Dedicated to Professor Iwo Białynicki-Birula on His 90th Birthday

Dynamical Quantum Phase Transitions from Quantum Optics Perspective

J. Zakrzewski[a,b,*]

[a] *Instytut Fizyki Teoretycznej, Wydział Fizyki, Astronomii i Informatyki Stosowanej, Uniwersytet Jagielloński, Profesora Stanisława Łojasiewicza 11, PL-30-348 Kraków, Poland*
[b] *Mark Kac Complex Systems Research Center, Uniwersytet Jagielloński, Kraków, Poland*

Published: 14.06.2023

Doi: 10.12693/APhysPolA.143.S179 *e-mail: jakub.zakrzewski@uj.edu.pl

In this work dedicated to Professor Iwo Białynicki-Birula on the occasion of his 90th birthday, I attempts to show that dynamical quantum phase transitions observed as singularities in the Loschmidt rate dynamics bear a close resemblance to the standard Rabi oscillations known from the dynamics of two-level systems. For some many-body systems, this analogy may go even further, and the behaviour observed for example transverse Ising chain can be directly mapped to such simple dynamics. A simple link between Loschmidt echo singularities and quantum scars is also suggested.

topics: phase transitions, Loschmidt echo singularities, Rabi oscillations, quantum scars

1. Introduction

The physics of complex systems may sometimes be understood (in particular limiting cases) in a simple, enlightening form. This has been often demonstrated in quantum optics, one of the many areas of Iwo Białynicki-Birula outstanding contributions. As a scientific grandson of Iwo Białynicki-Birula, I had relatively small overlap in scientific interests with him, our paths crossed for a moment in the studies of nonspreading wave-packets [1–4]. Still, however, I profited a lot from occasional conversations as well as participation, from time to time, in unusually vivid seminars with his active participation. Often his aim was to find a simple picture of the presented effects. In this contribution, I consider briefly two cases from studies of nonequilibrium dynamics of many-body systems which may be, in my opinion, understood in simple terms: dynamical quantum phase transitions (DQPT) [5, 6] and quantum many-body scars (QMBS) dynamics [7].

The simplest definition of DQPT consists of a sudden quench in which the system is prepared in the ground state $|\Psi\rangle$ of a parameter-dependent Hamiltonian $H(\lambda=0)$, and λ is suddenly changed to other value. It has been observed that often if a change of λ moves the Hamiltonian into a different phase, the time dynamics with $H(\lambda)$ of the now nonstationary state after quench reveals the so-called Loschmidt echo singularities. Their appearance is neither a necessary nor a sufficient condition for the phase transition between $H(0)$ and $H(\lambda)$. Still, a predominantly lack of singularities occurs if no phase transition is crossed while changing λ and vice versa.

Dynamical detection [7] of QMBS is in some sense similar. One prepares an initial nonstationary state for the many-body system described by $H(\lambda)$. When this initial state has a significant overlap with a few almost equally spaced in energy eigenstates of $H(\lambda)$, the time evolution of the observables reveals oscillations even in a weakly ergodic regime, i.e., when the dynamics of a typical generic state will lead to thermalization.

Both these phenomena, while of current interest, can be simply explained by identifying a "essential state model", i.e., a minimal approximate level scheme allowing one to simulate the dynamics. Let us first consider DQPT in the seminal example of the transverse Ising model.

2. DQPT in transverse Ising model

The first work on DQPT [5] considers the transverse Ising model with the Hamiltonian of the form

$$H = -\frac{1}{2}\sum_i \sigma_i^z \sigma_{i+1}^z - \frac{g}{2}\sum_i \sigma_i^x , \tag{1}$$

where g is the strength of the magnetic field pointing in the Ox direction. For small g, the interactions favor the ferromagnetic (FM) orientation (along Oz), with the two degenerate (in the thermodynamic limit) ground states given for $g \to 0$ by $|\psi(\pm)^z\rangle = \prod_i |\pm\rangle_i^z$, where $|\pm\rangle_i^z$ denotes the eigenvectors of σ_i^z. The phase transition from FM to paramagnetic order occurs for $g = 1$, for large g the unique ground state is well approximated by $|\varphi\rangle = \prod_i |+\rangle_i^x$ with $|\pm\rangle_i^x$ being the eigenvectors of σ_i^x.

Let g serve as the parameter λ and let us start with the ground state of (1) for small g, say with $|\Psi\rangle = |\psi(+)^z\rangle$, and abruptly change g to a large, positive value. In the new Hamiltonian, the term proportional to g will dominate, while the first term of the interaction will be a small perturbation. The initial state can then be decomposed in the basis of eigenvectors σ_i^x as

$$|\Psi(0)\rangle = \prod_i \frac{1}{\sqrt{2}}\Big(|+\rangle_i^x + |-\rangle_i^x\Big). \tag{2}$$

The initial state after a quench is therefore the product of two-state combinations with coefficients of equal magnitude (the phase does not affect the result). The subsequent time evolution, still neglecting interactions in the final Hamiltonian, yields

$$|\Psi(t)\rangle = \prod_i \frac{1}{\sqrt{2}}\Big(|+\rangle_i^x \, \mathrm{e}^{\mathrm{i}gt/2} \; + \; |-\rangle_i^x \, \mathrm{e}^{-\mathrm{i}gt/2}\Big). \tag{3}$$

By the survival probability (fidelity, return amplitude, or Loschmidt echo), one calls (depending on the context) the squared overlap of initial and time evolved state, $\mathcal{L}(t) \equiv |\langle \Psi(0)\,|\,\Psi(t)\rangle|^2$. Further one may define [6] the rate function $r(t)$ via $\mathcal{L}(t) = \exp(-Lr(t))$, where L is the system size (number of degrees of freedom). Such a measure has a good thermodynamic limit. Singularities in $r(t)$ time dependence, often referred to as Loschmidt echo singularities, are the defining features of DQPT.

Let us immediately consider the example above. The squared overlap $\mathcal{L}(t)$ becomes simply $\mathcal{L}(t) = \cos^{2L}(gt/2)$, and the size-independent rate $r(t)$ reveals singularities whenever the cosine function vanishes, i.e., for $t^* = (2k+1)\pi/g$ for an integer k. This example clearly shows that Rabi-type oscillations are the real origin of rate function singularities in this case.

One can complain that the situation described above is too simplified; singularities in the form of finite cusps appear also for smaller changes of g, where the approximations made by us would not work fully. Then, however, one can use the Jordan–Wigner transformation into a noninteracting fermion system, as in the original DQPT letter [5], and observe similar "two-level" dynamics for a given k as different k decouple.

3. Other examples

Our model, however, helps to explain also other situations. In fact, as reviewed in [6], 2-band topological noninteracting models lead to exactly the same dynamics. Again here, due to the lack of interactions, different k values can be treated independently, leading to a similar estimate of critical times at which singularities appear. Let us stress that while these singularities are essential for the phase-transition language application, they just seem to be due to the vanishing overlaps between the initial and time-evolved wavepacket.

Consider now a situation in which we make an abrupt quench within the same phase, then by definition the ground state changes slowly and continuously with the change of the parameter for a finite system. So it is quite justified to assume that the ground state at say $\lambda = 0$ expands in eigenstates $\{|\psi_k\rangle\}$ of $H(\lambda)$ as

$$|\Psi\rangle = \alpha_0\,|\psi_0\rangle + \sum_k \alpha_k\,|\psi_k\rangle\,, \tag{4}$$

with $|\alpha_0| \gg |\alpha_k|$ for $k > 0$. Then the survival probability (Loschmidt echo) is dominated by the large therm $|\alpha_0|^2$. The situation is more subtle in the thermodynamic limit due to the Anderson catastrophe. Still then, one may expect that many eigenstates at the final parameter value contribute to the initial wavepacket, leading to many superimposed oscillations at different frequencies. In such a situation, the rate function should not reveal strong maxima (not speaking of singularities).

Note that the situation is markedly different when the phase transition is crossed in λ because then, for the Ising system, via symmetry as described above, *two* eigenstates contribute significantly to the sum, leading to Rabi oscillations at half of their energy difference (per site).

The discussion up till now was concentrated on spin-1/2 models leading to simple Rabi oscillations. This might be a transverse Ising chain but also, e.g. a quantum dot dynamics [8]. As known from quantum optics, Rabi oscillations generalize to quantum revivals appearing when several equally spaced levels are populated [9]. Here again one may expect that between consecutive revivals, minima of the survival probability lead to maxima (and possibly cusps) of the Loschmidt rate functions. In a many-body system, an even more general situation was experimentally realized many years ago for interacting bosons in an optical lattice [10]. Initially, the bosons were kept in a shallow lattice, then abruptly the height of the lattice was increased dramatically, separating different lattice sites. Within each site, the initial almost coherent state was a superposition of states with different site occupations, separated by a quadratic progression in the interaction strength U (within the tight binding Bose–Hubbard description), and the revivals were observed. The corresponding Loschmidt rates reveal singularities (or maxima), as discussed in detail recently [11] in the DQPT language, at times when the overlap between initial and time evolved state is minimal, i.e., roughly in the middle between two consecutive revivals.

4. Quantum scars

Recently, an interesting manifestation of ergodicity breaking as persistent oscillations for certain initial states was discovered experimentally with ultracold Rydberg atoms [7]. This feature is due to the presence of few atypical, almost equally spaced eigenstates — so-called quantum many-body scars (QMBS) [12, 13], which are embedded in the otherwise thermal spectrum of a quantum many-body system. For initial states with a high overlap with a few QMBS, one observes long-lived oscillations of observables, whereas for generic initial conditions the system quickly approaches thermal equilibrium state. The same oscillations should be present in the survival probability leading, in turn, to maxima of the Loschmidt echo rate function if the data are interpreted in that way.

QMBS borrowed their name from the single-particle quantum chaos studies, where "quantum scar" described the enhanced probability of eigenstates or wavepackets in regions of space occupied by unstable periodic orbits [14] — in close relation to the semiclassical periodic orbits quantization [15, 16]. Then also the concept of scarring by symmetries was developed in the contex of hydrogen atom in magnetic field studies [17]. Similar symmetry concepts were used for the construction of nonergodic states in many-body case see, e.g. [18, 19].

Such QMBS may be easily imagined as having the origin in the approximate decoupling of a (not always apparent) single degree of freedom from other degrees of freedom. If this single degree is locally described by a harmonic oscillator (or an angular momentum), then the corresponding eigenstates are equidistant — their weak coupling to the remaining states preserves the energy structure. Now, if by accident (or cleverness) the initial state is prepared as a linear combination of those selected states (or if it has sufficiently large overlap on at least a few of them), one may naturally expect a persistent oscillation in the time dynamics. Let us mention also that quench dynamics and Rabi oscillations resulting from the excitations of two or more localized integrals of motion in the context of many-body localization have recently been studied [20, 21]. The localized, almost decoupled family of states may not be easy to identify, one may try to identify it e.g., by adiabatic following from some analytic limit [22] or via purely numerical approaches including artificial intelligence [23].

5. Conclusions

DQPT forms a very intriguing interpretation of rapid quantum quenches. On the other hand, signatures of DQPT in the form of singularities of the Loschmidt echo rate functions appear to a large extent due to the very definition of this rate. Survival probability (Loschmidt echo) itself reveals no singularities but rather smooth oscillations (or revivals in more complicated cases).

Let us stress that the mechanism presented above considers rather simple examples. For more complicated cases one may consider the Loschmidt echo as coming back not to a single ground state, but to the degenerate manifold, if it exists [6]. After the first draft of this note was completed, a related work appeared, giving a more general picture of DQPT [24]. It has been brought also to our attention that similar to DQPT cusp structures may appear in single particle dynamics [25, 26].

Acknowledgments

This note was born as a reflection on a seminar given in Krakow by Tadeusz Domański. His great presentation skills are acknowledged. I am grateful to Dominique Delande, Mateusz Łącki, Anatoly Polkovnikov, and Tommaso Roscilde for the discussions. The work of J.Z has been realized within the Opus grant 2019/35/B/ST2/00034, financed by National Science Centre (Poland). The research has also been supported by the Strategic Programme Excellence Initiative at Jagiellonian University.

References

[1] I. Bialynicki-Birula, M. Kaliński, J.H. Eberly, *Phys. Rev. Lett.* **73**, 1777 (1994).

[2] D. Delande, J. Zakrzewski, A. Buchleitner, *Europhys. Lett.* **32**, 107 (1995).

[3] D. Delande, J. Zakrzewski, *Phys. Rev. A* **58**, 466 (1998).

[4] A. Buchleitner, D. Delande, J. Zakrzewski, *Phys. Rep.* **368**, 409 (2002).

[5] M. Heyl, A. Polkovnikov, S. Kehrein, *Phys. Rev. Lett.* **110**, 135704 (2013).

[6] M. Heyl, *Rep. Prog. Phys.* **81**, 054001 (2018).

[7] H. Bernien, S. Schwartz, A. Keesling et al., *Nature* **551**, 579 (2017).

[8] K. Wrześniewski, I. Weymann, N. Sedlmayr, T. Domański, *Phys. Rev. B* **105**, 094514 (2022).

[9] J.H. Eberly, N.B. Narozhny, J.J. Sanchez-Mondragon, *Phys. Rev. Lett.* **44**, 1323 (1980).

[10] M. Greiner, O. Mandel, T.W. Hänsch, I. Bloch, *Nature* **419**, 51 (2002).

[11] M. Lacki, M. Heyl, *Phys. Rev. B* **99**, 121107 (2019).

[12] C.J. Turner, A.A. Michailidis, D.A. Abanin, M. Serbyn, Z. Papić, *Nat. Phys.* **14**, 745 (2018).

[13] C.J. Turner, A.A. Michailidis, D.A. Abanin, M. Serbyn, Z. Papić, *Phys. Rev. B* **98**, 155134 (2018).

[14] E.J. Heller, *Phys. Rev. Lett.* **53**, 1515 (1984).

[15] M.C. Gutzwiller, *J. Math. Phys.* **12**, 343 (1971).

[16] E. Bogomolny, *Physica D* **31**, 169 (1988).

[17] D. Delande, J.C. Gay, *Phys. Rev. Lett.* **59**, 1809 (1987).

[18] N. Shiraishi, T. Mori, *Phys. Rev. Lett.* **119**, 030601 (2017).

[19] S. Moudgalya, S. Rachel, B.A. Bernevig, N. Regnault, *Phys. Rev. B* **98**, 235155 (2018).

[20] L. Benini, P. Naldesi, R.A. Römer, T. Roscilde, *New J. Phys.* **23**, 023030 (2021).

[21] L. Benini, P. Naldesi, R.A. Römer, T. Roscilde, *Ann. Phys.* **435**, 168545 (2021).

[22] A.S. Aramthottil, U. Bhattacharya, D. González-Cuadra, M. Lewenstein, L. Barbiero, J. Zakrzewski, *Phys. Rev. B* **106**, L041101 (2022).

[23] T. Szołdra, P. Sierant, M. Lewenstein, J. Zakrzewski, *Phys. Rev. B* **105**, 224205 (2022).

[24] M. Van Damme, J.-Y. Desaules, Z. Papić, J.C. Halimeh, arXiv:2210.02453, 2022.

[25] J.M. Zhang, H.-T. Yang, *Europhys. Lett.* **114**, 60001 (2016).

[26] J.M. Zhang, Y. Liu, *Eur. J. Phys.* **37**, 065406 (2016).