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Instability of Super-Chandrasekhar White Dwarfs in Modified Gravity

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Super-Chandrasekhar white dwarfs are a timely topic in the last years in the scientific community due to its connection to supernovae type Ia. Some early studies tackled the possibility of white dwarfs surpassing the Chandrasekhar limit by means of a magnetic field. More recently modified gravity has been highlighted as the reason for these stars to surpass the Chandrasekhar limit and becoming a supernova progenitor. However, in general simple assumptions are considered for the stellar structure and equation of state, which can lead to unreliable conclusions. In this work we intend to be rigorous and consider a realistic equation of state to describe the white dwarfs in general relativity and modified gravity, taking into account a nuclear instability, that limits the maximum mass.

topics: modified gravity, super-Chandrasekhar white dwarfs, equation of state (EoS)

1. Introduction

White dwarfs (WD) are stars that can reach densities as high as $\sim 10^{11}$ g/cm^3 in their interiors with observed magnetic fields up to $\sim 10^9$ G. The masses are limited by the so-called Chandrasekhar mass limit $M_{\rm Ch} = 1.44\ M_\odot$, with $M_\odot$ representing the Solar mass. The radii of WDs are of the order of 10^4 km, which renders a surface gravity, $\log_{10}(g)$, in the range of 8–10. These extreme properties make WDs a laboratory of tests for strong gravity regimes, thus motivating their application to the study of modified gravity theories. In this way, we can constrain the parameter space of new theories.

On the other hand, some peculiar, overluminous type Ia supernovae have been linked to the possible existence of super-Chandrasekhar white dwarfs. The origin of type Ia supernovae is understood as the collapse of either a WD binary or a massive WD above the Chandrasekhar limit.

2. Hydrostatic equilibrium equations

To model massive stars, relativistic hydrostatic equilibrium equations are needed. For a perfect fluid energy–momentum tensor and for a static spherically symmetric spacetime, the Einstein's field equations $G^{\mu\nu} \equiv R^{\mu\nu} - 1/2 g^{\mu\nu} R = 8\pi T^{\mu\nu}$ lead to the Tolman–Oppenheimer–Volkoff (T.O.V.) equation [1, 2]. This equation reads in natural units as

$$p' = -(\rho + p)\frac{(4\pi p r + m/r^2)}{(1 - 2m/r)}, \tag{1}$$

where the prime indicates radial derivative and m is the gravitational mass enclosed within a surface of a radius, given by $m' = \mathrm{d}m/\mathrm{d}r = 4\pi\rho r^2$. To solve this system, one needs the equation of state $p(\rho)$ and the boundary conditions $m(r)|_{r=0} = 0$, $p(r)|_{r=0} = p_c$, and $\rho(r)|_{r=0} = \rho_c$, where p_c and ρ_c are the pressure and density at the center of the star, respectively. The numerical integration of (1), once the equation of state (EoS) is provided, gives the global properties of the stars.

When one considers a modification in the theory of gravity, the field equations changed. Generally, the symmetric spacetime/perfect fluid energy–momentum tensor is still used. In this case, one will have T.O.V.-like equations for the hydrostatic equilibrium equations that model relativistic stars.

For a specific theory called $f(R, L_m)$ gravity, which we considered earlier [3], the equations are

$$\alpha'\big(p + \rho\big) + 2z = 0, \tag{2}$$

$$p' - z = 0, \tag{3}$$

$$\frac{e^{-\beta}}{3r^2}\Big[2r^2\rho e^{\beta} + \Big(2R\,r^2\rho e^{\beta} + 3r^2 z\alpha' + 6pr\beta'$$
$$+2\big(2Rpr^2 + 3p\big)\,e^{\beta} - 6p\Big)\sigma$$
$$-\Big(\big(R - 3p\big)r^2 + 3\Big)\,e^{\beta} - 3r\beta' + 3\Big] = 0, \qquad (4)$$

$$\frac{e^{-\beta}}{3r^2}\Big[r^2\rho e^{\beta} + \Big(Rr^2\rho e^{\beta} + 3r^2 z\beta' + 6pr\alpha' - 6r^2 z'$$
$$-\big(Rpr^2 + 6p\big)\,e^{\beta} + 6p\Big)\sigma$$
$$+\Big(Rr^2 + 3\Big)\,e^{\beta} - 3r\alpha' - 3\Big] = 0. \qquad (5)$$

where α and β are metric potentials depending on the radial coordinate r, and z is an auxiliary variable which is the derivative of the pressure. For complete details, see [4, 5]. Once EoS is defined, global properties such as mass and radius can be found from (1)–(5).

3. Critical mass

The critical mass of white dwarfs has been known for a long time, when Stoner [7] considered special relativity to describe Fermi–Dirac statistics of stars. The mass was established as $M_{\rm crit} \approx K M_{\rm P}^3/\mu^2 m_n^2$, where $M_{\rm P}$ is the Planck mass, m_n is the neutron mass, and μ is the average molecular weight A/Z. The constant K was determined as $K = 3.72$. Later in the works of Chandrasekhar [7, 8], Landau [9], and Gamow [10], the value was corrected to $K = 3.09$ using the Lane–Emden equations. To reach this value, a simple EoS was used; it considers the model of the non-interacting relativistic Fermi gas of electrons. Although EoS can describe WDs very well, the corrections were derived by Hamada–Salpeter (HS) which accounts for electrostatic interactions, Thomas–Fermi deviations, exchange energy and spin–spin interactions [11, 12]. However, only the electrostatic corrections were found to be non-negligible. The Chandrasekhar EoS is dependent on μ, and the HS EoS — apart from the dependence on μ — depends on the nuclear composition of a homogeneous star, which slightly decreases the mass limit compared to Chandrasekhar results.

The electron pressure in HS EoS is lowered by electrostatic attraction of electrons and ions. Further and new developments when heavy elements are important were considered in the Thomas-Fermi [13] and Feynman–Metropolis–Teller models [14]. The role of the electron–ion interaction in these models started to be considered in more ways, i.e., inclusion of corrections of nuclear thresholds such as inverse β-decay and pycnonuclear reactions [15, 16], leading to the investigations of low mass neutron stars that could be generated by massive white dwarfs made of oxygen–neon–magnesium [17, 18], i.e., massive WDs near the Chandrasekhar limit [19].

3.1. Stability criteria for critical mass

3.1.1. Gravitational instability

When considering a one-parameter sequence of equilibrium stars with EoS for different central densities ρ_c, one can show that [20]

$$\frac{\partial M}{\partial \rho_c} > 0, \qquad \text{for stable equilibrium configurations,}$$

and

$$\frac{\partial M}{\partial \rho_c} < 0, \qquad \text{for unstable equilibrium configurations.}$$

If the $M(\rho_c)$ curve has only one critical point ($\partial M/\partial\rho_c = 0$), it will mark the onset of stability under radial oscillations, thus defining the maximum mass allowed due to gravity. In general, only these gravitational stability criteria were used in the works that studied white dwarfs in modified gravity, and in addition, a simplistic Chandrasekhar EoS was applied. When taking into account the improvements in EoS as shown in the previous section, the maximum mass decreases. Moreover, when considering the onset of nuclear instabilities, they are often reached before the onset of gravitational instability, which limits even the maximum mass in GR [21]. That is also important for modified gravity, i.e., the onset of nuclear instability should be taken into account since it will turn on before the gravitational one.

3.1.2. Nuclear instability

Corrections in EoS can arise due to nuclear reactions with the latter coming from the effects of inverse β-decay, which reduces the maximum mass M of white dwarfs [22]. As the star goes to higher density, the matter suffers compression and the electrons combine with the nuclei, generating another nucleus and a neutrino [23], ${}^A_Z X + e^- \to {}^A_{Z-1} Y + \nu_e$. Electron capture leads to global star instability, which can induce the core-collapse of the white dwarf. The undergoing collapse depends on the relation between electron capture and pycnonuclear reactions. The instability of a pure ^{12}C star, taking into account the general relativistic effects, has been calculated [24], leading to a maximum mass of $M \approx 1.366 M_\odot$. The maximum Fermi energy of electrons was computed to be 12.15 MeV, and for a heterogeneous WD with ^{12}C/^{16}O, the configuration becomes unstable when the ^{16}O concentration exceeds 0.06, leading to a maximum mass of $M \approx 1.365 M_\odot$.

For the reaction to occur, one needs the Gibbs energy per nucleon of the original nucleus to be higher than that of the newly produced nucleus, i.e., $g(p, A, Z) \geq g(p, A, Z-1)$. For a detailed discussion about neutronization, see Sect. V. in [23].

4. Results and discussions

Considering the Hamada–Salpeter EoS [12] for ^{4}He, ^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg, ^{32}S, ^{56}Fe and using the mass density threshold for electron capture [23], we constructed stellar sequences of equilibrium considering general relativity and $f(R, L_m)$ theories of gravity in order to explore the maximum mass allowed for stars.

In Fig. 1 we show the behavior of a star's mass versus the central energy density within general relativity. The pink triangles indicate the onset of gravitational instability (maximum mass point). From those points to the right, the stellar mass decreases with the increment of ρ_c, and thus this region is unstable under radial oscillations. Additionally, the black stars mark the onset of nuclear instabilities. From these points to the right side of the sequences, stars are unstable due to the electron capture reactions. As one can see, for light elements, gravitational instability limits the maximum mass of the star before the electron capture reactions occur. However, for elements heavier than oxygen, the electron capture reactions take place before the maximum mass point is reached. As a result, nuclear instabilities are the main factor in restricting the maximum stable mass.

In Fig. 2 we show the sequence of stellar masses versus central energy density within $f(R, L_m)$ gravity for white dwarfs composed of ^{4}He and ^{56}Fe. We have considered four values for the theory's parameter. The values are: 0.00, 0.05, 0.10 and 0.50 km^2. For $\sigma = 0.00$ the theory recovers results of general relativity.

In Fig. 2a, the element ^{4}He was considered. We can see an increment in masses according to an increase in the value of σ. One can observe that when $\sigma \neq 0$, the stability criterion is not applicable and the gravitational instability disappears, i.e., the instability criterion $\mathrm{d}M/\mathrm{d}\rho_c < 0$ is not met. Such behavior, in principle, could imply a white dwarf with arbitrarily large mass, which is an unrealistic result given the observational data. In this case, what constraints the maximum mass is the electron capture threshold marked by the black stars.

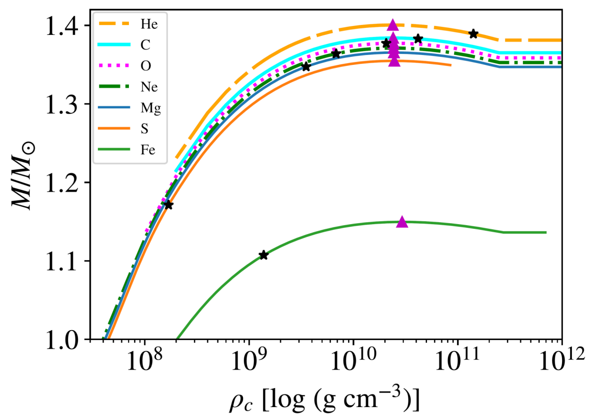

Fig. 1. Mass vs central energy density using the Hamada–Salpeter EoS for different star compositions.

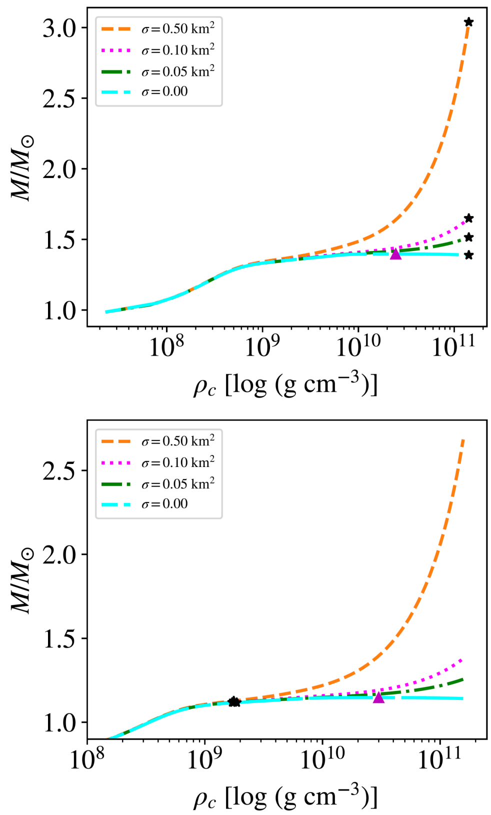

Fig. 2. Mass vs central energy density using the Hamada–Salpeter EoS. (a) ^{4}He WDs with four different values of the modified gravity parameter. (b) ^{56}Fe WDs with four different values modified gravity parameter.

In Fig. 2b, the element ^{56}Fe was considered. As in the previous case, increasing the theory's parameter also leads to an enhancement in the maximum masses. However, as the density threshold for electron capture in ^{56}Fe stars is remarkably smaller, the effects of the modified theory become negligible. Therefore, the density threshold for electron capture cannot be disregarded, and in particular it drastically reduces the maximum stable mass. This is important in the context of modified gravity theories used to generate high stellar masses. Once there is a limit in the density regime, it must be respected, otherwise misleading results will be obtained.

5. Conclusions

We have considered the stability of white dwarfs within the modified gravity theory. We found that the standard gravitational Chandrasekhar limit does not exist. Therefore, the maximum mass of a white dwarf is in the theory provided by nuclear instability, which is introduced due to electron capture.

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