The Effects of Single-Mode Coherent and Squeezed Lights on Nondegenerate Three-level Laser

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We consider a nondegenerate three-level laser in which the three-level atoms are in a cascade configuration, the laser cavity contains a degenerate parametric oscillator, and one of the cavity modes is driven by coherent light. We have analyzed the effects of the degenerate parametric oscillator and the driving coherent light on the laser using the expectation values of the cavity mode variables at a steady state. The results show that the two-mode light produced by the system under consideration exhibits quadrature squeezing. The presence of nonlinear crystal in the laser cavity generates a single-mode squeezed light and also enhances the degree of squeezing of the two-mode light. Although the driving coherent light has no effect on quadrature squeezing, it increases the intensity of the cavity modes.

topics: parametric oscillators, quadrature squeezing, three-level laser

1. Introduction

Three-level laser is a quantum optical system in which three-level atoms in a cascade configuration, initially prepared in a coherent superposition of the top and bottom levels, are injected into the cavity coupled to a vacuum reservoir via a single-port mirror. When a three-level atom makes a transition from the top to bottom level via the intermediate level, two photons are generated. If the generated light modes have the same frequencies, it is called a degenerate three-level laser; otherwise, it is called a nondegenerate three-level laser. The two generated photons are highly correlated, and this correlation is responsible for the non-classical future of the light produced by the system. Squeezed states are the non-classical future of light that cannot be explained using classical theories or characterized by the reduction of quantum fluctuations (noise) in one quadrature below that of the quantum standard limit or below that achievable in a coherent state at the expense of increased fluctuations in another conjugate quadrature, such that the product of these fluctuations still obeys the uncertainty relation.

An optical parametric oscillator is a quantum optical system and the most efficient source of squeezing. This quantum optical system consists of a nonlinear crystal pumped by coherent light and the cavity modes coupled to a vacuum reservoir via a single-port mirror. In a parametric oscillator, a pump photon of frequency 2ω is down-converted into a pair of correlated photons. It has achieved the best squeezing of 93% noise reduction relative to the vacuum [1–4].

Several studies have shown that three-level lasers can produce squeezed light under certain conditions: when the atoms are initially prepared in a coherent superposition of the top and bottom levels or when these levels are coupled by strong coherent light [5–13]. Moreover, Alebachew and Fesseha [5] have considered a degenerate three-level laser, the cavity of which contains a parametric amplifier, with the top and bottom levels of the injected atoms coupled by the pump mode emerging from the parametric amplifier. They have studied this system for the specific case in which the numbers of atoms initially in the top and bottom levels are equal. They have found that the system generates a highly squeezed light under certain conditions. The squeezing, in this case, is exclusively due to the parametric amplifier and the coupling of the top and bottom levels.

The main finding of this paper is to show the effects of single-mode driving coherent light and degenerate parametric oscillator on the nondegenerate three-level laser. Many studies have shown that nondegenerate three-level laser generates squeezed light [6, 7]. Squeezed light has potential and practical applications such as gravitational wave detection, noiseless communication etc. Our research indicates that there is a possibility to enhance the degree of squeezing and mean photon number of

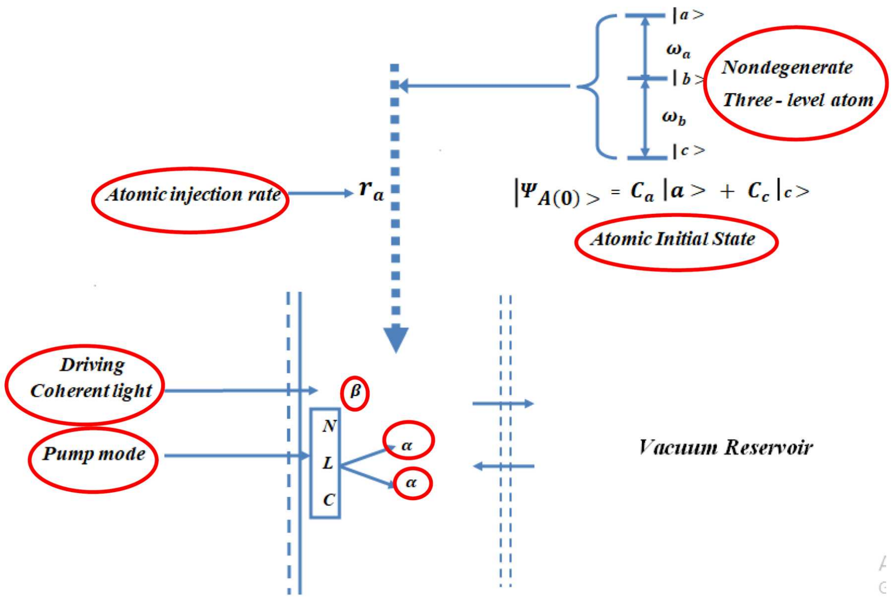

Fig. 1. Schematic representation of a nondegenerate three-level laser with a degenerate parametric oscillator, driven by coherent light, and coupled to vacuum reservoir.

the light generated by the nondegenerate three-level laser, if the laser cavity contains a single-mode degenerate parametric amplifier and one of the cavity modes driven by coherent light.

2. Materials and methods

Our system is a nondegenerate three-level laser in which the three-level atoms in a cascade configuration, initially prepared in a coherent superposition of the upper and bottom levels, are injected into the cavity at some constant rate r_a. It is assumed that the laser cavity contains a non-linear crystal that generates paired modes with the same frequencies. One of the cavity modes is driven by coherent light, and the cavity modes are coupled to a vacuum reservoir (as seen in Fig. 1). Using the master equation of the system under consideration, we have obtained the expectation values of the c-number cavity mode variables associated with the normal order at steady state. Making use of the resulting expectation values, the quadrature variances and mean photon numbers of the cavity modes have been calculated. Finally, with the help of the definition for quadrature squeezing relative to coherent/vacuum state and plots, we analyze the properties of the quadrature squeezing and the mean photon number using MATLAB software.

2.1. Master equation

A nondegenerate three-level laser with a degenerate parametric oscillator and one of the cavity modes driven by single-mode coherent light, as well as the cavity modes coupled to the vacuum reservoir, can be described by the Hamiltonian (as seen in Fig. 1)

$$\begin{aligned}\hat{H} = \mathrm{i}g\Big(|a\rangle\langle b|\hat{a} - \hat{a}^\dagger|b\rangle\langle a| + |b\rangle\langle c|\hat{b} - \hat{b}^\dagger|c\rangle\langle b|\Big) \\ +\mathrm{i}\varepsilon\Big(\hat{b}^\dagger - \hat{b}\Big) + \frac{\mathrm{i}}{2}\lambda\Big(\hat{a}^{\dagger 2} - \hat{a}^2\Big) \\ +\mathrm{i}\epsilon\left(\hat{a}^\dagger\hat{a}_{\mathrm{in}} - \hat{a}^\dagger_{\mathrm{in}}\hat{a} + \hat{b}^\dagger\hat{b}_{\mathrm{in}} - \hat{b}^\dagger_{\mathrm{in}}\hat{b}\right), \qquad (1)\end{aligned}$$

where g is the coupling constant for the interaction between the atom and cavity modes, ϵ is the coupling constant for the interaction between the cavity and reservoir modes, ε is the amplitude proportional to the driving mode, λ is the amplitude proportional to the pump mode, $\hat{a}$ and $\hat{b}$ are annihilation operators for the cavity modes, and $\hat{a}_{\mathrm{in}}$ and $\hat{b}_{\mathrm{in}}$ are the input operators. We take the atom to be initially in state

$$|\psi_A(0)\rangle = C_a|a\rangle + C_c|c\rangle. \qquad (2)$$

The density operator corresponding to this state for a single atom is

$$\begin{aligned}\hat{\rho}_A(0) = \rho^{(0)}_{aa}|a\rangle\langle a| + \rho^{(0)}_{ac}|a\rangle\langle c| \\ +\rho^{(0)}_{ca}|c\rangle\langle a| + \rho^{(0)}_{cc}|c\rangle\langle c|, \qquad (3)\end{aligned}$$

where $\rho^{(0)}_{aa} = C^*_a C_a$ and $\rho^{(0)}_{cc} = C^*_c C_c$ are the probability of atom finding in the upper level and the lower level, respectively, and $|\rho^{(0)}_{ac}|^2 = \rho^{(0)}_{aa}\rho^{(0)}_{cc}$ represents the atomic coherence. The equation of evolution of the density operator for the nondegenerate three-level laser is given by [14] as following

$$\begin{aligned}\frac{\mathrm{d}\hat{\rho}(t)}{\mathrm{d}t} = \frac{A\rho^{(0)}_{aa}}{2}\left(2\hat{a}^\dagger\hat{\rho}\hat{a} - \hat{a}\hat{a}^\dagger\hat{\rho} - \hat{\rho}\hat{a}\hat{a}^\dagger\right) \\ +\frac{A\rho^{(0)}_{cc}}{2}\Big(2\hat{b}\hat{\rho}\hat{b}^\dagger - \hat{\rho}\hat{b}^\dagger\hat{b} - \hat{b}^\dagger\hat{b}\hat{\rho}\Big) \\ -\frac{A\rho^{(0)}_{ac}}{2}\Big(2\hat{b}\hat{\rho}\hat{a} - \hat{a}\hat{b}\hat{\rho} - \hat{\rho}\hat{a}\hat{b}\Big), \qquad (4)\end{aligned}$$

in which

$$A = \frac{2g^2 r_a}{\gamma^2} \tag{5}$$

is the atomic linear gain coefficient, with r_a and γ being the atomic injection rate and the atomic decay constants, respectively. The equation of evolution of the density operator corresponding to the degenerate parametric oscillator and driving coherent light is

$$\frac{\mathrm{d}\hat{\rho}(t)}{\mathrm{d}t} = \frac{1}{2}\lambda\Big(\hat{a}^{\dagger 2}\hat{\rho} - \hat{a}^2\hat{\rho} - \hat{\rho}\hat{a}^{\dagger 2} + \hat{\rho}\hat{a}^2\Big)$$
$$+\varepsilon\Big(\hat{b}^{\dagger}\hat{\rho} - \hat{b}\hat{\rho} - \hat{\rho}\hat{b}^{\dagger} + \hat{\rho}\hat{b}\Big). \tag{6}$$

Moreover, the equation of evolution of the reduced density operator for cavity modes coupled to vacuum reservoir can be written as [14]

$$\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}t} = \frac{1}{2}\kappa\Big(2\hat{a}\hat{\rho}\hat{a}^{\dagger} - \hat{a}^{\dagger}\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger}\hat{a}$$
$$+2\hat{b}\hat{\rho}\hat{b}^{\dagger} - \hat{b}^{\dagger}\hat{b}\hat{\rho} - \hat{\rho}\hat{b}^{\dagger}\hat{b}\Big), \tag{7}$$

where κ is the cavity decay constant. With the aid of (4), (6) and (7), the master equation for the system under consideration can be written in the form

$$\frac{\mathrm{d}\hat{\rho}(t)}{\mathrm{d}t} = \frac{1}{2}A\rho_{aa}^{(0)}\Big(\hat{a}^{\dagger}\hat{\rho}\hat{a} - \hat{a}\hat{a}^{\dagger}\hat{\rho}\Big)$$
$$+\frac{1}{2}\kappa\Big(\hat{a}\hat{\rho}\hat{a}^{\dagger} - \hat{a}^{\dagger}\hat{a}\hat{\rho}\Big)$$
$$+\frac{1}{2}\Big(\kappa + A\rho_{cc}^{(0)}\Big)\Big(\hat{b}\hat{\rho}\hat{b}^{\dagger} - \hat{b}^{\dagger}\hat{b}\hat{\rho}\Big)$$
$$-\frac{1}{2}A\rho_{ac}^{(0)}\Big(2\hat{b}\hat{\rho}\hat{a} - \hat{a}\hat{b}\hat{\rho} - \hat{\rho}\hat{a}\hat{b}\Big)$$
$$+\frac{1}{2}\lambda\Big(\hat{\rho}\hat{a}^2 - \hat{\rho}\hat{a}^{\dagger 2}\Big) + \varepsilon\Big(\hat{\rho}\hat{b} - \hat{b}\hat{\rho}\Big) + \mathrm{C.C}, \tag{8}$$

in which C.C represents the complex conjugate of the equation, A is the linear gain coefficient of the laser, and κ is the dumping constant of the cavity modes.

2.2. The cavity-mode variables

We next proceed to determine the expectation values of the c-number cavity mode variables associated with the normal order at steady state. To this end, employing (8) and the relation $\frac{\mathrm{d}}{\mathrm{d}t}\langle\hat{A}\rangle = \mathrm{Tr}\big(\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}\hat{A}\big)$, we easily find

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\alpha\rangle = -\frac{1}{2}\mu_a\langle\alpha\rangle - \frac{1}{2}\nu\langle\beta^*\rangle + \lambda\langle\alpha^*\rangle, \tag{9}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\beta\rangle = -\frac{1}{2}\mu_c\langle\beta\rangle + \frac{1}{2}\nu\langle\alpha^*\rangle + \varepsilon, \tag{10}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\alpha^2\rangle = -\mu_a\langle\alpha^2\rangle - \nu\langle\beta^*\alpha\rangle + 2\lambda\langle\alpha^*\alpha\rangle + \lambda, \tag{11}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\beta^2\rangle = -\mu_c\langle\beta^2\rangle + \nu\langle\alpha^*\beta\rangle + 2\varepsilon\langle\beta\rangle, \tag{12}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\alpha^*\alpha\rangle = -\mu_a\langle\alpha^*\alpha\rangle - \frac{1}{2}\nu\left(\langle\alpha^*\beta^*\rangle + \langle\alpha\beta\rangle\right)$$
$$+\lambda\left(\langle\alpha^{*2}\rangle + \langle\alpha^2\rangle\right) + A\rho_{aa}^{(0)}, \tag{13}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\beta^*\beta\rangle = -\mu_c\langle\beta^*\beta\rangle + \frac{1}{2}\nu\left(\langle\alpha^*\beta^*\rangle + \langle\alpha\beta\rangle\right)$$
$$+\varepsilon\left(\langle\beta^*\rangle + \langle\beta\rangle\right), \tag{14}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\alpha\beta\rangle = -\frac{1}{2}(\mu_a + \mu_c)\langle\alpha\beta\rangle + \lambda\langle\alpha^*\beta\rangle$$
$$+\frac{1}{2}\nu\left(\langle\alpha^*\alpha\rangle - \langle\beta^*\beta\rangle\right) + \varepsilon\langle\alpha\rangle + \frac{1}{2}\nu, \tag{15}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\alpha^*\beta\rangle = -\frac{1}{2}\left(\mu_a + \mu_c\right)\langle\alpha^*\beta\rangle + \lambda\langle\alpha\beta\rangle$$
$$+\frac{1}{2}\nu\left(\langle\alpha^{*2}\rangle - \langle\beta^2\rangle\right) + \varepsilon\langle\alpha^*\rangle, \tag{16}$$

where

$$\mu_a = \frac{1}{2}(2\kappa + A\eta - A), \tag{17}$$

$$\mu_c = \frac{1}{2}(2\kappa + A\eta + A), \tag{18}$$

$$\nu = \frac{1}{2}A\sqrt{1-\eta^2}, \tag{19}$$

with

$$\eta = \rho_{cc}^{(0)} - \rho_{aa}^{(0)}. \tag{20}$$

Furthermore, we introduce new variables defined by

$$A_{\pm} = \langle\alpha^*\alpha\rangle \pm \langle\alpha^2\rangle, \tag{21}$$

$$B_{\pm} = \langle\beta^*\beta\rangle \pm \langle\beta^2\rangle, \tag{22}$$

$$C_{\pm} = \langle\alpha^*\beta\rangle \pm \langle\alpha\beta\rangle. \tag{23}$$

Taking into account (9)–(16) at steady state, we can write that

$$\langle\alpha\rangle = -\frac{2\varepsilon\nu}{\mu_c\mu_{a-} + \nu^2}, \tag{24}$$

$$\langle\beta\rangle = \frac{2\varepsilon\mu_{a-}}{\mu_c\mu_{a-} + \nu^2}. \tag{25}$$

$$\mu_{a\mp}A_{\pm} \pm \nu C_{\pm} = \frac{1}{2}\left(A(1-\eta) \pm 2\lambda\right), \tag{26}$$

$$\mu_c B_{\pm} \mp \nu C_{\pm} = 2(1\pm 1)\varepsilon\langle\beta\rangle, \tag{27}$$

$$(\mu_{a\mp} + \mu_c)C_{\pm} \mp \nu A_{\pm} \pm \nu B_{\pm} =$$
$$2(1\pm 1)\varepsilon\langle\alpha\rangle \pm \nu, \tag{28}$$

where

$$\mu_{a\mp} = \mu_a \mp 2\lambda. \tag{29}$$

With the aid of (26)–(28) we find

$$A_{\pm} = \frac{[A(1-\eta)\pm 2\lambda][\mu_c^2 + \mu_{a\mp}\mu_c + \nu^2] - 2\mu_c\nu^2}{2(\mu_{a\mp} + \mu_c)[\mu_{a\mp}\mu_c + \nu^2]}$$
$$+\frac{4(1\pm 1)\varepsilon^2\nu^2(\mu_{a-} \pm \mu_c)}{(\mu_c\mu_{a-} + \nu^2)(\mu_{a\mp} + \mu_c)[\mu_{a\mp}\mu_c + \nu^2]}, \tag{30}$$

$$B_\pm = \frac{\nu^2[\kappa \mp \lambda]}{(\mu_{a\mp} + \mu_c)[\mu_{a\mp}\mu_c + \nu^2]} + \frac{4(1 \pm 1)\varepsilon^2}{(\mu_c\mu_{a-} + \nu^2)} \times \frac{[\mu_{a-}\mu_{a\mp}[\mu_{a\mp} + \mu_c] + [\mu_{a-} \mp \mu_{a\mp}]\nu^2]}{(\mu_{a\mp} + \mu_c)[\mu_{a\mp}\mu_c + \nu^2]}, \quad (31)$$

$$C_\pm = \pm\frac{\mu_c\nu[\kappa \mp \lambda]}{(\mu_{a\mp} + \mu_c)[\mu_{a\mp}\mu_c + \nu^2]} - \frac{4(1 \pm 1)\varepsilon^2\mu_{a\mp}\nu(\mu_{a-} \pm \mu_c)}{(\mu_c\mu_{a-} + \nu^2)(\mu_{a\mp} + \mu_c)[\mu_{a\mp}\mu_c + \nu^2]}. \quad (32)$$

2.3. Quadrature squeezing

In Sect. 2.3 we seek to analyze the effects of the driving coherent light and the parametric amplifier on the quadrature squeezing of the cavity modes of nondegenerate three-level laser.

2.3.1. Quadrature squeezing of a single-mode light

The variances of the plus and minus quadratures for mode a and mode b defined by operators

$$\hat{a}_\pm = \sqrt{\pm 1}(\hat{a}^\dagger \pm \hat{a}),$$

$$\hat{b}_\pm = \sqrt{\pm 1}(\hat{b}^\dagger \pm \hat{b}), \quad (33)$$

can be written as

$$\Delta a_\pm^2 = \langle \hat{a}_\pm, \hat{a}_\pm \rangle,$$

$$\Delta b_\pm^2 = \langle \hat{b}_\pm, \hat{b}_\pm \rangle. \quad (34)$$

It can be easily verified that

$$[\hat{a}_+, \hat{a}_-] = [\hat{b}_+, \hat{b}_-] = 2\mathrm{i}. \quad (35)$$

A single-mode light is said to be in a squeezed state if one of the quadrature variances is less than that of the quadrature variance of the coherent/vacuum state with the satisfaction of the uncertainty principle. Employing (21) and (22), (34) can be expressed in terms of c-number variables associated with the normal order as

$$\Delta a_\pm^2 = 1 + 2A_\pm - 2(1 \pm 1)\langle\alpha\rangle^2,$$

$$\Delta b_\pm^2 = 1 + 2B_\pm - 2(1 \pm 1)\langle\beta\rangle^2. \quad (36)$$

On account of (24), (25), (30), and (31), we obtain

$$\Delta a_\pm^2 = 1 + \frac{[A(1-\eta) \pm 2\lambda][\mu_c^2 + \mu_{a\mp}\mu_c + \nu^2] - 4\mu_c\nu^2}{2(\mu_{a\mp} + \mu_c)[\mu_{a\mp}\mu_c + \nu^2]} + \frac{8(1 \pm 1)\varepsilon^2\nu^2(\mu_{a-} \pm \mu_c)}{(\mu_c\mu_{a-} + \nu^2)(\mu_{a\mp} + \mu_c)[\mu_{a\mp}\mu_c + \nu^2]} - \frac{8(1 \pm 1)\varepsilon^2\nu^2}{(\mu_c\mu_{a-} + \nu^2)^2}, \quad (37)$$

$$\Delta b_\pm^2 = 1 + \frac{2\nu^2[\kappa \mp \lambda]}{(\mu_{a\mp} + \mu_c)[\mu_{a\mp}\mu_c + \nu^2]} + \frac{8(1 \pm 1)\varepsilon^2\left[\mu_{a-}[\mu_{a\mp}^2 + \mu_{a\mp}\mu_c + \nu^2] \mp \mu_{a\mp}\nu^2\right]}{(\mu_c\mu_{a-} + \nu^2)(\mu_{a\mp} + \mu_c)[\mu_{a\mp}\mu_c + \nu^2]} - \frac{8(1 \pm 1)\varepsilon^2\mu_{a-}^2}{(\mu_c\mu_{a-} + \nu^2)^2}, \quad (38)$$

which represent the quadrature variances of mode a and mode b, respectively.

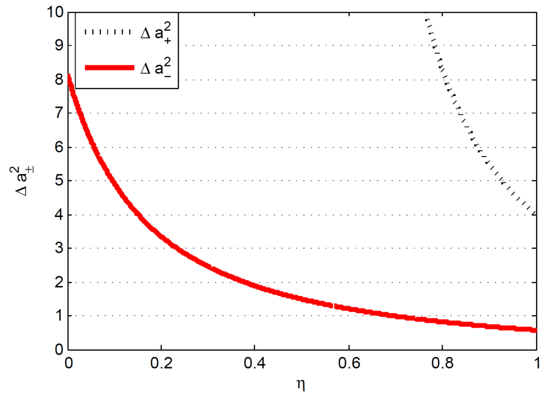

Fig. 2. Plots of the quadrature variances of mode a.

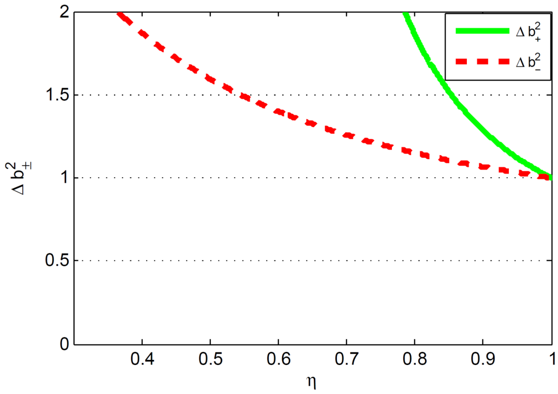

Fig. 3. Plots of the quadrature variances of mode b.

Figures 2 and 3 represent the plots of the quadrature variances of, respectively, mode a (37) and mode b (38) vs η, for $\kappa = 0.8$, $A = 10$, $\lambda = 0.3$, and $\varepsilon = 0.2$. We see in the figures that mode a exhibits squeezing in the minus quadrature for the value $\eta > 0.7$. Mode b, however, does not exhibit any squeezing in both quadratures.

The quadrature squeezing of mode a relative to the quadrature variance of coherent/vacuum state is defined by

$$S = \frac{(\Delta a_\pm^2)_{v/c} - (\Delta a_\pm^2)}{(\Delta a_\pm^2)_{v/c}}, \quad (39)$$

where $(\Delta a_\pm^2)_{v/c}$ is the quadrature variance of the coherent/vacuum state. We note that the quadrature variance of the coherent/vacuum state is one. In view of this, (39) can be written as

$$S = 1 - \Delta a_-^2. \quad (40)$$

Taking (37) into account, we obtain

$$S_a = -\frac{(A(1-\eta) - 2\lambda)[\mu_c^2 + \mu_{a+}\mu_c + \nu^2] - 4\mu_c\nu^2}{2(\mu_{a+} + \mu_c)[\mu_{a+}\mu_c + \nu^2]}. \quad (41)$$

Since the variable ε does not appear in (41), the driving coherent light has no effect on the quadrature squeezing of mode a.

2.3.2. Quadrature squeezing of two-mode light

In Sect. 2.3.2 we calculate the quadrature squeezing of a two-mode light produced by the system under consideration. We define the quadrature variance for a two-mode light as

$$\Delta c_{\pm}^2 = \langle \hat{c}_{\pm}, \hat{c}_{\pm} \rangle, \tag{42}$$

where

$$\hat{c}_{\pm} = \frac{\sqrt{\pm 1}}{2} \left(\hat{a}^{\dagger} \pm \hat{a} + \hat{b}^{\dagger} \pm \hat{b} \right) \tag{43}$$

is the quadrature operator for two-mode light. One can easily verify that

$$[\hat{c}_+, \hat{c}_-] = 2\mathrm{i}. \tag{44}$$

A two-mode light is said to be in the squeezed state if either $\Delta c_+^2 < 1$ or $\Delta c_-^2 < 1$, provided that $\Delta c_+^2 \Delta c_-^2 \geq 1$.

In view of (36), (42) can be written as

$$\Delta c_{\pm}^2 = 1 + A_{\pm} + B_{\pm} + C_{\pm} - (1 \pm 1)(\langle \alpha \rangle + \langle \beta \rangle)^2. \tag{45}$$

Up on substituting (24), (25), and (30)–(32) in (45), we have

$$\Delta c_{\pm}^2 = 1 + \frac{[A(1-\eta) \pm 2\lambda][\mu_c^2 + \mu_{a\mp}\mu_c + \nu^2]}{2(\mu_{a\mp} + \mu_c)[\mu_{a\mp}\mu_c + \nu^2]}$$
$$- \frac{\mu_c \nu [\kappa \mp \lambda] + \nu^2 [\mu_c - (\kappa \mp \lambda)]}{(\mu_{a\mp} + \mu_c)[\mu_{a\mp}\mu_c + \nu^2]}$$
$$+ \frac{4(1 \pm 1)\varepsilon^2 \nu \left[(\mu_{a-} \pm \mu_c)[\nu - \mu_{a\mp}] \mp \mu_{a\mp}\nu \right]}{(\mu_c \mu_{a-} + \nu^2)(\mu_{a\mp} + \mu_c)[\mu_{a\mp}\mu_c + \nu^2]}$$
$$+ \frac{4(1 \pm 1)\varepsilon^2 \mu_{a-}[\mu_{a\mp}^2 + \mu_{a\mp}\mu_c + \nu^2]}{(\mu_c \mu_{a-} + \nu^2)(\mu_{a\mp} + \mu_c)[\mu_{a\mp}\mu_c + \nu^2]}$$
$$- \frac{4\varepsilon^2 (1 \pm 1)[\nu^2 + \mu_{a-}^2]}{(\mu_c \mu_{a-} + \nu^2)^2}, \tag{46}$$

which represents the quadrature variances of the two-mode light produced by nondegenerate three-level laser with degenerate parametric oscillator, one of the cavity modes driven by coherent light, and the cavity modes coupled to a vacuum reservoir.

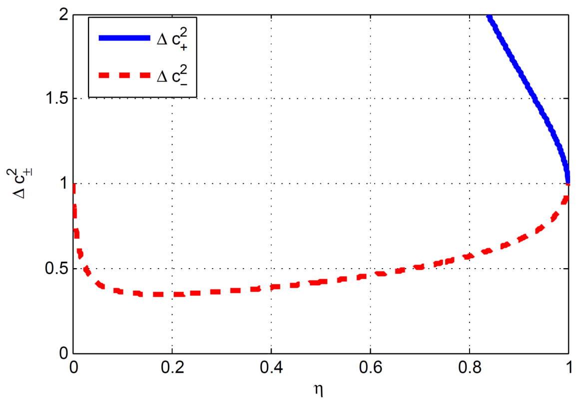

Fig. 4. Plots of the quadrature variances of two-mode light vs η.

Figure 4 presents the plots of quadrature variance of two-mode light (46) vs η for $\kappa = 0.8$, $\lambda = 0.3$, $\varepsilon = 0.2$, and $A = 100$. The plots show that the two-mode light exhibits squeezing in the minus quadrature.

The quadrature squeezing of the two-mode light relative to the quadrature variance of the vacuum state level on the basis of (39), is defined by

$$S = 1 - \Delta c_-^2. \tag{47}$$

In view of (46), we have

$$S_c = -\frac{[A(1-\eta) - 2\lambda][\mu_c^2 + \mu_{a+}\mu_c + \nu^2]}{2(\mu_{a+} + \mu_c)[\mu_{a+}\mu_c + \nu^2]}$$
$$+ \frac{\mu_c \nu (\kappa - \lambda) - \nu^2 [\mu_c - (\kappa + \lambda)]}{(\mu_{a+} + \mu_c)[\mu_{a+}\mu_c + \nu^2]}. \tag{48}$$

We note that as the variable ε does not appear in (48), the driving coherent light has no effect on quadrature squeezing of the two-mode light.

2.4. The photon number sum and difference

In Sect. 2.4 we calculate the mean photon number sum and difference of mode a and mode b produced by a nondegenerate three-level laser with a degenerate parametric oscillator and one of the cavity modes driven by coherent light. The mean photon number sum and difference are defined by

$$\bar{n}_{\pm} = \bar{n}_a \pm \bar{n}_b, \tag{49}$$

where

$$\bar{n}_a = \langle \alpha^* \alpha \rangle, \tag{50}$$

$$\bar{n}_b = \langle \beta^* \beta \rangle \tag{51}$$

are the mean photon numbers for mode a and mode b, respectively. Using (21) and (22), the mean of the photon number for mode a and mode b can be written as

$$\bar{n}_a = \frac{1}{2} \left(A_+ + A_- \right), \tag{52}$$

$$\bar{n}_b = \frac{1}{2} \left(B_+ + B_- \right). \tag{53}$$

Applying (30) and (31), the mean photon number can be found as

$$\bar{n}_a = \frac{[A(1-\eta) + 2\lambda][\mu_c^2 + \mu_{a-}\mu_c + \nu^2] - 2\mu_c \nu^2}{4(\mu_{a-} + \mu_c)[\mu_{a-}\mu_c + \nu^2]}$$
$$+ \frac{4\varepsilon^2 \nu^2}{(\mu_c \mu_{a-} + \nu^2)[\mu_{a-}\mu_c + \nu^2]}$$
$$+ \frac{[A(1-\eta) - 2\lambda][\mu_c^2 + \mu_{a+}\mu_c + \nu^2] - 2\mu_c \nu^2}{4(\mu_{a+} + \mu_c)[\mu_{a+}\mu_c + \nu^2]}, \tag{54}$$

$$\bar{n}_b = \frac{\nu^2 [\kappa - \lambda]}{2(\mu_{a-} + \mu_c)[\mu_{a-}\mu_c + \nu^2]}$$
$$+ \frac{4\varepsilon^2 \mu_{a-}^2}{(\mu_c \mu_{a-} + \nu^2)[\mu_{a-}\mu_c + \nu^2]}$$
$$+ \frac{\nu^2 [\kappa + \lambda]}{2(\mu_{a+} + \mu_c)[\mu_{a+}\mu_c + \nu^2]}. \tag{55}$$

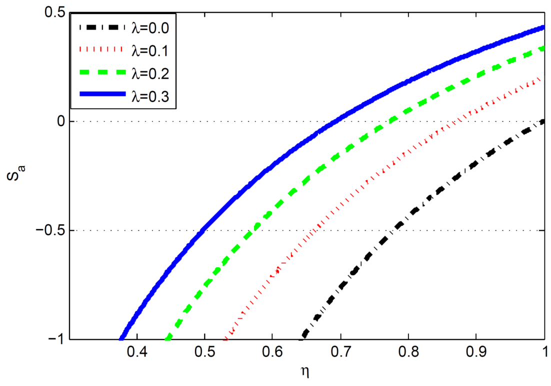

Fig. 5. Plots of the quadrature squeezing of mode a for different values of λ.

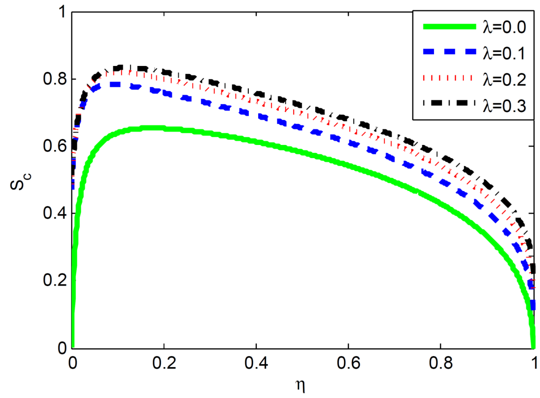

Fig. 6. Plots of the quadrature squeezing two-mode light relative to coherent light for different values of λ.

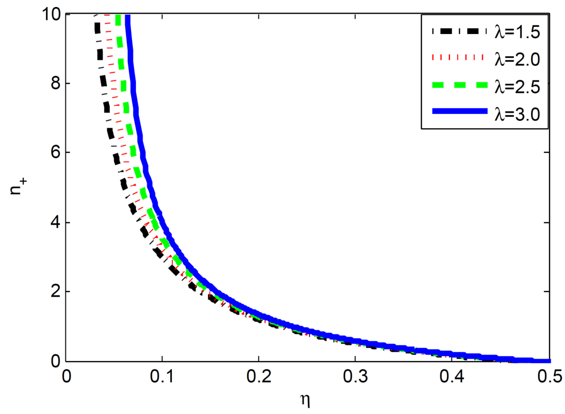

Fig. 7. Plots of the mean photon number sum (56) vs η for $\kappa = 0.8$, $\varepsilon = 0.3$, and $A = 100$ for different values of λ.

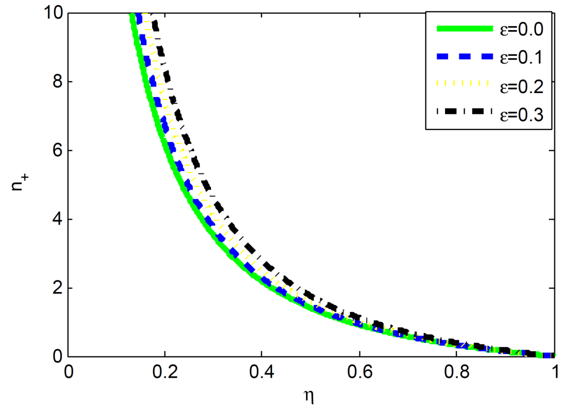

Fig. 8. Plots of the mean photon number sum (56) vs η for $\kappa = 0.8$, $\lambda = 0$, and $A = 10$ for different values of ε.

Hence Combination of (54) and (55) results in

$$\overline{n}_{\pm} = \frac{[A(1-\eta)+2\lambda][\mu_c^2+\mu_{a-}\mu_c+\nu^2]}{4(\mu_{a-}+\mu_c)[\mu_{a-}\mu_c+\nu^2]} - \frac{2\nu^2[\mu_c \mp (\kappa-\lambda)]}{4(\mu_{a-}+\mu_c)[\mu_{a-}\mu_c+\nu^2]} + \frac{4\varepsilon^2\left[\nu^2 \pm \mu_{a-}^2\right]}{(\mu_c\mu_{a-}+\nu^2)[\mu_{a-}\mu_c+\nu^2]} + \frac{[A(1-\eta)-2\lambda][\mu_c^2+\mu_{a+}\mu_c+\nu^2]}{4(\mu_{a+}+\mu_c)[\mu_{a+}\mu_c+\nu^2]} - \frac{2\nu^2[\mu_c \mp (\kappa+\lambda)]}{4(\mu_{a+}+\mu_c)[\mu_{a+}\mu_c+\nu^2]}, \tag{56}$$

which represents the mean photon number sum and difference of mode a and mode b produced by a nondegenerate three-level laser with a degenerate parametric oscillator; the cavity modes are driven by coherent light and coupled to a vacuum reservoir.

3. Results and discussion

In Fig. 5, we plot the quadrature squeezing of mode a relative to coherent state (41) vs η for $\kappa = 0.8$, $A = 100$, $\varepsilon = 0.2$, and for different values of the amplitude proportional to the pump mode $\lambda = 0.0$ (dash-dotted), $\lambda = 0.1$ (dotted), $\lambda = 0.2$ (dashed), and $\lambda = 3.0$ (solid). The plots indicate that the degree of squeezing of mode a increases with the amplitude proportional to the pump mode. We also observe that mode a can exhibit squeezing due to a presence of a degenerate parametric amplifier in the laser cavity. Moreover, we see that 43% of maximum quadrature squeezing can be obtained for the given values at $\eta = 1$.

Figure 6 represents the plots of the quadrature squeezing of two-mode light relative to coherent light (48) vs η for $A = 100$, $\varepsilon = 0.2$, $\kappa = 0.8$, and for different values of the amplitude proportional to pump mode $\lambda = 0.0$ (solid), $\lambda = 0.1$ (dashed), $\lambda = 0.2$ (dotted), and $\lambda = 0.3$ (dash-dotted). The plots indicate that a presence of a degenerate parametric amplifier in the laser cavity increases the degree of squeezing of the two-mode light. The maximum quadrature squeezing for $\varepsilon = 0.2$, $A = 100$, $\kappa = 0.8$ is found to be 83% below the coherent state level.

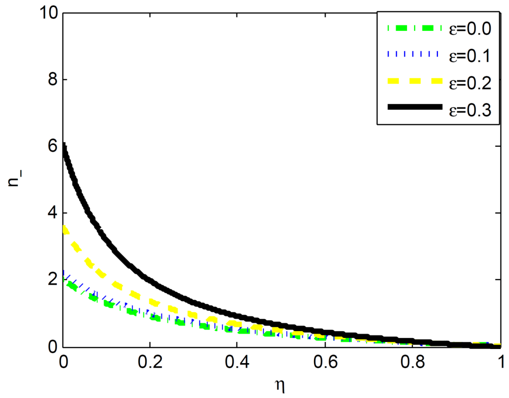

Fig. 9. Plots of the mean photon number difference (56) vs η for $\kappa = 0.8$, $\lambda = 0.1$, and $A = 2$ for different values of driving mode ε.

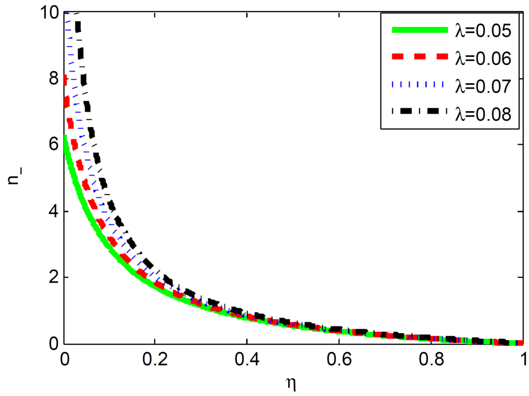

Fig. 10. Plots of the mean photon number difference (56) vs η for $\kappa = 0.8$, $\varepsilon = 0.2$, and $A = 5$ for different values of λ.

In Fig. 7, we plot the mean photon number sum (56) vs η for $\kappa = 0.8$, $\varepsilon = 0.3$, and $A = 100$ for different values of $\lambda = 1.5$ (dash-dotted), $\lambda = 2.0$ (dotted), $\lambda = 2.5$ (dashed), and $\lambda = 0.3$ (solid). In Fig. 8, we plot the mean photon number sum (56) vs η for $\kappa = 0.8$, $\lambda = 0$, and $A = 10$ for different values of $\varepsilon = 0$ (solid), $\varepsilon = 0.1$ (dash), $\varepsilon = 0.2$ (dotted), and $\varepsilon = 0.3$ (dash-dotted). The plots in both figures indicate that the mean of the photon number sum increases with the driving mode, and the amplitude is proportional to the pump mode.

Figure 9 represents the mean photon number difference (56) vs η for $\kappa = 0.8$, $\lambda = 1.0$, and $A = 2$ for different values of driving mode $\varepsilon = 0.0$ (dash-dotted), $\varepsilon = 0.1$(dotted), $\varepsilon = 0.2$ (dashed), and $\varepsilon = 0.3$ (solid). Figure 10 represents the mean photon number difference (56) vs η for $\kappa = 0.8$, $\varepsilon = 0.2$, and $A = 5$ for different values of $\lambda = 0.05$ (solid), $\lambda = 0.06$ (dashed), $\lambda = 0.07$ (dotted), and $\lambda = 0.08$ (dash-dotted). We see in both figures that the mean of the photon number difference increases as the driving coherent light mode and the amplitude proportional to the pump mode increase.

4. Conclusions

We have studied the effects of single-mode driving coherent light and degenerate parametric oscillator on a nondegenerate three-level laser. Using the steady state solutions of the cavity mode variables, we have calculated quadrature squeezing and mean photon number sum and difference. The results show that mode a and the two-mode light produced by the system under consideration exhibit squeezing in the minus quadrature. In addition, we observed that the presence of a degenerate parametric amplifier in the laser cavity enhances the quadrature squeezing of the two-mode light, the mean photon number sum and difference, and it also generates single-mode squeezed light. Although the driving light has no effect on the quadrature squeezing, it increases the mean photon number sum and difference.

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