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Pion–Nucleon Sigma Term by Deeply Bound Pionic Atoms

N. Ikeno[a,*], T. Nishi[b], K. Itahashi[b], N. Nose-Togawa[c], A. Tani[a] and S. Hirenzaki[d]

[a] *Department of Agricultural, Life and Environmental Sciences, Tottori University, Tottori 680-8551, Japan*
[b] *Nishina Center for Accelerator-Based Science, RIKEN, 2-1 Hirosawa, Wako, Saitama 351-0198, Japan*
[c] *Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki 567-0047, Japan*
[d] *Department of Physics, Nara Women's University, Nara 630-8506, Japan*

Doi: 10.12693/APhysPolA.142.342 *e-mail: ikeno@tottori-u.ac.jp

We report the results of research on the sensitivity of the observables of the deeply bound pionic atom to the pion–nucleon sigma term $\sigma_{\pi N}$. We calculate the uncertainties of determining the value of $sigma_{\pi N}$ with the accurate data of the deeply bound pionic atoms expected to be obtained at the RI Beam Factory (RIBF), RIKEN. We find that the energy gap of the $1s$ and $2p$ pionic states, $(B_\pi(1s) - B_\pi(2p))$, and the width of the $1s$ state for the lighter Sn isotope are expected to be important observables to precisely determine the $\sigma_{\pi N}$ value by taking into account the expected errors of the experiments.

topics: deeply bound pionic atoms, pion–nucleon sigma term

1. Introduction

The deeply bound pionic atom is known to be a very useful system to investigate the pion properties and aspects of chiral symmetry at finite density [1]. Recently, experimental studies of the deeply bound pionic atoms in Sn isotopes have been performed at the RI Beam Factory (RIBF) [2, 3]. In [2], the angular dependence of the formation spectra of the $(d, {}^3\mathrm{He})$ reaction was observed for the first time, and the binding energies and widths of the pionic $1s$ and $2p$ states were determined simultaneously. Then the value of the pion–nucleus isovector parameter b_1 was determined very precisely by the improved experimental analyses, and the reduction of the chiral condensate $|\langle \bar{q}q\rangle|$ was found concluded with a very small error in [3]. Further experimental information with quite good precision is also expected to be obtained for pionic atoms in ${}^{111,123}\mathrm{Sn}$ by the $(d, {}^3\mathrm{He})$ reaction for the ${}^{112,124}\mathrm{Sn}$ targets [4]. We also mention here a recent experimental achievement of high precision measurements of the kaonic ${}^3\mathrm{He}$ and ${}^4\mathrm{He}$ atoms [5]. Kaonic atoms are also interesting objects to investigate the aspects of the strong interaction symmetry in the flavor SU(3) at finite density.

The pion–nucleon sigma term $\sigma_{\pi N}$ has been studied by various research groups. However, the $\sigma_{\pi N}$ value has not been determined accurately enough. The reported $\sigma_{\pi N}$ values are different between phenomenological and lattice calculations and are distributed in the range of $\sigma_{\pi N} \simeq 30$–60 MeV (see for examples [6, 7]). Therefore, it seems very interesting to determine the $\sigma_{\pi N}$ value with regards to the precise data of deeply bound pionic atoms expected to be obtained in the near future [4]. In [8, 9], the $\sigma_{\pi N}$ value is reported to be $\sigma_{\pi N} = 57 \pm 7$ MeV by using the χ^2 fitting on the binding energies and widths of all existing pionic atom data including those in light nuclei. In our study, we especially focus on the observables of the deeply bound pionic $1s$ and $2p$ states, which are obtained precisely in the experiment of RIBF.

In this article, we discuss the sensitivity of the deeply bound pionic $1s$ and $2p$ state in Sn isotopes to the pion–nucleon sigma term $\sigma_{\pi N}$ in order to investigate the possibility of the precise determination of the value of $\sigma_{\pi N}$. Comprehensive reports of our studies can be found in [10]. We provide brief explanations based on [10] in this article.

2. Formalism

We calculate the structure of the deeply bound pionic atoms to see the $\sigma_{\pi N}$ term dependence of the observables of the deeply bound pionic atoms. To study the structure of the pionic atoms, we solve the Klein–Gordon equation [1, 11]

$$\Big[-\nabla^2 + \mu^2 + 2\mu V_{\mathrm{opt}}(r)\Big]\phi(\boldsymbol{r}) = \Big[E - V_{\mathrm{em}}(r)\Big]^2 \phi(\boldsymbol{r}), \tag{1}$$

where μ is the pion–nucleus reduced mass, E is the eigen energy written as $E = \mu - B_\pi - \frac{i}{2}\Gamma_\pi$ with the binding energy B_π and the width of atomic states Γ_π, $V_{\rm em}$ is the electromagnetic interaction, and $V_{\rm opt}$ is the pion–nucleus optical potential, which we explain in detail below.

In this article, we consider the standard potential widely used for a long time for the studies of pionic atoms. One of the standard optical potential, the so-called Ericson–Ericson type [12], is written as

$$2\mu V_{\rm opt}(r) = -4\pi \left[b(r) + \varepsilon_2 B_0 \rho^2(r)\right] + 4\pi\ \nabla\left[c(r) + \varepsilon_2^{-1} C_0 \rho^2(r)\right] L(r) \nabla, \tag{2}$$

with

$$b(r) = \varepsilon_1 \left[b_0\ \rho(r) + b_1\left(\rho_n(r) - \rho_p(r)\right)\right], \tag{3}$$

$$c(r) = \varepsilon_1^{-1} \left[c_0 \rho(r) + c_1\left(\rho_n(r) - \rho_p(r)\right)\right], \tag{4}$$

$$L(r) = \left[1 + \frac{4\pi}{3}\ \lambda\left(c(r) + \varepsilon_2^{-1} C_0 \rho^2(r)\right)\right]^{-1}, \tag{5}$$

where ε_1 and ε_2 are defined as $\varepsilon_1 = 1 + \mu/M$ and $\varepsilon_2 = 1 + \mu/(2M)$ with the nucleon mass M. The parameters b and c indicate the s-wave and p-wave πN interactions, respectively. Potential terms with parameters B_0 and C_0 are higher order contributions to the optical potential, and λ the Lorentz–Lorenz correction.

The parameters b_0 and b_1 in (3) are replaced by a density-dependent form with the $\sigma_{\pi N}$ term. We follow the form proposed in [13, 14] based on the Tomozawa [15]–Weinberg [16] and the Gell-Mann–Oakes–Renner [17] relations. We determine the value of the s-wave isovector potential parameter b_1 in terms of $\sigma_{\pi N}$ as

$$b_1(\rho) = b_1^{\rm free} \left[1 - \frac{\sigma_{\pi N}}{m_\pi^2 f_\pi^2} \rho\right]^{-1}, \tag{6}$$

where $b_1^{\rm free}$ is the isovector πN scattering length in a vacuum, $b_1^{\rm free} = -0.0861\ m_\pi^{-1}$ [8, 9, 19], and f_π is the pion decay constant in a vacuum, $f_\pi = 92.4$ MeV [13]. In the derivation of the s-wave isoscalar potential parameter b_0 [12], we take into account the double scattering effects with the density-dependent b_1 parameter in (6). As a result, one has

$$b_0(\rho) = b_0^{\rm free} - \frac{3}{2\pi}\varepsilon_1 \left[\left(b_0^{\rm free}\right)^2 + 2b_1^2(\rho)\right] \left(\frac{3\pi^2}{2}\rho\right)^{\frac{1}{3}}, \tag{7}$$

where $b_0^{\rm free}$ is the isoscalar πN scattering length in a vacuum, $b_0^{\rm free} = 0.0076\, m_\pi^{-1}$ [8, 9, 18]. Thus, the explicit $\sigma_{\pi N}$ term inclusion requires consideration of the density-dependent parameters b_0 and b_1 in the optical potential. We use the potential parameters obtained in [19], with the exception of b_0 and b_1, as shown in Table I.

In this article, we consider the Woods–Saxon form for the nuclear densities that appeared in the electromagnetic interaction $V_{\rm em}$ and the pion–nucleus optical potential $V_{\rm opt}$.

3. Results

In Fig. 1, we show the density dependence of the parameters $b_0(\rho)$ and $b_1(\rho)$ defined respectively in (7) and (6) for three different $\sigma_{\pi N}$ values, i.e., $\sigma_{\pi N} = 25$, 45, and 60 MeV. A larger $\sigma_{\pi N}$ value makes stronger dependence of density on parameters, and thus, it has a more repulsive pion–nucleus s-wave interaction.

In Fig. 2, we show the calculated pionic radial density distributions $|R_{nl}(r)|^2$ in ^{123}Sn with the $b_0(\rho)$ and $b_1(\rho)$ parameters for cases $\sigma_{\pi N} = 25$, 45, and 60 MeV. We can see a clear effect of $\sigma_{\pi N}$ to the pion wave function inside the nucleus. Densities are pushed more outwards for larger $\sigma_{\pi N}$ values due to the stronger repulsive effects of the potential.

In Fig. 3, the binding energies and widths of the deeply bound $1s$ and $2p$ states in ^{123}Sn are plotted as functions of $\sigma_{\pi N}$. We observe that each observable depends on the value of $\sigma_{\pi N}$ almost linearly within the range of the $\sigma_{\pi N}$ value considered here. Thus, as for the sensitivity of observable to $\sigma_{\pi N}$, we use the average slope of the line, namely the average size of the shift of each observable due to the 1 MeV variation of the $\sigma_{\pi N}$ value, $\Delta\sigma_{\pi N} = 1$ MeV. We note here that Fig. 3 should not be used directly to determine the value of $\sigma_{\pi N}$ by binding energy and/or width. The purpose of this figure is just to show the sensitivities of the observables to the value of $\sigma_{\pi N}$. We need a thorough analysis of the data in general to determine the absolute value of $\sigma_{\pi N}$.

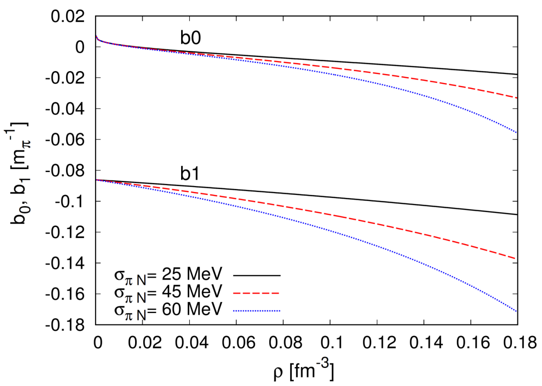

Fig. 1. The density dependence of the parameters $b_0(\rho)$ (7) and $b_1(\rho)$ (6) is shown for different $\sigma_{\pi N}$ values as indicated in the figure.

TABLE I

Pion–nucleus optical potential parameters are obtained in [19] for the so-called Ericson–Ericson potential [12].

Potential parameters	Values
c_0 $[m_\pi^{-3}]$	0.223
c_1 $[m_\pi^{-3}]$	0.25
B_0 $[m_\pi^{-4}]$	0.042 i
C_0 $[m_\pi^{-6}]$	0.10 i
λ	1.0

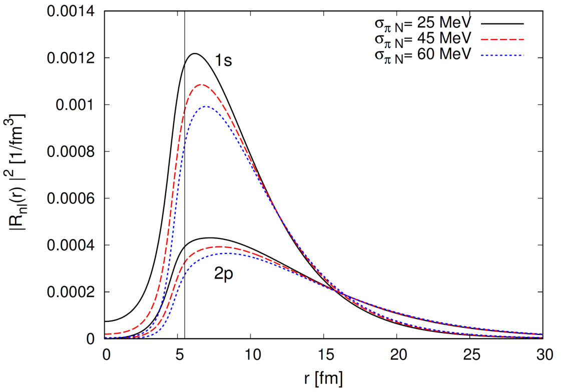

Fig. 2. The radial density distributions $|R_{nl}(r)|^2$ of the pionic $1s$, and $2p$ states in ^{123}Sn are plotted as the functions of the radial coordinate r for different $\sigma_{\pi N}$ values, as indicated in the figures. The density-dependent $b_0(\rho)$ and $b_1(\rho)$ parameters are used. The vertical line shows the radius of ^{123}Sn.

Based on Fig. 3, we evaluate that the average shift size of the $1s$ state binding energy $\Delta B_\pi(1s)$ in ^{123}Sn is $\Delta B_\pi(1s) = 6.2$ keV for the 1 MeV variation of the $\sigma_{\pi N}$ value, $\Delta\sigma_{\pi N} = 1$ MeV. For the $2p$ state, $\Delta B_\pi(2p) = 1.7$ keV. The shift of the width of the $1s$ pionic state $\Delta\Gamma_\pi(1s)$ in ^{123}Sn is 5.9 keV for the $\Delta\sigma_\pi = 1$ MeV variation. For the $2p$ state, $\Delta\Gamma_\pi(2p) = 2.5$ keV. We find that the sensitivities of the $1s$ state observables are stronger than those of the $2p$ states. We also evaluate the gap of the $1s$ and $2p$ states in ^{123}Sn as $|\Delta(B_\pi(1s) - B_\pi(2p))| = 4.5$ keV and $|\Delta(\Gamma_\pi(1s) - \Gamma_\pi(2p))| = 3.4$ keV, respectively. The sizes of the calculated sensitivity of the observables are compiled in Table I.

These calculated sensitivities of the observables can be compared with the accuracy of the most recent experimental data [2, 3]. Typical errors of up-to-date experiments for the deeply bound pionic atom observables by the $(d, {}^3\text{He})$ reactions in the Sn region are around 80 keV for the binding energy of the $1s$ state and around 40 keV for the width of the $1s$ state. In addition, the gap of the binding energies of the $1s$ and $2p$ states, $B_\pi(1s) - B_\pi(2p)$, is important as it can be determined so far more accurately and its error is expected to be $\sim$ 10–15 keV for the Sn region.

We consider the energy gap $B_\pi(1s) - B_\pi(2p)$ in ^{123}Sn and estimate the uncertainties of determining the $\sigma_{\pi N}$ value by the expected experimental errors and calculated sensitivities of the observables. The calculated sensitivity of the energy gap $|\Delta(B_\pi(1s) - B_\pi(2p))|$ for ^{123}Sn is 4.5 keV. In this case, the experimental error $\sim$ 10–15 keV of this energy gap can be interpreted as the uncertainty of the $\sigma_{\pi N}$ value $\sim$ 2.2–3.3 MeV, which is obtained by dividing the experimental error 10–15 keV by the observable sensitivity 4.5 keV for the 1 MeV change of the $\sigma_{\pi N}$ value.

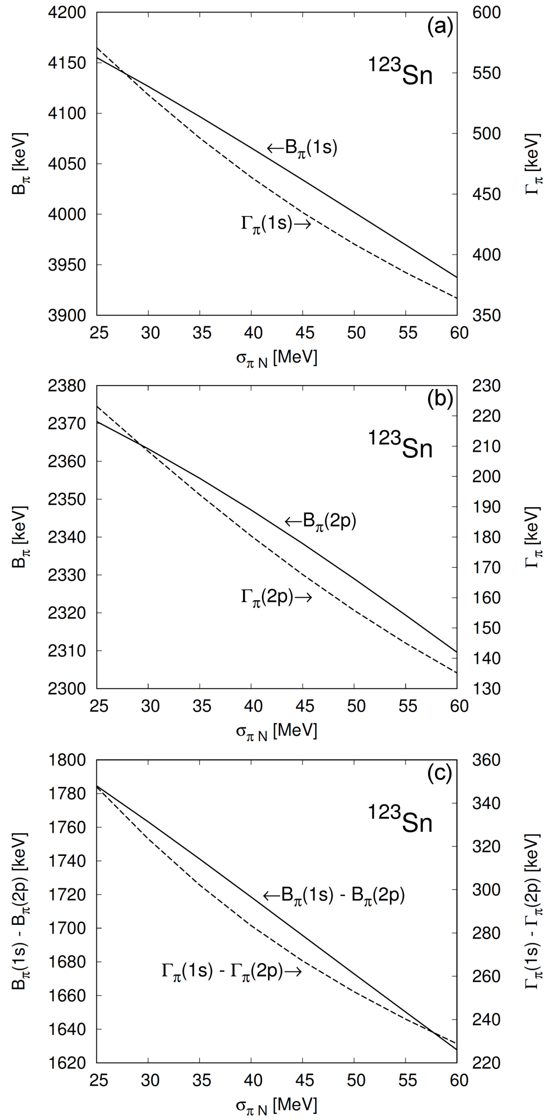

Fig. 3. The binding energies (B_π) and the widths (Γ_π) of the pionic $1s$ (a) and $2p$ (b) states, and (c) the differences of B_π and Γ_π between these states in ^{123}Sn are plotted as functions of $\sigma_{\pi N}$. The density-dependent $b_0(\rho)$ and $b_1(\rho)$ parameters are used.

On the other hand, the expected size of the experimental error 80 keV of $B_\pi(1s)$ and the calculated sensitivity 6.2 keV for the 1 MeV change of $\sigma_{\pi N}$ in ^{123}Sn allow concluding that the expected uncertainty of the $\sigma_{\pi N}$ value is large and can be equal 13 MeV, which is estimated as 80 keV$/(6.2$ keV$/\Delta\sigma = 1$ MeV). Similarly, the width of the $1s$ state for ^{123}Sn provides the expected $\sigma_{\pi N}$ uncertainty of 6.8 MeV for an experimental error 40 keV, which is estimated as 40 keV$/(5.9$ keV$/\Delta\sigma = 1$ MeV). Thus, we find from the typical size of the experimental errors and the calculated sensitivities of the observables that the energy gap between the $1s$ and $2p$ states has a larger possibility to provide important information to determine the $\sigma_{\pi N}$ value precisely.

TABLE II

Calculated average shifts of the observables of deeply bound pionic states are shown in the unit of keV for the 1 MeV change of the $\sigma_{\pi N}$ value, $\Delta\sigma_{\pi N} = 1$ MeV. Here, $\Delta(B_\pi(1s)-B_\pi(2p))$ and $\Delta(\Gamma_\pi(1s)-\Gamma_\pi(2p))$ indicate the average shifts of the differences of the binding energies and widths between the $1s$ and $2p$ states, respectively, for the $\sigma_{\pi N}$ change $\Delta\sigma_{\pi N} = 1$ MeV. These numerical results are taken from [10].

[keV]	^{123}Sn	^{111}Sn
$\lvert\Delta B_\pi(1s)\rvert$	6.2	7.5
$\lvert\Delta\Gamma_\pi(1s)\rvert$	5.9	12.9
$\lvert\Delta B_\pi(2p)\rvert$	1.7	1.7
$\lvert\Delta\Gamma_\pi(2p)\rvert$	2.5	3.6
$\lvert\Delta(B_\pi(1s)-B_\pi(2p))\rvert$	4.5	5.8
$\lvert\Delta(\Gamma_\pi(1s)-\Gamma_\pi(2p))\rvert$	3.4	9.3

We also calculate the average shifts of the observables of deeply bound pionic states in the lighter Sn isotope, i.e., ^{111}Sn. The results are summarized in Table II. Large sensitivities to $\sigma_{\pi N}$ are found for pionic states in the lighter Sn isotope ^{111}Sn. The shift of the width of the $1s$ pionic states, $\Delta\Gamma_\pi(1s)$, in ^{111}Sn is 12.9 keV for a $\Delta\sigma_\pi = 1$ MeV variation, which is more than twice of $\Delta\Gamma_\pi(1s)$ in ^{123}Sn.

Additionally, we estimate the uncertainties of determining the values of $\sigma_{\pi N}$ in the lighter Sn isotope ^{111}Sn as well as in the case of ^{123}Sn. As for the calculated sensitivity of the energy gap $\lvert\Delta(B_\pi(1s)-B_\pi(2p))\rvert$ for ^{111}Sn, the uncertainty of the value of $\sigma_{\pi N}$ is $\sim$ 1.7–2.6 MeV, which is estimated as $\sim$ 10–15 keV/(5.8 keV/$\Delta\sigma = 1$ MeV). The width of the $1s$ state $\Delta\Gamma_\pi(1s)$ for ^{111}Sn provides the expected uncertainty of $\sigma_{\pi N}$ of 3.1 MeV, which is estimated as 40 keV/(12.9 keV/$\Delta\sigma = 1$ MeV). Therefore, we also find that the width of the $1s$ state in lighter Sn isotopes has a relatively small uncertainty of the $\sigma_{\pi N}$ value.

In [10], we also calculated the formation spectra of the $(d,{}^3\mathrm{He})$ reactions in Sn isotope with the effective number approach [20–23]. We find that the shapes of the spectrum have a reasonable sensitivity to the $\sigma_{\pi N}$ value at any scattering angle. Especially the peak height of the pionic $1s$ state formation is clearly reduced for the smaller $\sigma_{\pi N}$ values.

4. Conclusions

Meson–nucleus bound states are known to be one of the best objects to investigate meson properties and strong interaction features at finite nuclear density under quasi-static circumstances. In addition to the pionic atoms considered in this article, the kaon– and η–nuclear bound states provide important information on the $\Lambda(1405)$ and $N^*(1535)$ baryon resonances and, for example, the $\eta(958)$–nuclear bound states are expected to provide information on the effects of the $U_A(1)$ anomaly at finite density [24–26]. In this article, we have studied the sensitivities of the observables of deeply bound pionic atoms to the value of the pion–nucleon sigma term $\sigma_{\pi N}$ and investigate their experimental feasibilities to determine the $\sigma_{\pi N}$ value precisely by considering the expected errors of up-to-date experiments. We improved the theoretical formula and implement the $\sigma_{\pi N}$ term in the optical potential to treat explicitly the density dependence on the potential parameters b_0 and b_1. We have calculated the various observables such as binding energies, widths, and cross-sections, and studied their sensitivities to the $\sigma_{\pi N}$ value for the deeply bound pionic atoms in ^{111}Sn and ^{123}Sn.

We found that the binding energies and widths of the pionic $1s$ states have the largest sensitivities to the $\sigma_{\pi N}$ value. Sensitivities tend to be even larger for the lighter Sn isotopes. Considering the expected errors of the experiments, we concluded that the energy gap of the $1s$ and $2p$ pionic states, $\lvert\Delta(B_\pi(1s)-B_\pi(2p))\rvert$, and the width of the $1s$ state for the lighter Sn isotope are expected to be good observables to accurately determine the $\sigma_{\pi N}$ value. The uncertainties to the $\sigma_{\pi N}$ value due to experimental errors to these observables are estimated to be around 3 MeV. Thus, we can say that it is very interesting to determine the value of $\sigma_{\pi N}$ based on precise data of the deeply bound pionic atoms. As for the next step, in order to perform an actual determination of the value of the $\sigma_{\pi N}$ term from the experimental data, we need to solve the expected difficulties, such as the well-known strong correlation between the potential parameters [19] and large uncertainties of the neutron distribution of the nucleus.

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