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Visualization of The Lorentz Force Using Fractal Structures

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In the following work, research was carried out on the visualization of Lorentz force using fractal structures. The fractal dimension of dendritic sediments and their morphological change under an external magnetic field generated by a permanent magnet were investigated. The formation of dendrites was determined by the diffusion limited aggregation process that occurs during the electrolysis of $CuSO_4$ solutions. The obtained experimental values of the fractal dimension correspond to the theoretical value. The morphology of the sediments obtained during the tests depends on the molar concentration of the electrolyte, as well as the voltage applied to the electrodes. The computer simulation of the deposits was carried out using an algorithm based on cellular automata. The theoretical values of the fractal dimension after the calculations gave a result of 1.7, and the experimentally obtained mean values of the fractal dimension for CuSO4 were 1.63.

topics: diffusion limited aggregation, cellular automata, Lorentz force, dendrites

1. Introduction

The Lorentz force is the force acting on an electrically charged particle moving in an electromagnetic field. It was discovered and described by Hendrik Lorentz. In the case of the movement of a charged particle in an external magnetic field, the relation that allowed for calculation of the Lorentz force is given as

$$\boldsymbol{F} = q\,(\boldsymbol{v} \times \boldsymbol{B})\,, \qquad (1)$$

where $\boldsymbol{F}$ is the Lorentz force and q is the charge moving with the velocity $\boldsymbol{v}$ in the external magnetic field $\boldsymbol{B}$.

As it was shown in [1], aggregation is the process of joining single molecules to other and they eventually form clusters. Such a process is well known for several decades. This process occurs in nature and in medicine (immunology) and materials engineering (i.e., polymers). Diffusion limited aggregation (DLA) opened a new way in the aggregation process [2], during which fractal structures were observed. Such structures were formed by molecules sticking to each other to construct dendritic fractal forms. As shown in [3–7], the electrodeposition process allowed to produce fractal dendritic structures. Previous studies presented simulation and aggregates produced from solutions of $CuSO_4$ and $ZnSO_4$ with different concentrations and voltages. In present studies, the same technique has been slightly modified by using a permanent magnet to produce magnetic field during the DLA process. Such a modification allowed to show the effect of the Lorentz force on the production of fractals. Moreover, the fractal dimension was calculated.

2. Simulation and experimental technique

The investigation of the electrolytic deposition was carried out using a cylindrical Petri dish with a diameter of about 50 mm and a depth of 1.5 mm. The dish was filled with a thin layer of an aqueous solution of copper sulfate ($CuSO_4$). The copper anode was prepared in the form of a ring mounted on the periphery of the vessel. The cathode made of graphite was placed in the dish center in order to provide contact with the electrolyte. Adjustable power supply allowed to control the voltage applied to the electrodes. The DLA process was filmed using the Sony DCR 106E camera and photographed with the Nikon Coolpix 3200 camera. The aggregates were formed at controlled voltage values within the range of 0–12 V and various molar concentration of the electrolyte (up to 1 M solution). Figure 1a–d presents a scheme of the experimental system, a Petri dish, a scheme of deposition in a magnetic field and an image of dendritic structure for copper deposition in a magnetic field, respectively.

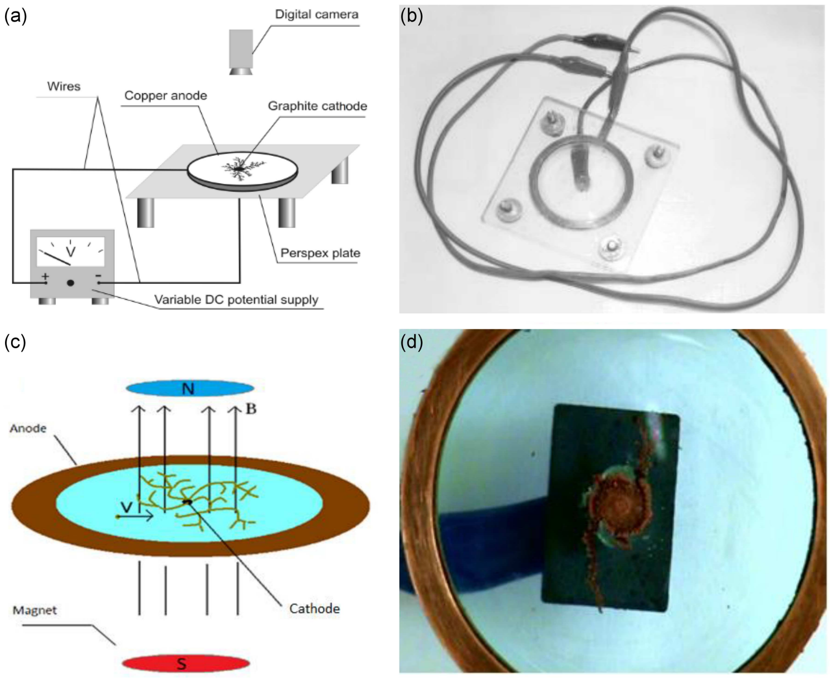

Fig. 1. Scheme of (a) the experimental system [1], (b) Petri dish with connected electrical wires [1], (c) deposition scheme in a magnetic field and (d) an example of the fractal structure of a copper deposit in a magnetic field (0.5 M solution of copper sulfate, voltage equal to 10 V).

As it was presented in [1], the simulation of electrolytic deposition of metallic fractal structures bases on chaotic (Brownian) motion. It was assumed that the copper ions can move randomly in the solution and be trapped by the cathode [3, 6]. During simulation, random walk model describes a single displacement. Taking into account the probability of attaching copper ions, it is highest in the closest area of the dendrites.

The procedure of simulation was given in [6] based on cellular automata. However, in the case of applying a magnetic field, the algorithm was slightly changed to the following form:

1. At the beginning, a square array of cells of size $n \times n$ is established. All cells are set to 0, except for the central element of the array. The value of this central point is 2 and it is treated as a nucleus of dendritic crystallization.
2. The particles run in a circular region, their motion is similar to the move of a knight in chess.
3. The free particle is treated as an element with the value of 1.
4. A free particle succumbs to two scenarios, i.e., (i) it leaves the area of the array — it is forgotten and a new particle is released at a randomly selected edge point, or (ii) the element stays in the area of the array, moving in a chaotic way until it encounters the crystallization nucleus and changes its value to 2. After that, it becomes a part of the immovable nucleus.
5. In the next step, another free element with value 1 is placed on the edge of the array at a randomly selected location and the procedure starts again.

3. Results and discussion

Figure 2 shows the results of simulation of the growth of the copper fractal structure in a magnetic field. Free particle is marked gray.

All technical problems with simulations were described in previous paper [1]. Based on the results of simulation, the fractal dimension was calculated based on the following relation

$$D = \frac{\ln(N)}{\ln(R_N)}, \tag{2}$$

where D denotes the fractal dimension, N is the number of particles joined to the structure, and R_N means the distance between the center of an array and the particle of the structure farthest from the center. The revealed value of the fractal dimension was 1.72 and was close to that calculated in [1].

Pictures of dendrites formed during the DLA process in the magnetic field are presented in Fig. 3. It is characteristic that all structures are bent due to acting of the Lorentz force. The structures were formed for the following parameters, respectively: (a) concentration 0.3 M, voltage 10 V, (b) concentration 0.5 M, voltage 12 V, and (c) concentration 1 M, voltage 10 V.

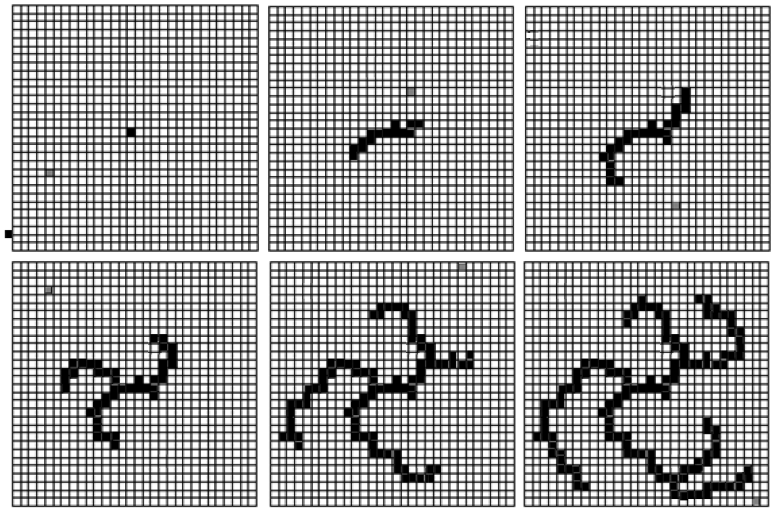

Fig. 2. Stages of fractal growth simulation realized for an array 30 × 30 size using the DLA model.

In order to determine the fractal dimension of the electrolytic deposits, the current flowing through the electrolyte and the length of the longest aggregate were measured.

As it was shown in [1], the calculation of the fractal dimension of the experimental forms was based on the first Faraday's law. The mass m of the deposition at the cathode is related to the electric charge q passing through the electrolyte, i.e.,

$$m = k\,q. \tag{3}$$

$$q = I\,t, \tag{4}$$

where I is the current and t is the time. Therefore,

$$m = k\,I\,t, \tag{5}$$

where k means the electrochemical equivalent of an element. The mass-produced deposit is extremely related to the deposition radius, according to [3]

$$m \propto R^D, \tag{6}$$

where m means the mass of the deposit, R is the length of the structure radius, and D means the fractal dimension.

The number of deposited ions is provided by [3]

$$N = C\,R^D, \tag{7}$$

where N is the number of particles in the structure and C is the proportionality constant. A calculation of the number of particles joined in specific time is described by [2]

$$\Delta N = \frac{I}{q}\,\Delta t. \tag{8}$$

In order to reveal fractal dimension directly, the relation (5) was rewritten in the following form

$$\log\big(N\big) = D\log\big(R\big) + \log\big(C\big). \tag{9}$$

The fractal dimension was determined from the slope of the linear regression. The calculated value of the fractal dimension is 1.63. This value is relatively close to the results obtained in theoretical prediction and in previous studies [1].

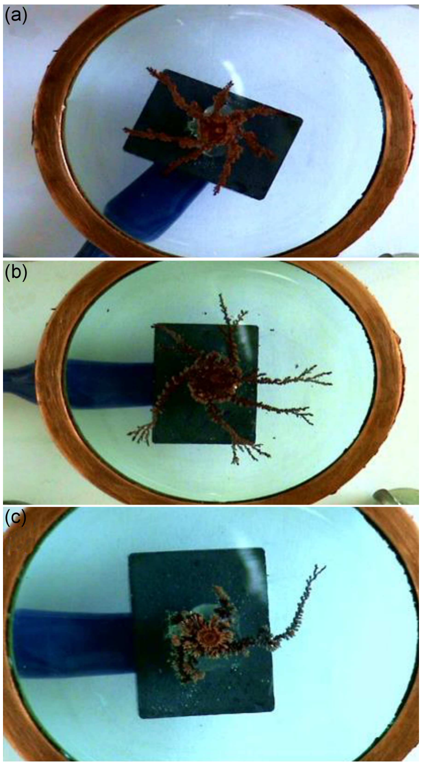

Fig. 3. Images of the copper electrodeposit structures for different molar concentrations of $CuSO_4$: (a) 0.3 M, $U = 10$ V, (b) 0.5 M, $U = 12$ V, (c) 1 M, $U = 10$ V.

4. Conclusions

As it was shown in the present paper, DLA structures observed in the electrodeposition process could easily visualize the Lorentz force. Moreover, such a simple experiment delivers information on such phenomena as aggregation, electrolysis, randomness in physical systems and fractal structures.

Moreover, the values of fractal dimension, revealed theoretically and experimentally, correspond quite well. The morphology of the obtained deposits is characteristically bent, which is the result of the Lorentz force acting on the ions moving in the electrolyte. The action of the Lorentz force was clearly visible for the voltage range 10–12 V and the electrolyte concentration higher than 0.3 M.

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