Resonances Expressed as Minima in Magnetoresistance of a Quantum Wire with Two Barriers Inside

V. Bezák*

Department of Experimental Physics, Comenius University in Bratislava, 84248 Bratislava, Slovakia

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Doi: 10.12693/APhysPolA.142.492 *e-mail: bezak@fmph.uniba.sk

The theory of the magnetoresistance of the InSb quantum wire is presented. The quantum wire is cylindrical in shape with a symmetric pair of delta-barriers inside. A tunable magnetic field parallel with the axis of the quantum wire is considered. The dependence of the Fermi energy on the magnetic field is calculated. The Landauer formula is used in the calculation of the resistance of a quantum wire. When parameters of the quantum wire (its radius, amplitude of delta-barriers, and distance between them, donor concentration) are appropriately chosen, the theory predicts the dependence of resistance on the magnetic field manifesting well-defined minima. The minima are attributed to the resonant tunnelling of the conduction electrons through the double delta-barrier.

topics: double delta-barrier, resonant tunnelling, Landauer formula, magnetoresistance

1. Introduction

Recently, in [1] we dealt with the problem of electron tunnelling between two qubits. Our exposition of the problem was based on a model of a quantum wire (QW) in which two equal delta-wells were considered. (We will use the acronym QW for quantum wire.) The topic of the present paper is QW, which has two delta-barriers. Thus, if the axis of the wire is oriented in the z-direction, then we assume the potential energy is

$$V(z) = \gamma \left[\delta\left(z-\frac{a}{2}\right)+\delta\left(z+\frac{a}{2}\right)\right] \tag{1}$$

with $\gamma > 0$. We say that this potential energy represents a symmetric, double barrier. The analysis of tunnelling through double barriers reveals one interesting phenomenon, namely resonant tunnelling. This means that there are some energy values $E = E^{(i)}$ for which the tunnelling probability is equal to one. The particle can cross the double barrier quite freely at resonant energies, as if there were no double barrier. We denote the absolute value of the wave number k_z as k and define the dimensionless quantity $\kappa = ka$. From the 1D Schrödinger equation with potential (1) it follows that the resonant energies are determined by the positive roots of the transcendental equation

$$\kappa = -\frac{\gamma m a^3}{\hbar^2}\tan(\kappa), \tag{2}$$

$$E^{(i)} = \frac{\hbar^2\kappa_i^2}{2ma^2}, \tag{3}$$

By using $\hbar^2/(2ma^2)$ as the energy unit, we can write the resonant energies as

$$\varepsilon^{(i)} = \kappa_i^2, \quad 0<\kappa_1<\kappa_2<\ldots. \tag{4}$$

The parameter $m > 0$ is the effective mass of the conduction electrons. The equations (2) and (3) hold if the dispersion function of the conduction electrons is quadratic, i.e., $E_c(k) = \hbar^2k^2/(2m)$. (Note, however, that (2) and (3) can be generalized. It is possible to derive analogues to them for non-quadratic dispersion functions $E_c(k_z)$, see [2]. Important non-quadratic functions $E_c(k_z)$ were derived for narrow-gap semiconductors, especially for InSb, by Kane [3].)

Figure 1 is a sketch of a nanostructure involving QW with two delta-barriers. We consider QW to be as a thin cylinder made of n-type InSb, interrupted at the positions $z = \pm a/2$ by very thin semiconductor layers other than InSb. We then formulate the Schrödinger equation for the conduction electrons in the effective-mass approximation [4], approximating the potential energy $V(x)$ as expression (1). We do not specify the material in which the InSb cylinder may be embedded. (It may be, as well as the layers forming the double delta-barrier, a semiconductor that is satisfactorily lattice-matched with InSb, such as $In_{0.9}Al_{0.1}Sb$ (cf., e.g., [5].) We assume that InSb is degenerate. This means that the donor concentration is higher than 10^{13} cm^{-3} [6]. The Fermi energy E_F in degenerate semiconductors lies inside their conduction band. Therefore, degenerate semiconductors resemble metals. However, the value of E_F in degenerate semiconductors can be a thousand

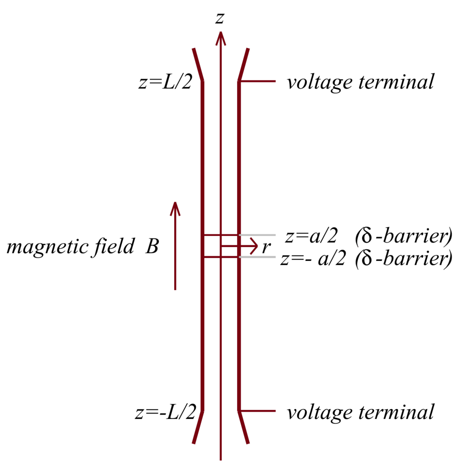

Fig. 1. Scheme of a quantum wire with two equal delta-barriers.

times lower than in metals. The funnelled shapes depicted in Fig. 1 at the ends of QW symbolize mesoscopic leads. Horizontal arrows at $z = \pm L/2$ suggest voltage terminals. Let b be the radius of the considered QW. If there are no delta barriers in QW, its conductance would be equal to $\pi b^2 \mu_{\rm e} n_{\rm e}/L$ ($\mu_{\rm e}$ and $n_{\rm e}$ are, respectively, the mobility and the concentration of the conduction electrons). According to the transport theory of solids [7], mobility depends on the relaxation time τ, which characterizes the collisions of the conduction electrons with defects and with phonons inside QW. We may estimate mobility using the Drude formula $\mu_{\rm e} = e\tau/m$. We denote the QW resistance with no delta-barriers as R_τ. When including the delta-barriers into account, we have to consider the additional resistance R_δ. Assuming the resistances additivity when the resistors are connected in series, we expect that the total resistance of QW can be written as

$$R_{\rm QW} = R_\delta + R_\tau. \tag{5}$$

(Doubt about the validity of this equation might arise if the lengths L and a were comparable. We assume, however, that $L \gg a$.) It therefore remains to determine the term R_δ. At enough low temperatures, ideally at $T = 0$, the Landauer formalism is applicable, presuming a ballistic regime. (Note that there exist two Landauer's formulae. Here we have in mind the formula concerning the "four terminal definition" of the conductance [8, 9].) In our case, the Landauer formula involves the probability $\mathcal{T}$ with which the conduction electrons can tunnel through the double delta-barrier. Thanks to the metallic character of the charge transport, the relevant energies of the conduction electrons in QW are close to the Fermi energy.

In terms of the corresponding conductances, which we denote as $G_{\rm QW}$, G_δ, G_τ, the formula (5) reads

$$\frac{1}{G_{\rm QW}} = \frac{1}{G_\delta} + \frac{1}{G_\tau}. \tag{6}$$

There are good reasons to argue that G_δ and G_τ are independent quantities. That is why we may say that (6) confirms the Matthiessen rule [7].

The vertical arrow depicted on the left side in Fig. 1 indicates the magnetic field B applied in the direction of the QW axis. As we will prove in Sect. 3, the Fermi energy is a growing function of B. This fact motivates us to foresee that the magnetic field may serve as a control tool for the value of the tunnelling probability $\mathcal{T}$ in the Landauer formula, and then also for the value of the resistance $R_{\rm QW}$. The possibility to control the QW resistance by the longitudinal magnetic field becomes interesting, as we will show exactly, owing to the Zeeman effect.

2. Mathematical formulation of the problem

2.1. Probability of tunnelling through the double delta-barrier

Our first task is to solve the Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi_k(x)}{\partial x^2} + \gamma\Big[\delta\left(x-\frac{a}{2}\right) + \delta\left(x+\frac{a}{2}\right)\Big]\psi_k(x) = E_k\psi_k(x), \quad \gamma > 0. \tag{7}$$

There are two solutions — we denote them as $\psi_k^{>}(x)$ and $\psi_k^{<}(x)$, both with the eigen-energy $E_k = \frac{\hbar^2 k^2}{2m}$. We take $k>0$ and $a>0$. The wave function $\psi_k^{>}(x)$ expresses the motion of the conduction electron from left to right

$$\psi_k^{>}(x) = \begin{cases} {\rm e}^{{\rm i}kx} + D^{\leftarrow}\,{\rm e}^{-{\rm i}kx}, & x < -\frac{a}{2}, \\ M_1\cos(kx) + M_2\sin(kx), & -\frac{a}{2} < x < \frac{a}{2}, \\ D^{\rightarrow}\,{\rm e}^{{\rm i}kx}, & \frac{a}{2} < x. \end{cases} \tag{8}$$

The coefficients $D^{\leftarrow}$, M_1, M_2, and $D^{\rightarrow}$ can be uniquely determined [2]. The quantities $|D^{\leftarrow}|^2$ and $|D^{\rightarrow}|^2$ are the reflection and transmission (tunnelling) coefficient, respectively. They represent complementary probabilities, i.e.,

$$|D^{\leftarrow}|^2 + |D^{\rightarrow}|^2 = 1. \tag{9}$$

We prefer to consider tunnelling probability as a function of

$$\varepsilon_\kappa = \kappa^2. \tag{10}$$

(Recall that $\kappa = ka$.) According to our calculations, the tunnelling probability is given by the formula

$$\mathcal{T}(\varepsilon_\kappa) = \frac{(E_k')^4}{(E_k')^4 + 4\gamma^2\big[E_k'\cos(ka) + \gamma\sin(ka)\big]^2}, \tag{11}$$

where

$$E_k' = \frac{\partial E_k}{\partial k} = \frac{\hbar^2 k}{m}. \tag{12}$$

We define the dimensionless parameter

$$\alpha = \frac{m\gamma a}{\hbar^2}. \tag{13}$$

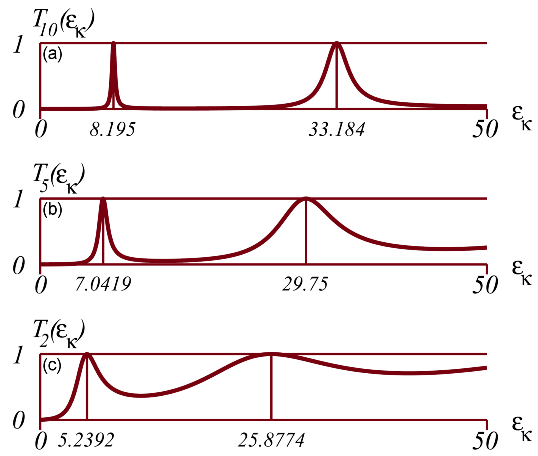

Fig. 2. Tunnelling probabilities $\mathcal{T}_\alpha(\varepsilon_\kappa)$ for α equal to (a) 10, (b) 5, and (c) 2. (The parameter α characterizes the strength of the delta-barriers $\gamma\delta(z \pm a/2)$.) For $a = 20$ nm, the energy $\varepsilon_\kappa = 1$ corresponds to 0.6744 eV.

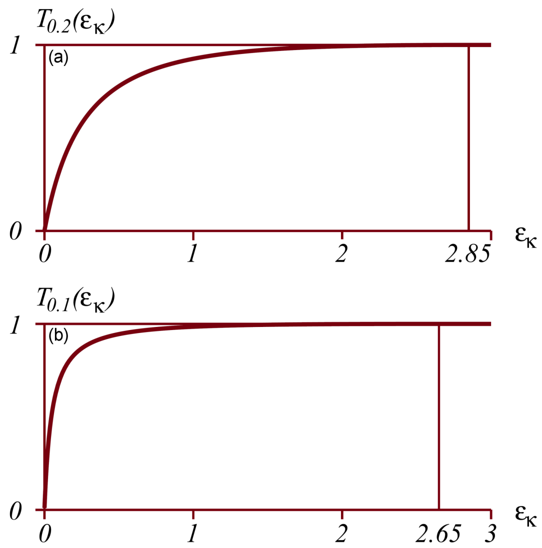

Fig. 3. Tunnelling probabilities $\mathcal{T}_\alpha(\varepsilon_\kappa)$ for α equal to (a) 0.2 and (b) 0.1. The first derivative $\mathcal{T}'_\alpha(\varepsilon_\kappa)$ for $\varepsilon_\kappa \to +0$ is 3.125 and 12.5, respectively.

Then expression (11) is transformed as

$$\mathcal{T}_\alpha(\varepsilon_\kappa) = \frac{1}{1 + 4\left(\frac{\alpha}{\kappa}\right)^2 \left[\cos(\kappa) + \frac{\alpha}{\kappa}\sin(\kappa)\right]^2} = \frac{1}{1 + 4\frac{\alpha^2}{\varepsilon_\kappa}\left[\cos(\sqrt{\varepsilon_\kappa}) + \frac{\alpha}{\sqrt{\varepsilon_\kappa}}\sin(\sqrt{\varepsilon_\kappa})\right]^2}. \tag{14}$$

The tunnelling probabilities $\mathcal{T}_\alpha(\varepsilon_\kappa)$ for some values of α are presented in Figs. 2–4.

2.2. Resonant maxima

The derivative $\mathrm{d}\mathcal{T}_\alpha(\varepsilon)/\mathrm{d}\varepsilon$ is positive for $\varepsilon \to +0$, see Fig. 3. For higher values of α this derivative can be very small. For instance, if $\alpha = 2$, then $\mathrm{d}\mathcal{T}(\varepsilon)/\mathrm{d}\varepsilon\big|_{+0} = 1/144 \approx 0.007$. The function $\mathcal{T}_\alpha(\varepsilon)$ exhibits an infinite number of resonant maxima. Their positions $\varepsilon_{\alpha,\max}^{(i)}$ are defined by the equation $\mathcal{T}_\alpha(\varepsilon_{\alpha,\max}^{(i)}) = 1$. This equation is tantamount to the transcendental equation

$$\cos(\kappa) + \frac{\alpha}{\kappa}\sin(\kappa) = 0, \quad \kappa = \sqrt{\varepsilon_{\alpha,\max}^{(i)}}. \tag{15}$$

When applied to double delta-barriers in InSb, (14) becomes approximate for high values of ε_κ. This is because the dispersion function of conduction electrons in InSb is not exactly quadratic if the values of E_k are higher than about 0.2 eV.

We want to expound an analysis focused mainly on values of the function $\mathcal{T}_\alpha(\varepsilon)$ around its first maximum. To get a small value of $\varepsilon_{\alpha,\max}^{(1)}$, we ought to choose a small value of α. The lowest possible value of $\varepsilon_{\alpha,\max}^{(1)}$ is $\pi/2 = 1.57$ and it corresponds to $\alpha \to +0$. However, it is not suitable to choose a value that is too small for α. Namely, when approximating the delta-barrier as a limiting case of a rectangular barrier of width w and height V_0, i.e., taking $\gamma \approx wV_0$, the parameter α is, according to (13), proportional to V_0. With $\alpha \ll 1$, the height V_0 is unacceptably small. Therefore, as a compromise, we opt for $\alpha = 1$. Figure 4 is a plot of the function $\mathcal{T}_1(\varepsilon_\kappa)$ in an interval embracing the position $\varepsilon_{1,\max}^{(1)} = 4.116$.

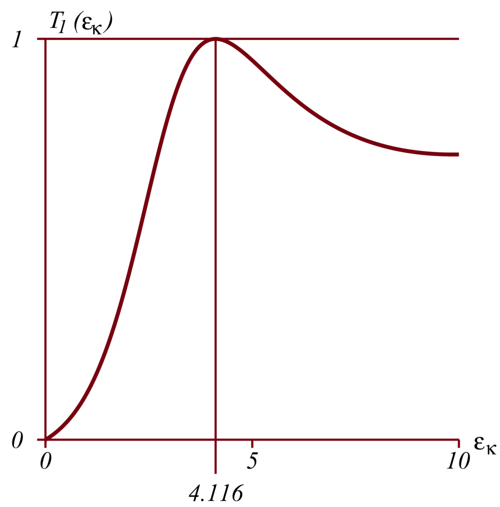

Fig. 4. For $a = 15$ nm, the unity in the horizontal axis corresponds to 0.0121 eV. Then the position of the maximum of the depicted curve, $\varepsilon_\kappa = 4.116$, corresponds to 0.0498 eV.

3. Exemplification of parameters of a model and derivation of Fermi energy

3.1. The case $B = 0$

Our option of the material of QW is n-InSb. Correspondingly, we consider the effective mass m of the conduction electrons in QW equal to $0.014\,m_0$

(m_0 is the electron mass in vacuum). Let us first estimate the value of V_0 defining the amplitude of the delta-barrier, $\gamma \approx wV_0$. As a result, (13) gives us

$$V_0 \approx \frac{\hbar^2 \alpha}{mwa}. \tag{16}$$

When inserting here $w = 1$ nm, $\alpha = 1$, and $a = 15$ nm, we obtain $V_0 \approx 0.363$ eV. This is a reasonable value. We will consider throughout the present paper the donor concentration $n_\mathrm{D} = 10^{16}$ cm^{-3} = 10^{22} m^{-3}. Since all donors are ionized, the concentration of the conduction electrons is equal to n_D at low temperatures. (Formally, we consider the zero temperature, $T = 0$.) The average distance between the conduction electrons is $n_\mathrm{D}^{-1/3} \approx 46.4$ nm. If no magnetic field is applied, the Fermi energy E_F is fully determined by the value of n_D. As we take QW as a cylinder of radius b, the number of conduction electrons in it is $\pi b^2 L\, n_\mathrm{D}$. We will consider $b = 40$ nm and $L = 1000$ nm. Then $\pi b^2 L\, n_\mathrm{D} \approx 50$. We can conclude that the transport of the conduction electrons along QW is ballistic with few occasional collisions of electrons with point defects.

Taking the effective mass approximation and assuming a constant potential energy in the radial direction, we can write the Schrödinger equation as

$$-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}\right]\psi_{k_z,\nu,\mu}(r,\phi,z)$$
$$+\gamma\left[\delta\left(z+\frac{a}{2}\right) + \delta\left(z-\frac{a}{2}\right)\right]\psi_{k_z,\nu,\mu}(r,\phi,z) =$$
$$E_{k_z,\nu,\mu}\psi_{k_z,\nu,\mu}(r,\phi,z). \tag{17}$$

Here, $k_z \in (-k_\mathrm{F}, k_\mathrm{F})$, ν=1, 2, ... and μ=0, $\pm1, \pm2, \ldots$. One thing should be emphasized in connection with (17) — the eigenfunctions $\psi_{k_z,\nu,\mu}(r,\phi,z)$ depend on the amplitude γ of the double delta-barrier, but the eigenvalues $E_{k_z,\nu,\mu}$ do not! Therefore, even if the calculation of the tunnelling probability concerning QW shown schematically in Fig. 1 must be based on (17), the determination of the Fermi energy, which is our goal in this section, may be based on a simpler Schrödinger equation where $\gamma = 0$, i.e.,

$$-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}\right]\psi_{k_z,\nu,\mu}(r,\phi,z) =$$
$$E_{k_z,\nu,\mu}\psi_{k_z,\nu,\mu}(r,\phi,z). \tag{18}$$

Thus, although the functions $\psi_{k_z,\nu,\mu}$ and $\widetilde{\psi}_{k_z,\nu,\mu}$ are different, the set $\{E_{k_z,\nu,\mu}\}$ of the eigenvalues of (17) and (18) is the same.

The energies $E_{k_z,\nu,\mu}$ lie inside the conduction band of InSb. Owing to the finite value of the radius b, these energies are organized in subbands. The subbands are enumerated by the indexes ν and μ. The lowest subband has the indexes $\nu = 1$ and $\mu = 0$. Each eigenfunction $\widetilde{\psi}_{k_z,\nu,\mu}(r,\phi,z)$ of (18) may be considered as a product $R(r)\,\Phi(\phi)\chi(z)$. Clearly, $\Phi(\phi) \sim \mathrm{e}^{\mathrm{i}\mu\phi}$. We denote the R-function as $R_{k_z,\nu,\mu}(r)$. It obeys the equation

$$-\frac{\hbar^2}{2m}\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r} - \frac{\mu^2}{r^2}\right]R_{k_z,\nu,\mu}(r) = E\,R_{k_z,\nu,\mu}, \tag{19}$$

where

$$E = E_{k_z,\nu,\mu} - \frac{\hbar^2 k_z^2}{2m}. \tag{20}$$

Let us take $k_z = 0$. The first task is to calculate the eigenenergies $E_{0,1,0}$ and $E_{0,1,1}$. The solution of the equation

$$\frac{\mathrm{d}^2 f(\xi)}{\mathrm{d}\xi^2} + \frac{1}{\xi}\frac{\mathrm{d}f(\xi)}{\mathrm{d}\xi} + \left(1 - \frac{\mu^2}{\xi^2}\right)f(\xi) = 0, \tag{21}$$

which is finite in the point $\xi = 0$, is the Bessel function $J_\mu(\xi)$. The first roots of $J_0(\xi)$ and $J_1(\xi)$ are, respectively, $\xi_{1,0} = 2.4048$ and $\xi_{1,1} = 3.8317$. (Recall that $J_{-1}(\xi) = -J_1(\xi)$.) When comparing (19) and (21), we observe that

$$\xi = \sqrt{\frac{2mE_\perp}{\hbar^2}}\,r, \quad 0 < r < b, \tag{22}$$

where the symbol $E_\perp$ indicates the "perpendicular part" of the eigenenergy. Hence,

$$E_{0,1,0} = \frac{\hbar^2}{2mb^2}\,\xi_{1,0}^2, \quad E_{0,1,1} = \frac{\hbar^2}{2mb^2}\,\xi_{1,1}^2. \tag{23}$$

Choosing $b = 40$ nm, we obtain $E_{0,1,0} \approx 9.837$ meV and $E_{0,1,1} \approx 24.974$ meV. In the next section, we will deal with the Schrödinger equation for the presence of the magnetic field B. Then the eigenenergies under consideration will be functions $E_{0,1,0}(B)$ and $E_{0,1,1}(B)$. Fermi energy, as we will show, is an increasing function of the magnetic field. We assume that the magnetic field is not too strong and so the condition

$$E_{0,1,0}(B) < E_{0,1,1}(0) \tag{24}$$

is fulfilled. Under the condition (24), the problem we are dealing with is essentially one-dimensional if we assess it from an "orbital" viewpoint. (Of course, when the problem is assessed from the viewpoint of the Zeeman effect, the condition $B < 15$ T need not be required.)

The number of states of the conduction electrons in the interval $(k_z, k_z + \mathrm{d}k_z)$ is $2\,\mathrm{d}k_z/(2\pi)$. (The spin degeneracy gives a factor of 2 in the numerator.) The wave numbers k_z of the conduction electrons lie in the interval $(-k_\mathrm{F}, k_\mathrm{F})$. At the zero temperature, clearly $2k_\mathrm{F} = \pi^2 b^2 n_\mathrm{D}$, and the Fermi energy is

$$E_\mathrm{F}(0) = E_{0,1,0}(0) + \Delta E_\mathrm{F}(0), \quad \Delta E_\mathrm{F}(0) = \tfrac{\hbar^2 k_\mathrm{F}^2}{2m}. \tag{25}$$

With $b = 40$ nm, and $n_\mathrm{D} = 10^{22}$ m^{-3}, we find that $k_\mathrm{F} \approx \pi^2 b^2 n_\mathrm{D}/2 \approx 7.8957 \times 10^7$ m^{-1} and $\Delta E_\mathrm{F}(0) \approx 0.017$ eV. When $\hbar^2/(2ma^2)$ is chosen as the energy unit, the Fermi energy for $B = 0$ is

$$\varepsilon_\mathrm{F}(0) = \varepsilon_{0,1,0}(0) + \Delta\varepsilon_\mathrm{F}(0) = \left(\tfrac{a\xi_{1,0}}{b}\right)^2 + \left[\tfrac{\pi^2 a b^2 n_\mathrm{D}}{2}\right]^2. \tag{26}$$

Considering $a = 15$ nm, $b = 40$ nm, we obtain $\varepsilon_{0,1,0}(0) = 0.1406\,\xi_{1,0}^2 \approx 0.8132$, $\Delta\varepsilon_\mathrm{F}(0) \approx 1.4027$ and $\varepsilon_\mathrm{F}(0) \approx 2.2159$. In the context of the problem that we are solving, it is important that $\varepsilon_\mathrm{F}(0) < \varepsilon_{1,\max}^{(1)}$.

3.2. The case $B > 0$

The objective of this section is to calculate the function $E_{\mathrm{F}}(B)$. When the magnetic field is parallel to the axis of QW, this simplifies the problem solving.

3.2.1. Calculation ignoring spin of the conduction electrons

At first, let us treat the problem as if the conduction electrons were spinless particles. Our intention is to solve the Schrödinger equation

$$\frac{1}{2m}\big(-\mathrm{i}\hbar\nabla + e\boldsymbol{A}(\boldsymbol{r})\big)^2\widetilde{\psi}_E(\boldsymbol{r}) = E\,\widetilde{\psi}_E(\boldsymbol{r}). \quad (27)$$

(We posit that $e > 0$; so the charge of the electron is $-e$.) Let $\boldsymbol{r}^0$, $\boldsymbol{\phi}^0$ and $\boldsymbol{z}^0$ be the unit vectors of the cylindrical system of coordinates. The vector potential can be defined as $\boldsymbol{A}(\boldsymbol{r}) = \boldsymbol{\phi}^0 A_\phi(r) = \frac{1}{2}rB\boldsymbol{\phi}^0$. The operator nabla is $\nabla = \boldsymbol{r}^0\frac{\partial}{\partial r} + (\boldsymbol{\phi}^0/r)\frac{\partial}{\partial\phi} + \boldsymbol{z}^0\frac{\partial}{\partial z}$. Then $\nabla\times\boldsymbol{A}(\boldsymbol{r}) = \boldsymbol{z}^0[A_\phi(r)/r + \mathrm{d}A_\phi(r)/\mathrm{d}r] = \boldsymbol{z}^0 B$ and $[-\mathrm{i}\hbar\nabla + \boldsymbol{A}(\boldsymbol{r})]^2 = -\hbar^2\nabla^2 - \mathrm{i}\hbar eB\frac{\partial}{\partial\phi} + \frac{1}{4}e^2B^2r^2$. Thus, we can write (27) as

$$\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\phi^2} + \frac{\partial^2}{\partial z^2}\right) - \mathrm{i}\,\frac{\hbar\omega_{\mathrm{L}}}{2}\frac{\partial}{\partial\phi} + \frac{m\omega_{\mathrm{L}}^2}{8}r^2 - E_{k_z,\nu,\mu}\right]\psi_{k_z,\nu,\mu}(r,\phi,z) = 0, \quad (28)$$

where

$$\omega_{\mathrm{L}} = \frac{eB}{m} \quad (29)$$

is the frequency. (See the “Problem” added to paragraph 111 in [10]. The subscript 'L' should suggest that the frequency ω_{L} is related with the Landau problem of free motion of a charged particle in a constant magnetic field. Synonymously, we may call ω_{L}, like many authors, the effective cyclotron frequency.)

As in the case when B=0, we seek the solution of (27) in the form $\widetilde{\psi}_{k_z,\nu,\mu}(r,\phi,z) \simeq R_{k_z,\nu,\mu}(r)\mathrm{e}^{\mathrm{i}\mu\phi}\mathrm{e}^{\mathrm{i}k_z z}$. By omitting the indexes in the radial function R and in the energy E, we can write down the equation

$$-\frac{\hbar^2}{2m}\left[\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}R}{\mathrm{d}r}\right) - \frac{\mu^2}{r^2}R\right] - \frac{\hbar\omega_{\mathrm{L}}\mu}{2}R + \left(\frac{m\omega_{\mathrm{L}}^2}{8}r^2 + \frac{\hbar^2k_z^2}{2m} - E\right)R = 0. \quad (30)$$

Eigenfunctions and eigenvalues of (30) are defined by three quantum numbers, i.e., k_z, ν, and μ. So we write $R \equiv R_{k_z,\nu,\mu}(r)$ and $E \equiv E_{k_z,\nu,\mu}$. The quantum number ν is determined by the boundary condition $R_{k_z,\nu,\mu}(b) = 0$. As in Sect. 3.1, we limit the further calculation to the special case when $\mu = 0$. At first, we take also $k_z = 0$. The equation (30) with $k_z = 0$ and $\mu = 0$ is transformed to

$$\frac{\hbar^2}{2m}\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}R}{\mathrm{d}r}\right) + \left(E_\perp - \frac{m\omega_{\mathrm{L}}^2}{8}r^2\right)R = 0. \quad (31)$$

We introduce the dimensionless variable u and the dimensionless energy $\mathcal{E}_\perp$, respectively,

$$u = -\frac{m\omega_{\mathrm{L}}}{2\hbar}r^2 \quad \text{and} \quad \mathcal{E}_\perp = \frac{E_\perp}{\hbar\omega_{\mathrm{L}}}. \quad (32)$$

Then we use the denotation

$$R(r) \equiv R_{0,\nu,0}(r) = \rho(u). \quad (33)$$

It is easy to verify that

$$r\,\frac{\mathrm{d}R}{\mathrm{d}r} = 2u\,\frac{\mathrm{d}\rho}{\mathrm{d}u}, \quad \frac{1}{r}\,\frac{\mathrm{d}}{\mathrm{d}r} = -\frac{m\omega_{\mathrm{L}}}{\hbar}\,\frac{\mathrm{d}}{\mathrm{d}u},$$

$$\frac{\hbar^2}{2m}\,\frac{1}{r}\,\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}R}{\mathrm{d}r}\right) = -\hbar\omega_{\mathrm{L}}\left(u\,\frac{\mathrm{d}^2\rho}{\mathrm{d}u^2} + \frac{\mathrm{d}\rho}{\mathrm{d}u}\right),$$

$$-\frac{1}{8}\,m\omega_{\mathrm{L}}^2r^2 = \frac{1}{4}\,\hbar\omega_{\mathrm{L}}u. \quad (34)$$

Hence, the function $\rho(u)$ obeys

$$u\,\frac{\mathrm{d}^2\rho}{\mathrm{d}u^2} + \frac{\mathrm{d}\rho}{\mathrm{d}u} - \left(\mathcal{E}_\perp + \frac{u}{4}\right)\rho = 0. \quad (35)$$

The substitution

$$\rho(u) = \exp(-u/2)f(u) \quad (36)$$

in (34) gives

$$u\,\frac{\mathrm{d}^2f}{\mathrm{d}u^2} + (1-u)\,\frac{\mathrm{d}f}{\mathrm{d}u} - \left(\mathcal{E}_\perp + \frac{1}{2}\right)f = 0. \quad (37)$$

The equation

$$u\,\frac{\mathrm{d}^2f}{\mathrm{d}u^2} + (\zeta - u)\frac{\mathrm{d}f}{\mathrm{d}u} - \eta f = 0 \quad (38)$$

was first studied by Kummer [11, 12]. Its basic solutions, denoted as $M(\eta,\zeta,u)$ and $U(\eta,\zeta,u)$ in [11] (or as $\Phi(\eta,\zeta;u)$ and $\Psi(\eta,\zeta;u)$ in [12]), are confluent hypergeometric functions. (Maple calls these functions KummerM and KummerU. Note that there exists also the denotation ${}_1F_1(\eta,\zeta;u)$ for $M(\eta,\zeta;u)$. Users of Mathematica can find the function Hypergeometric1F1[η,ζ,u] in the list of special functions.) We are considering $\eta = \mathcal{E}_\perp + 1/2$, $\zeta = 1$. The function $U(\mathcal{E}_\perp + 1/2, 1; u)$ is irrelevant in our problem. Thus, the radial function is

$$\rho(u) = \exp(-u/2)\,M(\eta,1,u). \quad (39)$$

(We do not need to calculate the normalization coefficient.) Let us introduce the dimensionless variable $\beta > 0$ proportional to the magnetic field B

$$\beta = |u|_{r=b} = \frac{eb^2}{2\hbar}\,B. \quad (40)$$

The eigenenergies $\mathcal{E}_\perp$ are solutions of the equation

$$M(\mathcal{E}_\perp + 1/2, 1, -\beta) = 0. \quad (41)$$

We now focus our attention on the $\mathcal{E}_\perp(\beta)$ function corresponding to the lowest eigenenergy $E_\perp(B)$. The problem is explained in detail in Appendix. The function $\mathcal{E}_\perp(\beta)$ is approaching the asymptotic value $\mathcal{E}_\perp(\infty) = 1/2$, while $E_\perp(B) \approx \hbar\omega_{\mathrm{L}}/2$ for high values of B. This means that if the magnetic field is strong, the energy $E_\perp$ in the quantum wire is the same as in the bulk material. If the variable β lies in the interval $0 - -7$, the function $\beta\,\mathcal{E}(\beta)$ differs remarkably from $\beta/2$. This unequivocally manifests the quantum size effect.

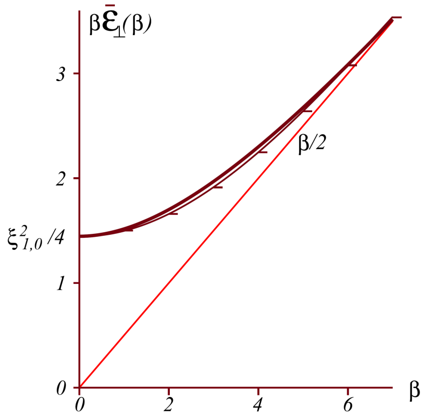

Fig. 5. The functions $\beta\,\overline{\mathcal{E}}_{\perp}(\beta)$ (bold curved line) and $\beta\,\mathcal{E}_{\perp}(\beta)$ (thin curved line). The short horizontal bars correspond to the exact values of the lowest positive root of the equation $M(\mathcal{E}_{\perp}(j)+1/2,1,-j)=0$, where $j=\beta_j$, $(j=1,2,\ldots)$. The thin curved line was obtained by interpolation between the points $(\beta_j,\beta_j\mathcal{E}(\beta_j))$. The function $\beta\,\overline{\mathcal{E}}_{\perp}(\beta)$ is defined by (42).

We can now derive the dependence of the Fermi energy E_{F} on the magnetic field. The function $E_{\mathrm{F}}(B)$ depends uniquely on $E_{\perp}(B)$. Although using the Kummer function $M(\eta,1,-\beta)$ allowed us to compute exactly the $E_{\perp}(B)$ function, basing the Fermi energy calculation on something what must be defined as the root of the transcendental equation is simply impractical. Instead, it is useful to replace $\beta\mathcal{E}_{\perp}(\beta)$ with a simple explicit function $\beta\overline{\mathcal{E}}_{\perp}(\beta)$. A very satisfactory function for this purpose is

$$\beta\,\overline{\mathcal{E}}_{\perp}(\beta)=\begin{cases}1.4458-1.389\,(1-\mathrm{e}^{-0.31\,\beta})+0.5\,\beta, \\ \quad \text{if} \quad 0\le\beta<7, \\ 0.5\,\beta, \quad \text{if} \quad 7<\beta.\end{cases} \tag{42}$$

The results regarding the functions $\beta\,\overline{\mathcal{E}}_{\perp}(\beta)$ and $\beta\,\mathcal{E}_{\perp}(\beta)$ are shown in Fig. 5. Because $\overline{E}_{\perp}(B)=(\hbar eB/m)\,\overline{\mathcal{E}}_{\perp}(\beta)$, we have got

$$\overline{E}_{\perp}(B)=\begin{cases}\frac{2\hbar^2}{mb^2}\left[1.4458-1.389\left(1-\mathrm{e}^{-0.155\,eb^2B/\hbar}\right)\right] \\ \quad +\frac{\hbar eB}{2m}, \quad \text{if} \quad 0\le B<14\,\hbar/(eb^2), \\ \frac{\hbar eB}{2m}, \quad \text{if} \quad 14\,\hbar/(eb^2)<B.\end{cases} \tag{43}$$

(We assume that the effective mass m depends negligibly on the magnetic field. We also ignore the enhancement of the value of m due to the confinement of InSb in the nanostructure. According to [13], this enhancement can be about 20%.)

3.2.2. The Zeeman splitting

If $B\neq 0$, we have to dichotomize the conduction electrons, taking the spin up and spin down electrons separately. Each energy $E_{\perp}(B)$, which has been calculated in Sect. 3.2.1 splits into two energies,

$$E_{\perp}^{\uparrow}(B)=E_{\perp}(B)-g^*\mu_{\mathrm{B}}B,$$
$$E_{\perp}^{\downarrow}(B)=E_{\perp}(B)+g^*\mu_{\mathrm{B}}B. \tag{44}$$

where $\mu_{\mathrm{B}}=\hbar e/(2m_0)$ is the Bohr magneton. (Recall that m_0 is the electron rest mass.) The factor g^* (known as the effective Landé or the gyromagnetic factor) is dimensionless. For a free electron (in a vacuum), $g^*=-2$. The value of $|g^*|$ of the conduction electrons in InSb is much higher than 2 due to the very small effective mass m and the strong spin–orbit interaction. Nowadays, the reliable determination of g^* in InSb nanostructures is a hot topic. The generally accepted value of g^* for bulk InSb is -51.3 (see e.g. [14]). However, if InSb is a component in nanostructures, thorough investigations [15, 16] revealed that the effective g-factor may be more than 40% higher than its bulk value. The value of $|g^*|\simeq 50$ is high and some authors have called the effective g-factor of InSb “giant”. Two peculiarities of the factor g^* of InSb are worth mentioning. First, it is level-dependent. The level dependence of g^* is not interesting in our case, because we are not considering other quantum numbers than $\nu=1$ and $\mu=0$. The second peculiarity of the factor g^* of InSb is its conspicuous anisotropy [13, 17]. For instance, the authors of [13] came to the conclusion that $|g^*|$ is equal to 52 in one direction and 26 in the other (perpendicular). The anisotropy of g^*, which was later on reported in [17], was somewhat nearer to isotropy. Reliable determination of the g^* factor for a given orientation of the crystalline lattice inside InSb QWs is undoubtedly difficult and we do not want to discuss it here. In our calculations, we will use the bulk value of the effective g-factor, $g^*=-51.3$.

The dependence of the Fermi energy E_{F} on the magnetic field results from

$$\pi b^2 n_{\mathrm{D}}=\int_{-\infty}^{\infty}\frac{\mathrm{d}k_z}{2\pi}\left\{\left[\exp\left(\frac{E^{\uparrow}_{k_z,1,0}(B)-E_{\mathrm{F}}}{k_{\mathrm{B}}T}\right)+1\right]^{-1}+\left[\exp\left(\frac{E^{\downarrow}_{k_z,1,0}(B)-E_{\mathrm{F}}}{k_{\mathrm{B}}T}\right)+1\right]^{-1}\right\}. \tag{45}$$

We will calculate the Fermi energy at the zero temperature.

3.2.3. The Fermi energy $E_{\mathrm{F}}^{(0)}(B)$ in the approximation neglecting the Zeeman splitting ($g^*=0$)

If we take formally $g^*=0$, then (43) at $T=0$ is simplified as

$$\pi b^2 n_{\mathrm{D}}=\frac{1}{\pi}\int_{-k_{\mathrm{F}}}^{k_{\mathrm{F}}}\mathrm{d}k_z=\frac{2}{\pi}\int_{0}^{k_{\mathrm{F}}}\mathrm{d}k_z=\frac{2}{\pi}\sqrt{\frac{2m}{\hbar^2}}\sqrt{E_{\mathrm{F}}^{(0)}-E_{0,1,0}(B)}. \tag{46}$$

This equation gives the function

$$E_{\mathrm{F}}^{(0)}(B) = \frac{\hbar^2}{2m}\left(\frac{\pi^2 b^2 n_{\mathrm{D}}}{2}\right)^2 + E_{0,1,0}(B) =$$

$$\Delta E_{\mathrm{F}}(0) + \overline{E}_{\perp}(B). \qquad (47)$$

(We have neglected the difference between $E_{0,1,0}(B)$ and $\overline{E}_{\perp}(B)$.) On the scale where $\hbar^2/(2ma^2)$ is the energy unit (see Fig. 2), (45) reads

$$\varepsilon_{\mathrm{F}}^{(0)}(\beta) = \left(\frac{\pi^2 a b^2 n_{\mathrm{D}}}{2}\right)^2 + \frac{2ma^2}{\hbar^2}\,\overline{E}_{\perp}(B) =$$

$$\Delta\varepsilon_{\mathrm{F}}(0) + \frac{2ma^2}{\hbar^2}\,\overline{E}_{\perp}(B). \qquad (48)$$

3.2.4. The Fermi energy E_{F} calculated for $T = 0$ with $g^* \neq 0$

We define two wave numbers, $k_{\mathrm{F}}^{\uparrow}$ and $k_{\mathrm{F}}^{\downarrow}$, related to the Fermi energy E_{F} as follows

$$k_{\mathrm{F}}^{\uparrow} = \sqrt{\tfrac{2m}{\hbar^2}}\sqrt{E_{\mathrm{F}} - E_{0,1,0}(B) + g^*\mu_{\mathrm{B}}B},$$

$$k_{\mathrm{F}}^{\downarrow} = \sqrt{\tfrac{2m}{\hbar^2}}\sqrt{E_{\mathrm{F}} - E_{0,1,0}(B) - g^*\mu_{\mathrm{B}}B}. \qquad (49)$$

For $T = 0$, (45) is simplified to

$$\pi b^2 n_{\mathrm{D}} = \frac{1}{\pi}\left\{\int_0^{k_{\mathrm{F}}^{\uparrow}} \mathrm{d}k_z + \int_0^{k_{\mathrm{F}}^{\downarrow}} \mathrm{d}k_z\right\} = \frac{k_{\mathrm{F}}^{\uparrow} + k_{\mathrm{F}}^{\downarrow}}{\pi} =$$

$$\frac{1}{\pi}\sqrt{\frac{2m}{\hbar^2}}\left\{\sqrt{E_{\mathrm{F}} - E_{0,1,0}(B) + g^*\mu_{\mathrm{B}}B}\right.$$

$$\left. + \sqrt{E_{\mathrm{F}} - E_{0,1,0}(B) - g^*\mu_{\mathrm{B}}B}\right\}. \qquad (50)$$

Using the denotation $A = \pi^2\hbar b^2 n_{\mathrm{D}}/\sqrt{2m}$, $C = E_{\mathrm{F}} - E_{0,1,0}(B)$ and $Q = g^*\mu_{\mathrm{B}}$, we observe that (50) is of type $A = \sqrt{C + QB} + \sqrt{C - QB}$ which is an algebraic equation for C. The solution is $C = A^2/4 + Q^2B^2/A^2$. In this way, we derived the dependence of the Fermi energy on the magnetic field

$$E_{\mathrm{F}}(B) = E_{0,1,0}(B) + \frac{A^2}{4} + \frac{Q^2B^2}{A^2} =$$

$$\overline{E}_{\perp}(B) + \frac{A^2}{4} + \frac{Q^2B^2}{A^2}. \qquad (51)$$

(The function $\overline{E}_{\perp}(E)$ was defined by (43).) As the magnetic field B increases, the spin-dependent term in (51) is expected to be more and more important. Nevertheless, our estimate yields a relatively small value of the ratio

$$\frac{Q^2}{A^2} = 2m\left(\frac{g^*\mu_{\mathrm{B}}}{\pi^2\hbar b^2 n_{\mathrm{D}}}\right)^2. \qquad (52)$$

Indeed, with the parameter values we use, we have $A^2 \approx 0.068$ eV, $|Q| = 2.97 \times 10^{-3}$ eV/T. Thus, $Q^2/A^2 = 1.297 \times 10^{-4}$ eV/T^2. Hence, if we take $B = 10$ T, the spin-dependent term $Q^2B^2/A^2 \approx 0.013$ eV is comparable with $\Delta E_{\mathrm{F}}(0) \approx 0.017$ eV. On this occasion, however, we have to emphasize that if $n_{\mathrm{D}} \ll 10^{22}$ m^{-3}, the spin-dependent term in (51) (taken with the same magnetic field) becomes utterly dominant.

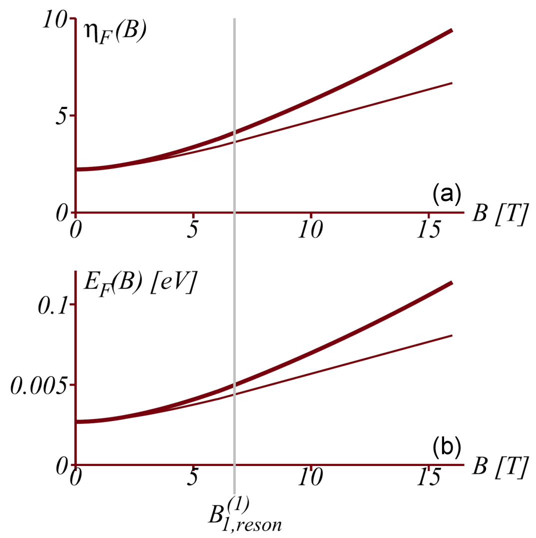

Fig. 6. (a) The thin curved line is the function $\eta_{\mathrm{F}}^{(0)}(B) = \Delta\varepsilon_{\mathrm{F}}(0) + \eta_{\perp}^{(0)}(B)$ calculated for $g^* = 0$. After correction with respect to the Zeeman effect, the Fermi energy $\eta_{\mathrm{F}}(B)$ is plotted as a bold line. (The calculation used $a = 15$ nm, $b = 40$ nm, $n_{\mathrm{D}} = 10^{22}$ m^{-3}, $g^* = -51.3$.) (b) The same curves as in panel (a), but the vertical axis is rescaled in electronvolts.

Thus, the dependence of the Fermi energy on the magnetic field reads

$$E_{\mathrm{F}}(B) = E_{\mathrm{F}}^{(0)}(B) + 2m\left(\frac{g^*\mu_{\mathrm{B}}}{\pi^2\hbar b^2 n_{\mathrm{D}}}\right)^2 B^2. \qquad (53)$$

When replacing B by $2\hbar\beta/(eb^2)$ and multiplying (53) by $2ma^2/\hbar^2$, we can obtain the function $\varepsilon_{\mathrm{F}}(\beta)$. However, β is only a formal variable and we actually need to pay attention to the dependence of ε_{F} on the magnetic field B. We express this dependency as the function $\eta_{\mathrm{F}}(B)$, taking into account that $\eta_{\mathrm{F}}(B) = \varepsilon_{\mathrm{F}}(\beta)$. So we prefer to define dimensionless Fermi energy as the function

$$\eta_{\mathrm{F}}(B) = \varepsilon_{\mathrm{F}}\left(\frac{eb^2B}{2\hbar}\right) = \frac{2ma^2}{\hbar^2}\,E_{\mathrm{F}}(B). \qquad (54)$$

Similarly $\eta_{\mathrm{F}}^{(0)}(B) = (2ma^2/\hbar^2)\,E_{\mathrm{F}}^{(0)}(B)$ and $\overline{\eta}_{\perp}^{(0)}(B) = (2ma^2/\hbar^2)\,\overline{E}_{\perp}^{(0)}(B)$. We proved above that $E_{\mathrm{F}}^{(0)}(B) = \Delta E_{\mathrm{F}}(0) + \overline{E}_{\perp}(B)$. So in dimensionless formalism we can write (51) as

$$\eta_{\mathrm{F}}(B) = \Delta\varepsilon_{\mathrm{F}}(0) + \overline{\eta}_{\perp}(B) + \left(\frac{2mg^*a\mu_{\mathrm{B}}}{\pi^2\hbar^2 b^2 n_{\mathrm{D}}}\right)^2 B^2. \qquad (55)$$

Numerically, when taking $n_{\mathrm{D}} = 10^{22}$ m^{-3}, $a = 15$ nm, $b = 40$ nm, $m/m_0 = 0.014$, $|g^*| = 51.3$, we obtain $\Delta\varepsilon_{\mathrm{F}}(0) = 1.4027$ (recall (26)) and

$$\left(\frac{2mg^*a\mu_{\mathrm{B}}}{\pi^2\hbar^2 b^2 n_{\mathrm{D}}}\right)^2 = \frac{0.01074}{T^2}. \qquad (56)$$

It remains to carry out the transformation of $\overline{E}_{\perp}(B)$ to $\overline{\eta}_{\perp}(B)$. The factor $2\hbar^2/(2m)$ in (43) is transformed as $4(a/b)^2$. Then

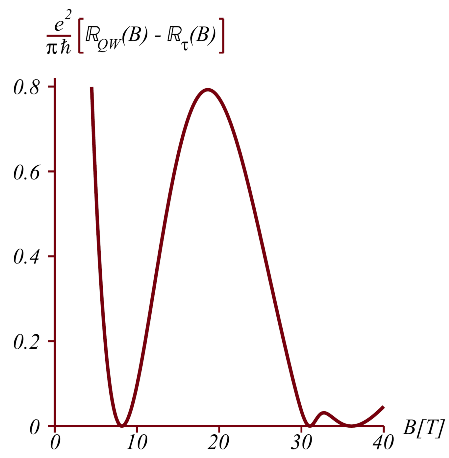

Fig. 7. Resistance given by the relation $[e^2/(\pi\hbar)](R_{\mathrm{QW}}(B)-R_\tau(B))$. The same parameters were used as in Fig. 6.

$4(a/b)^2 \times 1.4458 = 0.8133$. By scrutinizing function (43), we can also specify the exponent in its exponential function, $0.155 \times eb^2/\hbar = 0.363\ \mathrm{T}^{-1}$, and the term which is linear in B, i.e., $(2ma^2/\hbar^2)(e\hbar/(2m)) = ea^2/\hbar = 0.329\ \mathrm{T}^{-1}$. The function $\overline{\eta}_\perp(B)$ is shown as a thin line in Fig. 6.

So, we can propound the final form of the dimensionless function of our concern

$$\eta_{\mathrm{F}}(B) = \begin{cases} 1.4027 + \left[0.8133 - 0.91\,(1-\mathrm{e}^{-0.363\,B}) \right. \\ \quad \left. +0.329\,B\right] + 0.0107\,B^2, \\ \quad \text{if} \quad 0 < B < 6, \\ 1.4027 + 0.329\,B + 0.0107\,B^2, \\ \quad \text{if} \quad 6 < B. \end{cases} \tag{57}$$

(This function is depicted as a bold curved line in Fig. 6. The values of B are given in Tesla. The two branches of function (57) are sewn together in the point $B = 6$; hence the factor 0.91 in front of $(1 - \mathrm{e}^{-0.363\,B})$.) Note also that for $b = 40$ nm, condition (24) is surely fulfilled if $B < 15$ T (see Fig. 6b).

4. Magnetoresistance

Using the parameters of the previous sections ($\alpha = 1$, $a = 15$ nm, $b = 40$ nm, $n_{\mathrm{D}} = 5 \times 10^{22}\ \mathrm{m}^{-3}$, $g^* = -51.3$, $m = 0.014\,m_0$), we will now aim to determine the dependence of the conductance G_{QW} and the resistance $R_{\mathrm{QW}} = 1/G_{\mathrm{QW}}$ on the magnetic field B. The key issue in our theory is that the ballistic resistance of QW can become zero at well-defined magnetic fields. We especially have in mind a pair of values, which we denote (for $\alpha = 1$) as $\{B^<_{\mathrm{reson}}, B^>_{\mathrm{reson}}\}$. These two values reflect the resonant tunnelling corresponding to the first maximum of the function $\mathcal{T}_1(\varepsilon_\kappa)$. The two values of the magnetic field correspond to the two electron energies, the first due to spin up and the second due to spin down. It is also possible to consider an additional (third) value of the magnetic field at which the ballistic resistance becomes equal to zero, see Fig. 7.

4.1. Ballistic approximation

The ballistic approximation in our case means that no reason for the conductance value other than the presence of the double delta-barrier is taken into account. The ballistic theory has two ingredients: (i) the calculation of the tunnelling probability $\mathcal{T}_1$ (recall that the subscript 1 specifies the strength of the delta-barrier), and (ii) the calculation of the Fermi energy. The calculation of $\mathcal{T}_1$ is a task of *mechanics* and the calculation of the Fermi energy is a task of *kinetics*. The calculation of the Fermi energy has nothing to do with the quantum-mechanical calculation of the probability function $\mathcal{T}_1(\varepsilon_\kappa)$. The role of the Fermi energy is only to fix two values of κ; we will denote them as $\kappa_{\mathrm{F}}^\uparrow$ and $\kappa_{\mathrm{F}}^\downarrow$.

4.1.1. How the situation would appear if the conduction electrons were treated as spinless quasiparticles

It is useful to briefly discuss what our theory would predict if the gyrofactor were zero. The analysis for $g^* = 0$ was presented in Sect. 3.2.1. Generally, $(\kappa(B))^2 = \varepsilon_{\kappa,1,0}(\beta) - \varepsilon_{0,1,0}(\beta)$, and so

$$(\kappa_{\mathrm{F}}(B))^2 = \epsilon_{\mathrm{F}}^{(0)}(\beta) - \varepsilon_{0,1,0}(\beta) = \Delta_{\mathrm{F}}(0). \tag{58}$$

(Here we took (26) into account.) The value of κ_{F} is independent of the B magnetic field. Taking into account the probability function $\mathcal{T}_1(\varepsilon_\kappa)$ (see Fig. 4), we may employ the Landauer formula for the electrical conductance

$$G_\delta = \frac{e^2}{\pi\hbar}\,\frac{\mathcal{T}_1(\kappa_{\mathrm{F}}^2)}{1-\mathcal{T}_1(\kappa_{\mathrm{F}}^2)}. \tag{59}$$

The value of κ_{F} is fully determined by the donor density n_{D}. The study of resonant tunnelling would presume having many samples of equal QWs, differing only in the donor density in them. If the probability $\mathcal{T}_1(\kappa_{\mathrm{F}}^2)$ for the value of n_{D} was one, the ballistic conductance of QW would be infinite.

4.1.2. Formula regarding the Zeeman effect

As explained above, the possibility to control the conductance by a longitudinal magnetic field is less efficient for $|g^*| = 0$ than for $|g^*| \gg 1$. According to (49), we can employ the functions

$$(\kappa_{\mathrm{F}}^\uparrow)^2 = (ak_{\mathrm{F}}^\uparrow)^2 \equiv \eta_{\mathrm{F}}^\uparrow(B) =$$

$$\frac{2ma^2}{\hbar^2}\left[E_{\mathrm{F}}(B) - E_{0,1,0}(B) + g^*\mu_{\mathrm{B}}B\right],$$

$$(\kappa_{\mathrm{F}}^\downarrow)^2 = (ak_{\mathrm{F}}^\downarrow)^2 \equiv \eta_{\mathrm{F}}^\downarrow(B) =$$

$$\frac{2ma^2}{\hbar^2}\left[E_{\mathrm{F}}(B) - E_{0,1,0}(B) - g^*\mu_{\mathrm{B}}B\right]. \tag{60}$$

As in the previous section, the function $E_{0,1,0}(B) \approx E_\perp(B)$ is cancelled

$$\frac{2ma^2}{\hbar^3}\left[E_\mathrm{F}(B) - E_{0,1,0}(B)\right] = \Delta\varepsilon_\mathrm{F}(0) + \left(\frac{2mg^* a\mu_\mathrm{B} B}{\pi^2\hbar^2 b^2 n_\mathrm{D}}\right)^2 . \tag{61}$$

With the chosen numerical values of the parameters of our model, we have got the functions

$$\eta_\mathrm{F}^\uparrow(B) = 1.4027 + 0.2445\,B + 0.0107\,B^2,$$

$$\eta_\mathrm{F}^\downarrow(B) = 1.4027 - 0.2445\,B + 0.0107\,B^2 . \tag{62}$$

Now we define two probability functions

$$\mathcal{P}_1^\uparrow(B) = \mathcal{T}_1(\eta_\mathrm{F}^\uparrow(B)), \quad \mathcal{P}_1^\downarrow(B) = \mathcal{T}_1(\eta_\mathrm{F}^\downarrow(B)). \tag{63}$$

We can imagine QW as a conductor composed of two channels distinguished by the spin number. The conductance of QW can be expressed as the sum

$$G_\delta(B) = G_\delta^\uparrow(B) + G_\delta^\downarrow(B) = \frac{e^2}{2\pi\hbar}\left[\frac{\mathcal{P}_1^\uparrow(B)}{1-\mathcal{P}_1^\uparrow(B)} + \frac{\mathcal{P}_1^\downarrow(B)}{1-\mathcal{P}_1^\downarrow(B)}\right]. \tag{64}$$

Clearly,

$$\lim_{g^*\to 0} G_\delta(B) = G_\delta^{(0)}. \tag{65}$$

Taking the inverse of expression (64), we obtain the resistance of QW

$$R_\delta(B) = \frac{\pi\hbar}{e^2}\,\frac{2\left[1-\mathcal{P}_1^\uparrow(B)\right]\left[1-\mathcal{P}_1^\downarrow(B)\right]}{\mathcal{P}_1^\uparrow(B) + \mathcal{P}_1^\downarrow(B) - 2\mathcal{P}_1^\uparrow(B)\mathcal{P}_1^\downarrow(B)}. \tag{66}$$

This function is shown in Fig. 7. The minima of $R_\delta(B)$ obviously correspond to the roots of the equations $\mathcal{P}^\uparrow(B) = 1$ and $\mathcal{P}^\downarrow(B) = 1$. Inside the interval of magnetic fields chosen in Fig. 7, there are two roots of the equation $\mathcal{P}^\uparrow(B) = 1$ (namely, $B_{1,\,\mathrm{reson}}^\uparrow \approx 8.2$ T, $B_{2,\,\mathrm{reson}}^\uparrow \approx 36$ T) and one root of the equation $\mathcal{P}^\downarrow(B) = 1$ (namely, $B_\mathrm{reson}^\downarrow \approx 31$ T).

4.2. Correction regarding residual resistance of QW

If the residual resistance R_τ of the quantum wire under consideration were negligible, the resistance R_QW might be identified with the value of R_δ. However, the residual resistance is not small. That is why it is more convenient to plot the dependence $R_\mathrm{QW} - R_\tau$ vs B, as in Fig. 7, rather than R_QW vs B. The residual resistance of the quantum wire can be easily estimated as

$$\frac{e^2}{\pi\hbar}\,R_\tau = \frac{L}{\pi^2 b^2 n_\mathrm{D}}\,\frac{m}{\hbar\tau}. \tag{67}$$

When inserting a typical value of the relaxation time, $\tau \approx 10^{-12}$ s, we find that $[e^2/(\pi\hbar)]R_\tau$ is roughly 0.77 in our case.

5. Conclusions

We have derived analytically the resistance of the cylindrically shaped InSb quantum wire (QW). Our theory deals with a special case: we assume that QW hosts two equal delta-barriers. We have shown that Fermi energy can be potently controlled by a longitudinal magnetic field. When the quantum wire parameters (i.e., radius b of QW, donor density n_D, strength γ of delta-barriers, and distance a between them) are adequately tuned, the resistance of QW can manifest an unusual dependence on magnetic field. We presented a theory based on the Landauer formula. We proved that when the magnetic field grows from zero, the resistance of QW initially decreases until it reaches a minimum value. Afterwards the resistance increases with the growth of the magnetic field. The situation repeats itself: after reaching a maximum, the resistance decreases again as B grows and reaches the second minimum. A third minimum is also predicted, see Fig. 7.

We recommend mapping the vicinity of the first minimum since experimentation using relatively high DC magnetic fields for the mapping of the vicinity of the second and third minimum can be difficult. (Note that nowadays it is not possible to use higher DC magnetic fields than 45.5 T [18].)

The phenomenon described in this paper can be used in the design of the spin selector. Indeed, let us imagine two metallic current terminals joint on the side with a vertical nanostructure shown schematically in Fig. 1: an input terminal joint at the position $z_\mathrm{in} = -L/1 - \Delta$ and the output terminal at $z_\mathrm{out} = L/2 + \Delta$. We can consider the experimental arrangement in which the magnetic field B is localized in the cylinder region $0 < r < b$, but is much weaker for $r = R \gg b$. Considering the electron spins, the picture following from our theory is like this: the input terminal hosts in the position $r = R$ a mixture of spins ↑ and ↓. On the other hand, the electron spins at $r = R$ in the output terminal are *polarized* at Fermi energy, just like the spins ↑.

There is a possibility to utilize resonant tunnelling of electrons through double barriers in the design of resonant tunnel devices. For instance, recently Castro et al. [19] studied resonant tunnelling of electrons through the AlSb/GaInAsSb double barriers. However, their samples were not thin wires, and they did not focus on magnetoresistance.

The question arises whether the non-parabolicity of the conduction band of InSb [3], which we have neglected in our calculations, is important enough. Incorporating non-parabolicity in the theory of the present paper may lead to some quantitative (not qualitative) improvements that (we believe) are not very important. (We do not exclude that Fig. 7, which shows our main result, may be somewhat changed.)

For the sake of simplicity, we assumed in our theory that the temperature is zero. So, the question arises whether the experimental confirmation of the result manifested in Fig. 7 does not require the use of too low temperatures. At the temperature of liquid helium $T = 4.2$ K, the thermal energy is $k_\mathrm{B}T \approx 0.00036$ eV and this value is one order of magnitude lower than the Fermi energy in the magnetic field at ~ 10 T. Thus, the Fermi–Dirac distribution function may be approximated as a step function. This means that the resistance $R_{\mathrm{QW}}(B)$ at $T \sim 1$ K should be practically the same as at $T = 0$.

One can also pose another question: what happens if we replace the two delta-barriers in QW with a *single* rectangular barrier? Can we expect that the function $R_{\mathrm{QW}}(B)$ exhibits at least one minimum? Our answer is affirmative if we consider the Fermi energy *above* the barrier. Of course, the outcome depends on the choice of the width a and the height U_0 of the barrier (along with the choice of donor density in QW). Indeed, the function $\mathcal{T}(\varepsilon_\kappa)$, meaning the probability of the transmission of electrons through a rectangular barrier, if $\varepsilon_\kappa > 2ma^2U_0/\hbar)$, resembles the function depicted in Fig. 2, with some resonant energies at which the probability $\mathcal{T}(\varepsilon_\kappa)$ is equal to one (cf. e.g. [20]). We can apply the Landauer formula. And the Zeeman effect should work just like it does for the double delta-barrier.

Experiments with InSb quantum wires attract nowadays much attention in connection with the idea that they should be able to generate (quasi)particles known as majoranas. (Majorana was the author of the theory allowing the existence of particles equal to their antiparticles.) In particular, one can mention the ambitious project in which the use of InSb quantum wells was decisive. Attempts to create Majorana fermions in this project were not successful [21, 22]. Nevertheless, the goal to produce Majorana fermions is alluring, and hope of their potential use in a future quantum computer is still alive.

Two properties of InSb are extraordinary, namely the small effective mass of conduction electrons and their large (negative) effective gyrofactor. Further inquiries for details related to InSb quantum wells are most desirable.

Appendix

In this Appendix, we consider eigenfunctions and eigenvalues with $\mu = 0$ and $k_z = 0$. The Kummer function $M(\eta, 1, -|u|)$ is defined as the sum

$$M(\eta,1,-|u|) = 1 + \sum_{n=1}^{\infty} (-1)^n \tfrac{\eta(\eta+1)\ldots(\eta+n-1)}{(n!)^2} |u|^n = 1 + \sum_{n=1}^{\infty} (-1)^n \tfrac{(1+1/\eta)\ldots(1+(n-1)/\eta)}{(n!)^2} (\eta|u|)^n. \quad (68)$$

In our case

$$\eta = \mathcal{E}_{\mathrm{L}}(B_z) + \frac{1}{2}, \quad |u| = \frac{m\omega_{\mathrm{L}}}{2\hbar} r^2,$$
$$\mathcal{E}_\perp(B_z) = \frac{1}{\hbar\omega_{\mathrm{L}}} E_\perp(B_z), \quad \omega_{\mathrm{L}} = \frac{eB_z}{m}. \quad (69)$$

In the literature (e.g. [11, 12]), the Kummer function $M(a, b, z)$ was depicted and tabulated for positive values of z. If $z < 0$, we can employ the transformation $M(a, b, -z) = \mathrm{e}^{-z} M(b - a, b, z)$. In our case,

$$M(\eta,1,-|u|) = \mathrm{e}^{-|u|} M\big(1-\eta,1,|u|\big). \quad (70)$$

Our aim is to derive the dependence of the eigenenergy on the magnetic field. In other words, we will calculate the function $E_\perp(B_z)$.

A. The lowest eigenenergy in the fieldless case, $B = 0$

Let us consider the limit

$$\lim_{\omega_{\mathrm{L}}\to+0} M(\eta,1,-|u|) = \lim_{\omega_{\mathrm{L}}\to+0} M(1-\eta,1,|u|) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n!)^2} \left(\frac{mE_\perp(0)r^2}{2\hbar^2}\right)^n. \quad (71)$$

(We have used (70). Clearly, $|u| \to +0$ for $\omega_{\mathrm{L}} \to +0$.) When using the variable

$$\xi = \sqrt{\frac{2mE_\perp(0)}{\hbar^2}}\, r \quad (72)$$

(cf. the definition (22) in Sect. 3.1) and recalling that

$$J_0(\xi) = \sum_{n=0}^{\infty} \tfrac{(-1)^n}{(n!)^2} \left(\frac{\xi^2}{4}\right)^n, \quad (73)$$

we find that

$$\lim_{B_z\to+0} R(r) \equiv \lim_{\omega_{\mathrm{L}}\to+0} \rho(u) = J_0(\xi). \quad (74)$$

We define the quantity

$$\xi_b = \sqrt{\frac{2mE_\perp(0)}{\hbar^2}}\, b. \quad (75)$$

We denote the positive roots of

$$J_0(\xi_b) = 0 \quad (76)$$

as $\xi_{\nu,0}$. (The second subscript equal to zero means that the magnetic number is zero, $\mu = 0$.) The first positive root of $J_0(\xi)$ is $\xi_{1,0} \approx 2.4048$. The corresponding eigenenergy according to Sect. 3.1. is

$$E_\perp(0) \equiv E_{0,1,0}(0) = \frac{\hbar^2}{2mb^2} \xi_{1,0}^2. \quad (77)$$

B. The lowest eigenenergy for the given value of $B > 0$

We define the dimensionless quantity

$$\beta = \frac{m\omega_{\mathrm{L}} b^2}{2\hbar} = \frac{eb^2 B}{2\hbar}. \quad (78)$$

Hence

$$\hbar\omega_{\mathrm{L}} = \frac{2\hbar^2\beta}{mb^2}. \quad (79)$$

We will calculate the function $E_\perp(B)$ expressing the B-dependence of the lowest eigenenergy of the Schrödinger equation under consideration. The transcendental equation $M(\mathcal{E}_\perp + 1/2, 1, -\beta) = 0$ gives the function $\mathcal{E}_\perp(\beta)$ which diverges for $\beta \to +0$. However, the function $\beta\mathcal{E}_\perp(\beta)$ converges. It is shown in Fig. 3 in Sect. 2. One can prove that

$$\lim_{\beta\to+0} \beta\mathcal{E}_\perp(\beta) = \frac{1}{4}\xi_{1,0}^2. \quad (80)$$

This is in agreement with (77). If the magnetic field B is weak, then

$$E_{\perp}(B) = \frac{\hbar^2 \xi_{1,0}^2}{2mb^2} + \mathcal{O}(B^2). \tag{81}$$

On the other hand, if the magnetic field is strong, then

$$E_{\perp}(B) \approx \frac{\hbar\omega_{\mathrm{L}}}{2} = \frac{\hbar eB}{2m}. \tag{82}$$

In terms of the classical mechanics, we can consider a cyclotron radius r_{c} determined by the energy $E_{\perp}$. The equations (81) and (82) correspond to the limiting cases when $r_{\mathrm{c}} \gg b$ and $r_{\mathrm{c}} \ll b$, respectively.

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