. Comput. Phys. 109, 150 (1993).
- [30] Y.Y. Yeung, C. Rudowicz, Comput. Chem. 16, 207 (1992).