Rayleigh–Taylor Instability of Superposed Dusty Jeffrey Fluids through Porous Medium with Interfacial Surface Tension

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The linear Rayleigh–Taylor instability of two immiscible superposed non-Newtonian fluids saturated by a porous medium in the simultaneous presence of interfacial surface tension and suspended dust particles is investigated. The non-Newtonian behavior of fluids is described by the Jeffrey model. The set of coupled partial differential equations satisfying the appropriate boundary conditions is solved by applying the linear theory and normal mode technique. The exact solutions are found for both regimes with constant density, and the dispersion relation (between the growth rate and the wave number) is obtained. The physical system is found to be stable for a bottom-heavy configuration density-wise, such as in the Newtonian viscous fluid using the Routh–Hurwitz criterion. However, for the unstable configuration, the surface tension, medium porosity, dynamic viscosity, and density of dust particles have a stabilizing impact on the growth rate of the unstable Rayleigh–Taylor mode; whereas the density difference between the fluids and the Jeffrey parameter has a destabilizing effect on the growth rate of the unstable Rayleigh–Taylor mode.

topics: Rayleigh–Taylor instability, suspended particles, Jeffrey model, interfacial surface tension

1. Introduction

Interfacial instabilities in fluid systems are characterized by interpenetration of material and mixing at molecular scales due to perturbations occurring at the interfaces of the material. The Rayleigh–Taylor instability (RTI) is buoyancy-driven and occurs in fluid flows that contain species of differing molecular masses in the presence of acceleration, such as accelerating fronts or gravity [1]. The detailed understanding of the processes and the consequences of interfacial instabilities has vast implication for real systems, including the designs of efficient, high-gain capsules for inertial confinement fusion (ICF) and modeling of supernova implosions and explosions. Such understanding is available due to current computational efforts. A simple explanation for the occurrence of the Rayleigh–Taylor instability (RTI) deals with a normal room existing on Earth with normal gravitational effects. Suppose this room contains the volume of air above the floor, which supports a uniform layer of water that lies beneath the ceiling. Similar to pouring of water into oil, the heavier fluid, once perturbed, streams to the bottom, pushing light fluid aside. The effect of viscosity on the stability of the plane interface separating two incompressible superposed fluids of uniform densities in the presence of a uniform horizontal magnetic field has been investigated by Bhatia [2]. The importance of surface tension with regard to the problem of plasma instability might be of industrial and astrophysical importance. Considerable study has been given to the presence of surface tension in fluid instabilities studies. The effect of surface tension and viscosity on RTI has been studied by Mikaelian [3]. The effect of surface tension on the immiscible Rayleigh–Taylor turbulence has been investigated by Chertov et al. [4] and it was observed that surface tension leads to the formation of an emulsion whose typical drop size decreases over time.

In the recent study of spacecraft observations, we note that suspended dust particles play an important role in the dynamics of the Martian atmosphere. In geophysical situations, the fluid is permeated with dust particles instead of pure conducting fluid. In order to study the role of suspended dust particles on RTI in such media, Sharma and Chhajlani [5] have investigated the effect of suspended dust particles and rotation on RTI of two superposed magnetized conducting plasmas, and they showed that rotation and suspended particles do not affect RTI. The problem of hydromagnetic transverse instability of two highly viscous fluid particle flow with FLR (finite Larmor radius) corrections was discussed by Sayed [6] and it was showed that FLR and kinematic viscosity have a stabilizing effect for a vortex sheet. The effect of surface tension and rotation on RTI of two superposed incompressible fluids with suspended dust particles has been analyzed by Sharma et al. [7]. There, it was observed that the growth rate of RTI decreases with a rise in values of the mass concentration and relaxation frequency of the suspended dust particles, and the substantial stabilizing effect of these quantities was depicted. RTI of two superposed fluids with suspended particles and in the presence of rotation was studied by Hoshoudy and Kumar in [8], and demonstrated greater stability due to the simultaneous impact of the relaxation frequency of suspended dust particles and rotation on the growth rate of the unstable configuration.

Additionally, the investigation of fluid flow through porous media has become more advantageous due to the variety of applications in industry, laboratories, geophysics and in industry in lieu of the recovery of crude oil from that of reservoir rock pores. In this light, Sharma and Spanos [9] have established the Rayleigh–Taylor instability of streaming superposed fluids with surface tension saturating a porous medium. The effect of surface tension on the hydromagnetic Rayleigh–Taylor instability of two superposed fluids in a porous medium with suspended dust particles immersed in a uniform horizontal magnetic field was studied by Sharma et al. [10]. The authors found that the relaxation frequency, mass concentration of dust particles have stabilizing effect on RTI. The stability of two superposed viscous, streaming horizontally fluids lying one over the other in the presence of suspended dust particles flowing through a porous medium has been analyzed by Prajapati and Chhajlani [11]. The effect of boundary roughness on the saturation of electro–hydrodynamic RTI in two superposed fluids in the presence of nanostructured porous layer have been studied by Chavaraddi and Awati [12]. The authors found that surface tension stabilizes the system while electromagnetic fields, boundary roughness decrease the growth rate of RTI. The impact of magnetic and electric fields on RTI in a power-law fluid has been investigated by Chavaraddi et al. [13], and the study revealed that the magnetic field, electric field, power-law fluid and layer thickness stabilizes the system. The fluids have been assumed to be Newtonian in all the above-mentioned studies.

Nevertheless, little attention has been given in the literature to studying RTI of superposed fluids with non-Newtonian behavior characterized by a variety of models in the universe. Furthermore, the mechanism and great potential of non-Newtonian fluids, characterized by various model flows, have attracted many researchers in the present arena due to the existence of this behavior in the chemical industries as well as in astrophysical situations during the past few decades. The Newtonian or non-Newtonian behavior of a fluid is characterized by a linear or non-linear relationship between the stress and rate-of-strain components, termed the constitutive equations. We are interested therein one of the non-Newtonian fluids characterized by the Jeffrey model (Joseph and Preziosi [14]), which signifies the ratio of relaxation and retardation and exhibits a linear viscoelastic feature, yield stress, shear thinning properties and high shear viscosity. The Jeffrey model is described by the following constitutive relations

$$T_{ij} = -p\,\delta_{ij} + \tau_{ij}, \tag{1}$$

with

$$\tau_{ij} = \frac{2\mu}{1+\lambda}\left[1 + \lambda_1\left(\frac{\partial\,(2e_{ij})}{\partial t} + \frac{\partial u_i}{\partial x_j}\right)\right] e_{ij}, \tag{2}$$

where T_{ij}, τ_{ij}, e_{ij}, p, μ, and δ_{ij} denote the stress tensor, the viscous stress tensor, the rate-of-strain tensor, the isotropic pressure, the coefficient of viscosity, and Kronecker delta, respectively. The Jeffrey parameter (λ) is defined as the ratio of the stress relaxation time (λ_0) to the strain retardation time (λ_1).

For incompressible fluids, the above relation (2) reduces to

$$\tau_{ij} = \frac{2\mu}{1+\lambda}\,e_{ij}. \tag{3}$$

The two time representations are of great importance in the possibility of wave propagation in the earth's mantle in lieu of the strain retardation time, whilst the stress relaxation time assigns the time taken by a fluid to retain a position from its disturbed position to its basic stable state. Such a fluid also finds application in environmental engineering, with the inclusion of polypropylene coalescence sintering, geological flows and blood etc. Some experimental methods have been proposed for measuring rheology of polymeric solution [15]. The novelty of such calculations was carried out for the movement of microorganisms in weakly non-Newtonian fluids [16]. The ill-posed second-order (retardation) fluid model exhibiting stress relaxation has been utilized by Christov and Christov [17] to establish the stress retardation against stress relaxation in linear viscoelasticity using Volterra functional series. The problem of Jeffrey fluid in the rotating system with suspended dust particles in the presence of volume fraction and Hall effect was analyzed by Dey [18] and the retarding impact of the non-Newtonian behavior of the fluid was observed on the considered system. The impact of the magnetic field on RTI in a coupled stress fluid was established by Chavaraddi and Awati [19]. The instability of two Oldroydian viscoelastic rotating superposed fluids with a uniform vertical angular velocity through a porous medium was analyzed by Mathur and Kumar [20]. The influence of the boundary roughness on RTI at the interface of the superposed coupled stress fluid was demonstrated by Chavaradi and Gouder [21] and the boundary roughness and couple-stress parameter are found to stabilize the system.

Therefore, this paper aims to investigate the impact of suspended dust particles on the Rayleigh–Taylor instability superposed Jeffrey fluids flowing through a porous medium to see how the criterion of instability/stability changes with variations in the distinct parameters involved.

2. Formulation of the problem and perturbation equations

The initial stationary state whose stability we wish to examine is that of two infinite horizontal layers of homogeneous Jeffrey fluids superimposed on one another, separated by a plane interface at $z = 0$, saturated by a porous medium. The porous medium is assumed to be of homogeneous medium porosity (ε) and medium permeability (k_1). Two regimes $z < 0$ and $z > 0$ consist of two distinct layers of Jeffrey fluids with densities ρ_1 and ρ_2. The system is acted on by the gravity force $\boldsymbol{g} = (0, 0, -g)$. Suppose that at some prescribed level (z_s), the density may change discontinuity and bring into play an effect due to the effective interfacial tension (T_s).

The modified conservation equations of the governing problem for Jeffrey fluid in a porous medium (Joseph and Preziosi [14]; Prajapati and Chhajlani [11]) are

$$\frac{\rho}{\varepsilon}\left[\frac{\partial}{\partial t}+\frac{1}{\varepsilon}\left(\nabla\cdot\boldsymbol{q}\right)\right]\boldsymbol{q} = -\nabla p+\rho\boldsymbol{g}-\frac{\mu}{k_1\left(1+\lambda\right)}\boldsymbol{q} + \sum_s\left[T_s\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)z_s\right]\delta(z-z_s)+\frac{KN}{\varepsilon}(\boldsymbol{V}-\boldsymbol{q}), \tag{4}$$

$$\nabla\cdot\boldsymbol{q} = 0, \tag{5}$$

$$\frac{\rho}{\varepsilon}\frac{\partial\rho}{\partial t}+\left(\boldsymbol{q}\cdot\nabla\right)\rho = 0, \tag{6}$$

where $\boldsymbol{q}$, $\boldsymbol{V}$, ρ, p, μ, and λ represent the filter velocity of Jeffrey fluid, suspended particle velocity, density, hydrodynamic pressure, viscosity, and Jeffrey parameter, respectively. In (4), $\delta(z-z_s)$ denotes the Dirac delta function. The density of each particle remains unchanged as we follow it with its motion, as revealed from (6).

Suspended dust particles are assumed to be nonconducting, of spherical shape, and uniform in size. Due to the dust particles, the fluid exerts a force on particles that is equal and opposite to that of the particles. Consequently, an extra force term given by $\frac{KN}{\varepsilon}\left(\boldsymbol{V}-\boldsymbol{q}\right)$ is added in the equations of motion, where N is the particle number density; K is the Stoke coefficient of resistance, given by $K = 4\pi a\mu$ for spherical particles; a is the particle radius. Inter-particle reactions are ignored due to the large enough distance among the particles.

Therefore, the equations of motion and continuity of suspended particles are given by

$$mN\left[\frac{\partial\boldsymbol{V}}{\partial t}+\frac{1}{\varepsilon}\left(\boldsymbol{V}\cdot\nabla\right)\boldsymbol{V}\right] = KN\left(\boldsymbol{q}-\boldsymbol{V}\right), \tag{7}$$

$$\varepsilon\frac{\partial N}{\partial t}+\nabla\cdot\left(N\boldsymbol{V}\right) = 0, \tag{8}$$

where m is the mass of the dust particles and mN is the mass of particles per unit volume.

The basic flow and suspended particles are supposed to be motionless and quiescent (no settling of suspended particles), and the physical state variables (pressure, density) are dependent only on the z-axis only.

Therefore, the basic state solutions are given by

$$q = (0,0,0)\,, \quad V = (0,0,0)\,, \quad \rho = \rho\left(z\right). \tag{9}$$

To investigate the stability of hydrodynamic motion, infinitesimal perturbations are superimposed to each of the physical quantities of the initial state solutions as

$$q = q_0 + q', \quad V = V_0 + V', \quad p = p_0 + p', \quad \rho = \rho_0 + \rho', \quad z_s = z_{s_0} + {z_s}', \tag{10}$$

where $\boldsymbol{q}(u,v,w)$, $\boldsymbol{V}(l,r,s)$, p', ρ', and ${z_s}'(x,y,t)$ represent perturbations in the fluid velocity, particle velocity, pressure, fluid density, and surface of separation, respectively. The subscript $'$ denotes an equilibrium state.

Using perturbations (10) and linear theory, (4)–(8) become

$$\frac{\rho}{\varepsilon}\frac{\partial\boldsymbol{q}}{\partial t} = -\nabla p'+\boldsymbol{g}\rho'-\frac{\mu}{k_1(1+\lambda)}\boldsymbol{q}+\frac{KN}{\varepsilon}\left(\boldsymbol{V}-\boldsymbol{q}\right) + \sum_s\left[T_s\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right){z_s}'\right]\delta\left(z-z_s\right), \tag{11}$$

$$\nabla\boldsymbol{q} = 0, \tag{12}$$

$$\frac{\rho}{\varepsilon}\frac{\partial\,\delta\rho}{\partial t}+\left(\boldsymbol{q}\cdot\nabla\right)\rho = 0, \tag{13}$$

$$\tau\left(\frac{\partial}{\partial t}+1\right)\boldsymbol{V} = \boldsymbol{q}, \tag{14}$$

where $\tau = m/k$ is the relaxation time for suspended dust particles.

3. Methodology adopted

To examine the stability of the system, perturbations are analyzed in terms of normal modes by ascribing a wave number whose dependence on space (x, y, z) and time t is of the form

$$f'\left(x,y,z,t\right) = f\left(z\right)\mathrm{e}^{\mathrm{i}\left(k_x x+k_y y-nt\right)}, \tag{15}$$

where $f(z)$ is some function of z, $k = \sqrt{k_x^2+k_y^2}$ is the resultant real wave number, and n is the growth rate, in general, a complex number.

Using expression (14), (10)–(13) in Cartesian notation are given by

$$\frac{\mathrm{i}\rho n}{\varepsilon}u = -\mathrm{i}k_x p' - \frac{\mu}{k_1(1+\lambda)}u, \tag{16}$$

$$\frac{\mathrm{i}\rho n}{\varepsilon}v = -\mathrm{i}k_y p' - \frac{\mu}{k_1(1+\lambda)}v, \tag{17}$$

$$\frac{\mathrm{i}\rho n}{\varepsilon}w = -Dp' - g\rho' - \frac{\mu}{k_1(1+\lambda)}w + k^2\sum_s T_s {z_s}'\delta(z-z_s), \tag{18}$$

$$\mathrm{i}k_x u + \mathrm{i}k_y v + Dw = 0, \tag{19}$$

$$\mathrm{i}\varepsilon n\rho' = -wD\rho, \tag{20}$$

$${z_s}' = \frac{w_s}{\mathrm{i}\varepsilon n}, \tag{21}$$

$$(\tau \mathrm{i}n + 1)V = u. \tag{22}$$

The perturbation in the surface of separation $({z_s}')$ is defined in terms of the normal component of the velocity w_s at z_s (see (21)).

Eliminating variables u, v, p', ρ', ${z_s}'$ from (16)–(22), we get the characteristic equation for w as

$$\left[\frac{\mathrm{i}n}{\varepsilon} + \frac{v}{k_1(1+\lambda)} + \frac{\alpha_0}{\varepsilon}\left(\frac{\mathrm{i}n}{\tau\mathrm{i}n+1}\right)\right]\left(D\rho(Dw) - \rho k^2 w\right) = \mathrm{i}gk^2\left[(D\rho) - \frac{k^2}{g}\sum_s T_s\delta(z-z_s)\right]\frac{w}{\varepsilon n}, \tag{23}$$

where $\nu = \mu/\rho$ is the kinematic viscosity, and $\alpha_0 = mN/(\rho_1+\rho_2)$ is the mass concentration of suspended dust particles. It is noteworthy that the density of the embedded dust particles in two regimes $z > 0$ and $z < 0$ are taken the same.

Let two uniforms superposed Jeffrey fluids of densities (ρ_1) of the lower fluid and (ρ_2) of the upper fluid be separated by a horizontal boundary at $z = 0$ Then, in each of the two regions of the constant densities, (22) becomes

$$\left(D^2 - k^2\right)w = 0. \tag{24}$$

The boundary condition across the interface of two fluids are as follows (Chandrasekhar [1]):

(i) The velocity w should vanish when $z \to +\infty$ (for upper fluid) and $z \to -\infty$ (for lower fluid).

(ii) The function $w(z)$ is continuous at $z = 0$.

(iii) Integrating (23) between $z_s - \varepsilon'$ and $z_s + \varepsilon'$ passing to the limit $\varepsilon' = 0$, we obtain the jump condition

$$\Delta_s\left[\frac{\mathrm{i}n}{\varepsilon} + \frac{v}{k_1(1+\lambda)} + \frac{\alpha_0}{\varepsilon}\left(\frac{\mathrm{i}n}{\tau\mathrm{i}n+1}\right)Dw\right] = \mathrm{i}gk^2\left[\Delta_s(\rho) - \frac{k^2T}{g}\right]\left(\frac{w}{\varepsilon n}\right)_s, \tag{25}$$

where $\Delta_s(f) = f(z_s+0) - f(z_s-0)$ is the jump experienced by a physical quantity at the interface $z = z_s$.

Applying the boundary condition (i) and (ii), the appropriate solutions of w are

$$w_1 = A\mathrm{e}^{+kz}, \quad (z<0), \tag{26}$$

$$w_2 = A\mathrm{e}^{-kz}, \quad (z>0), \tag{27}$$

where the constant A is chosen to ensure the continuity of w.

Using the solutions (26) and (27), after some algebraic simplifications the jump condition (25) is

$$n^3 - \left[\frac{\mathrm{i}(1+\alpha_1\beta_1+\alpha_2\beta_2)}{\tau} + \frac{\mathrm{i}\varepsilon}{k_1}\left(\frac{\alpha_1\nu_1}{1+\lambda_1} + \frac{\alpha_2\nu_2}{1+\lambda_2}\right)\right]n^2 - \left[\frac{\varepsilon}{\tau k_1}\left(\frac{\alpha_1\nu_1}{1+\lambda_1} + \frac{\alpha_2\nu_2}{1+\lambda_2}\right) - gk\left((\alpha_1-\alpha_2) + \frac{k^2T}{g(\rho_1+\rho_2)}\right)\right]n + \frac{\mathrm{i}gk}{\tau}\left[(\alpha_1-\alpha_2) + \frac{k^2T}{g(\rho_1+\rho_2)}\right] = 0, \tag{28}$$

Therefore, we obtain the following dispersion relation (using $\mathrm{i}\,n = \sigma$)

$$\sigma^3 + \sigma^2\left[f_s\left(1 + \frac{2mN}{\rho_1+\rho_2}\right) + \frac{\varepsilon}{k_1}\left(\frac{\alpha_1\nu_1}{1+\lambda_1} + \frac{\alpha_2\nu_2}{1+\lambda_2}\right)\right] - \sigma\left[\left(gk(\alpha_1-\alpha_2) - \frac{k^2T}{g(\rho_1+\rho_2)}\right) - \frac{\varepsilon f_s}{k_1}\left(\frac{\alpha_1\nu_1}{1+\lambda_1} + \frac{\alpha_2\nu_2}{1+\lambda_2}\right)\right] - gkf_s\left[(\alpha_1-\alpha_2) - \frac{k^2T}{g(\rho_1+\rho_2)}\right] = 0, \tag{29}$$

where $f_s = 1/\tau$ is the relaxation frequency parameter, and $\alpha_1 = \frac{\rho_1}{\rho_1+\rho_2}$ and $\alpha_2 = \frac{\rho_2}{\rho_1+\rho_2}$.

The dispersion relation of the RT configuration for two superposed Jeffrey fluids of different densities, taking into account surface tension, medium porosity, permeability, suspended dust particles, and Jeffrey parameters, is given by (29) and is also similar to Prajapati and Chhajlani [11] in the absence of Jeffrey parameters.

Now, we assume that both fluids have equal viscosities and the same values of Jeffrey parameter, i.e., $\nu_1 = \nu_2 = \nu$ and $\lambda_1 = \lambda_2 = \lambda$ (for the sake of simplicity).

Therefore, (29) yields

$$\sigma^3 + \sigma^2 \left[f_s \left(1 + 2\alpha_0\right) + \frac{2\varepsilon\nu}{k_1\left(1+\lambda\right)} \right] - \sigma \left[gk(\alpha_1 - \alpha_2) - \frac{k^2 T}{g\left(\rho_1+\rho_2\right)} - \frac{2\varepsilon f_s \nu}{k_1\left(1+\lambda\right)} \right] - gk\, f_s \left[(\alpha_1 - \alpha_2) - \frac{k^2 T}{g\left(\rho_1+\rho_2\right)} \right] = 0, \tag{30}$$

Now, the condition of instability is evaluated from the constant term in (30) as

$$(\alpha_2 - \alpha_1) - \frac{k^2 T}{g\left(\rho_1+\rho_2\right)} > 0 \tag{31}$$

or

$$k < \sqrt{\tfrac{(\rho_2-\rho_1)g}{T}}. \tag{32}$$

The arrangement remains stable for all disturbances $k > k_c$, where the cutoff wave number k_c or equivalently a cutoff wavelength $\Lambda_c = \frac{2\pi}{k_c}$ is given by

$$k_c = \frac{2\pi}{\Lambda_c} = \sqrt{\tfrac{(\rho_2-\rho_1)g}{T}}. \tag{33}$$

The configuration is unstable for $k < k_c$ and after this cutoff wave number the perturbations turned out to be stabilized. Now, we consider two cases from (30).

3.1. Case I: Stable configuration ($\alpha_1 > \alpha_2$)

As (30) does not allow any real positive or complex root with the real positive part showing thereby, the stability of the system implies that the system fulfills the necessary stability condition of the Routh–Hurwitz criterion, since the principal diagonal minors from (30) are given by

$$\Delta_1 = 1 + \frac{2mN}{\rho_1+\rho_2} + \frac{\varepsilon\tau\left(\alpha_1 \upsilon_1 + \alpha_2 \upsilon_2\right)}{k_1\left(1+\lambda\right)} > 0, \tag{34}$$

$$\Delta_2 = 1 + \frac{\varepsilon\left(\alpha_1 \upsilon_1 + \alpha_2 \upsilon_2\right)}{k_1\left(1+\lambda\right)} \left[\frac{2mN}{\rho_1+\rho_2} + \frac{\varepsilon\tau\left(\alpha_1 \upsilon_1 + \alpha_2 \upsilon_2\right)}{k_1\left(1+\lambda\right)} \right] \left[\tau gk(\alpha_2-\alpha_1) + \frac{k^2 T}{g\left(\rho_1+\rho_2\right)} \right] + \frac{\varepsilon\left(\alpha_1 \upsilon_1 + \alpha_2 \upsilon_2\right)}{k_1\left(1+\lambda\right)} > 0, \tag{35}$$

$$\Delta_3 = gk \left[(\alpha_2-\alpha_1) + \frac{k^2 T}{g\left(\rho_1+\rho_2\right)} \right] \left\{ 1 + \frac{\varepsilon\left(\alpha_1 \nu_1 + \alpha_2 \nu_2\right)}{k_1} + \left(\frac{2mN}{\rho_1+\rho_2} + \frac{\varepsilon\tau\left(\alpha_1 \nu_1 + \alpha_2 \nu_2\right)}{k_1} \right) + \left(\tau gk(\alpha_2-\alpha_1) + \frac{k^2 T}{g\left(\rho_1+\rho_2\right)} \right) + \frac{\varepsilon\left(\alpha_1 \nu_1 + \alpha_2 \nu_2\right)}{k_1} \right\} > 0. \tag{36}$$

Hence, when $\alpha_1 > \alpha_2$, all Δ_i $(i = 1, 2, 3)$ are positive satisfying the sufficient condition of Routh–Hurwitz criterion. Thus, a necessary and sufficient condition of the Routh–Hurwitz criterion is satisfied with $\rho_1 > \rho_2$.

3.2. Case II: Unstable case ($\alpha_1 < \alpha_2$)

If we assume the case where the upper fluid is heavier than the lower fluid, the system will be unstable. There is one real positive root of (30), which leads to configuration instability. By using the Routh–Hurwitz criterion to (30), we find the condition for instability as

$$(\alpha_2 - \alpha_1) - \frac{k^2 T}{g\left(\rho_1+\rho_2\right)} < 0, \tag{37}$$

It is noteworthy from (37) that the stability condition remains unaffected due to the presence of porosity, suspended dust particle and Jeffrey parameters.

Thus, the configuration is stable for all disturbances with $k > k_c$ (given by (30)) and the surface tension leads to the stabilization of the potentially unstable configuration of all sufficiently short wave lengths; however, the configuration remain unstable for all long wave lengths.

Now, introducing the non-dimensional physical quantities: $\sigma^* = \frac{\sigma}{\sqrt{gk}}$, $f_s^* = \frac{f_s}{\sqrt{gk}}$ and $\nu^* = \frac{\nu}{\sqrt{gk}}$ (asterisks are dropped for the sake of convenience), (30) transforms to the non-dimensional form (Prajapati and Chhajlani [11])

$$\sigma^3 + \sigma^2 \left[f_s \left(1 + 2\alpha_0\right) + \frac{2\varepsilon\nu}{k_1\left(1+\lambda\right)} \right] - \sigma \left[(\alpha_1-\alpha_2)\left(1 - \frac{k^2}{k_c^2}\right) - \frac{2\varepsilon f_s \nu}{k_1\left(1+\lambda\right)} \right] - f_s(\alpha_1-\alpha_2)\left(1 - \frac{k^2}{k_c^2}\right) = 0. \tag{38}$$

which is the required dispersion relation to examine stability/instability criterion.

4. Numerical results and discussion

To investigate the effects of various parameters on the stability of the system under consideration, numerical calculations were performed for the dispersion relation (38) and analyzed by computing the values of the RTI growth rate numerically using software Mathematica version 12. The aim was to demonstrate the influence of a variety of fluid properties accounting for perspective physical variables on the stability of the system.

In Fig. 1 the behavior of kinematic viscosity on the growth rate of unstable perturbed mode (σ) versus relaxation frequency of suspended dust particles (f_s) for different distinct values of kinematic viscosity $\nu = 0, 0.1, 0.2, 0.3$ has been emphasized. It is depicted from the curves that the value of the growth rate descends with the increase in the value of kinematic viscosity. Thus, the kinematic viscosity has a damping effect on the growth rate of RTI.

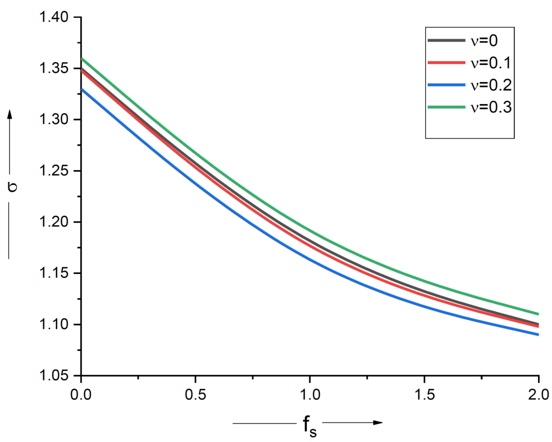

Fig. 1. Plot of the growth rate of unstable RT mode σ versus relaxation frequency of dust particles f_s, for different values of kinematic viscosity v.

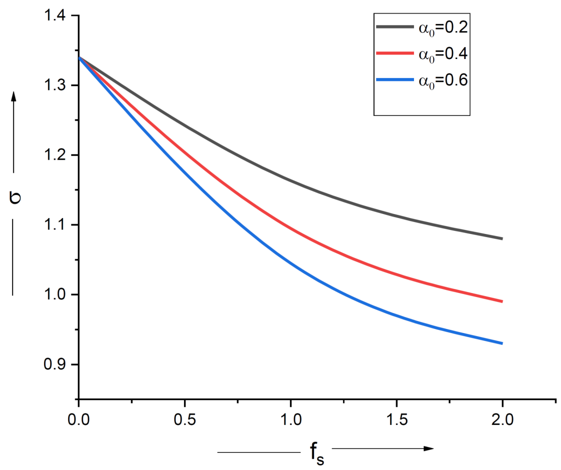

Fig. 2. Plot of growth rate of unstable RT mode σ versus relaxation frequency of dust particles f_s, for different values of mass concentration α_0.

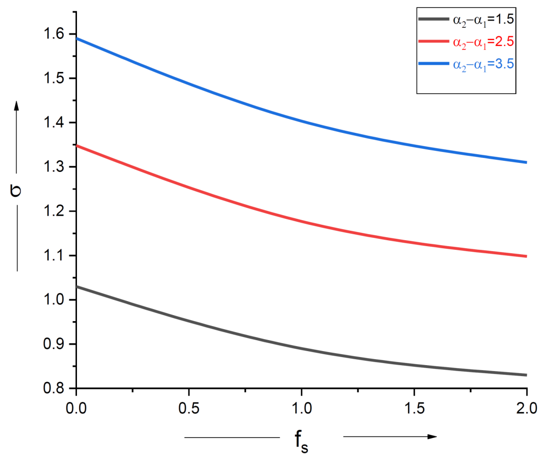

Fig. 3. Plot of growth rate of unstable RT mode σ versus relaxation frequency of dust particles f_s, for different values of density difference between fluids $(\alpha_2 - \alpha_1)$.

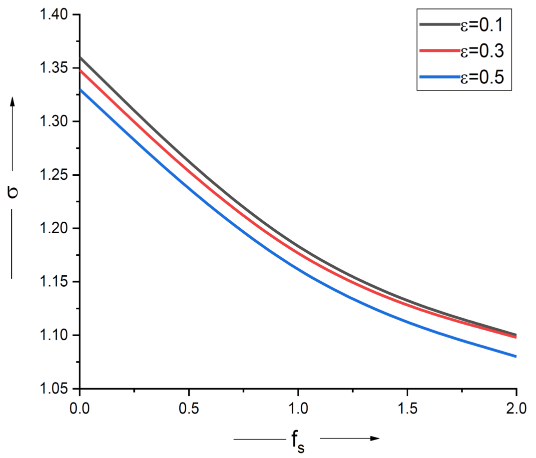

Fig. 4. Plot of growth rate of unstable RT mode σ versus relaxation frequency of dust particles f_s, for different values of porosity ε.

It is also noticed that the maximum value taken by the growth rate is higher for the system with $\nu = 0$. The impact of the mass concentration of dust particles on the growth rate of unstable perturbed mode (σ) versus relaxation frequency of suspended dust particle (f_s) for different values of the mass concentration $\alpha_0 = 0.2, 0.4, 0.6$ has been displayed in Fig. 2. From the curves, it is assessed that the RTI growth rate decreases with an increase in the value of the mass concentration of dust particles. Henceforth, the level of mass concentration of dust particles has a substantial stabilizing impact on the growth rate of the RT configuration.

The growth rate of unstable perturbed mode (σ) versus relaxation frequency of suspended dust particles (f_s) for different values of density difference between fluids, $(\alpha_2 - \alpha_1) = 1.5, 2.5, 3.5$, is plotted in Fig. 3. The curves illustrate an increment in the

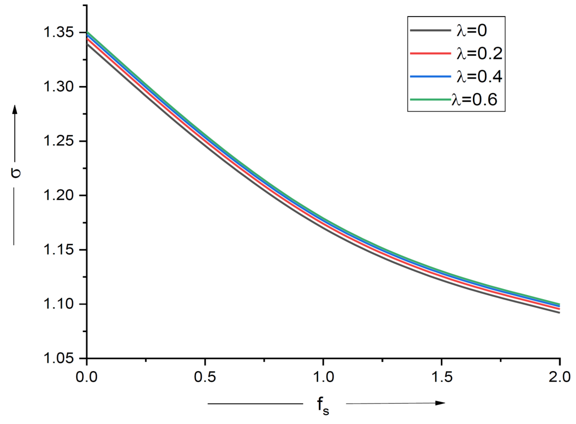

Fig. 5. Plot of growth rate of unstable RT mode σ versus relaxation frequency of dust particles f_s, for different values of the Jeffrey parameter λ.

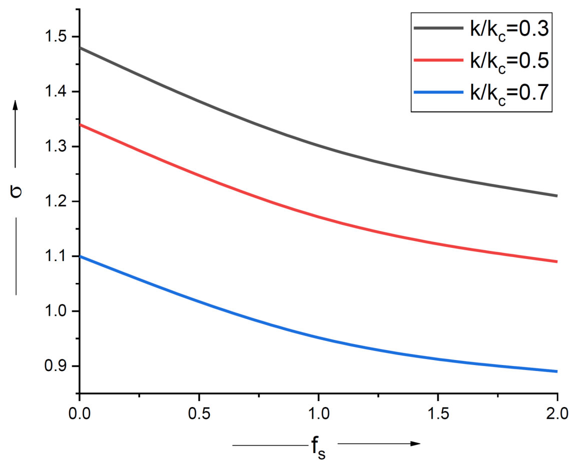

Fig. 6. Plot of growth rate of unstable RT σ versus relaxation frequency of dust particles f_s, for different values of surface tension between fluids k/k_c.

growth rate with an increment in the density difference between the fluids. Thus, the growth rate for a system of two Jeffrey fluids tends to a maximum value with a large density difference. It is also observed that the maximum value of the growth rate increases with the increase in the density difference between the fluids.

Figure 4 displays the effect of the medium porosity on the growth rate of unstable perturbed mode (σ) versus relaxation frequency of suspended dust particles (f_s) for distinct values of porosity $\varepsilon = 0.1, 0.3, 0.5$.

From the curves, it is assessed that an increment in porosity leads to a decrement in the growth rate. Thus porosity has a stabilizing impact on the growth rate of the unstable mode of RTI.

The influence of different values of the Jeffrey parameter $\lambda = 0, 0.2, 0.4, 0.6$ on the growth rate of unstable perturbed mode (σ) versus relaxation frequency of suspended dust particles (f_s) has been displayed in Fig. 5. The curves show an increase in the growth rate of the unstable mode of RTI upon increasing the value of the Jeffrey parameter. Hence, the Jeffrey parameter has a destabilizing impact on the growth of RTI. It is also noticed from the curve that the maximum value attained by the growth rate is smaller for λ values considered. The influence of different values of surface tension $\frac{k}{k_c} = 0.3, 0.5, 0.7$ on the growth rate of unstable perturbed mode (σ) versus relaxation frequency of suspended dust particles (f_s) has been shown in Fig. 6. The curves depict a fall in the values of the growth rate with an increase in the surface tension values. Hence, the surface tension has a substantial stabilizing impact, thereby, suppressing RTI.

5. Conclusions

The Rayleigh–Taylor instability (RTI) of the interface between two superposed Jeffrey fluids saturated by a porous medium embedded by suspended dust particles and the inclusion of interfacial surface tension is analyzed. The criterion of this instability and the cutoff wave numbers are found to be uninfluenced due to the Jeffrey parameter, permeability, and suspended dust particles. However, they are modified by the presence of surface tension. The surface tension shows a stabilizing role on the growth rate of the considered configuration, thereby, decreasing the cutoff wave number. The medium porosity, dynamic viscosity and density of dust particles stabilize rhe Rayleigh–Taylor instability. Furthermore, the density difference between the fluids and the Jeffrey parameter describing the non-Newtonian behavior of fluid tend to destabilize the system. The maximum value of the growth rate is also found to depend on the surface tension, dynamic viscosity, and embedded dust particles. The Routh–Hurwitz criterion is applied to discuss the stability of the system under consideration.

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