ACTA PHYSICA POLONICA A No. 1 Vol. **142** (2022)

Papers submitted to the Conference "Applications of Physics in Mechanical and Material Engineering"

Transmission in Two-Element Quasi One-Dimensional Acoustic Barriers

S. Garus*, J. Garus and W. Sochacki

Department of Mechanics and Fundamentals of Machinery Design, Faculty of Mechanical Engineering and Computer Science, Czestochowa University of Technology,
Dąbrowskiego 73, 42-201 Częstochowa, Poland

Doi: 10.12693/APhysPolA.142.92 *e-mail: sebastian.garus@pcz.pl

The study analyzed the mechanical wave transmission for a source with a sound pressure level of 90 dB through all possible distributions of a finite multilayer structure made of air and polylactic acid. The structure was surrounded by air. The influence of thickness and spatial distribution of layers on the shifts of transmission peaks and the reduction of its value are presented. The analyzed structures showed the existence of bandgaps, i.e., wave frequencies, where, due to destructive interference, no propagation through the structure occurs. The transmission was determined using the transfer matrix method algorithm.

topics: acoustic barriers, transfer matrix method, phononic band gap, multilayers

1. Introduction

Phononic structures consist of mechanical wave diffusing elements embedded in the matrix material. The mechanical wave, interacting with the elements of the phononic structure due to diffraction and destructive interference, may not propagate in the structure for the given frequency ranges. This phenomenon is called the phononic band gap (PhBG) [1–9]. It enables the manipulation of mechanical waves, and thus the design of devices such as, for example, acoustic filters [10–12], selective filters [13–15], medical devices [16, 17], acoustic cloaking [18], mechanical wave lenses [19–23], waveguides [24–26], sensors [27], acoustic diodes [28], noise suppression devices [29], acoustic barriers [30], or energy harvesting [31–33].

The study investigated the influence of the arrangement of layers, their thickness, and number on the formation of the band gap in one-dimensional structures in terms of designing multi-layer acoustic partitions.

2. Transfer matrix method (TMM)

The transmission matrix algorithm is used to obtain the frequency response of the mechanical wave propagating in one-dimensional phononic structures, thanks to which it is possible to determine the width and location of the gaps [13, 34]. The theoretical foundations of the method are presented in [35–37].

Propagation of the disturbance in the pressure field $p(\boldsymbol{x},t)$ in the multilayer medium is determined by the partial differential equation of the hyperbolic type (Helmholtz equation) in the form

$$\frac{1}{c_i^2}\frac{\partial^2 p\left(\boldsymbol{x},t\right)}{\partial t^2} - \nabla^2 p\left(\boldsymbol{x},t\right) = 0, \tag{1}$$

where c_i is the phase velocity of the mechanical wave in the i layer of the given material. The solution of (1) for the one-dimensional case in the layer i takes the form

$$p_i\left(x,t\right) = \left(A_i\, \mathrm{e}^{\mathrm{i}\,k_i x} + B_i\, \mathrm{e}^{-\mathrm{i}\,k_i x}\right)\mathrm{e}^{-\mathrm{i}\,\omega t}, \tag{2}$$

where A_i is the amplitude of the component of the wave propagating in the direction of the incident wave, and B_i is the amplitude of the component of the wave propagating in the direction opposite to the direction of the incident wave. The wavenumber k_i for a given layer i is defined by the circular frequency ω or its frequency f as

$$k_i = \tfrac{\omega}{c_i} = \tfrac{2\pi f}{c_i}. \tag{3}$$

The incident mechanical wave with the amplitude p_{in}^{+} is transmitted to the layer structure made of materials A and B. The amplitude of the mechanical wave leaving the structure is marked as p_{out}^{+}, while the amplitude of the reflected wave is marked as p_{in}^{-}. The relationship between the incident wave passing through the structure and the reflected wave is described by the matrix equation

$$\begin{pmatrix} p_{\mathrm{in}}^{+} \\ p_{\mathrm{in}}^{-} \end{pmatrix} = M \begin{pmatrix} p_{\mathrm{out}}^{+} \\ 0 \end{pmatrix}, \tag{4}$$

where M is the matrix characteristic of an N layered structure, the form of which is influenced by the type of materials used and the topology of the structure and is defined as

$$M = \Phi_{in,1} \left(\prod_{i=1}^{N-1} P_i \, \Phi_{i,i+1} \right) P_N \, \Phi_{N,\,\mathrm{out}}. \tag{5}$$

The characteristic matrix M consists of the propagation matrix P_i, inside a given layer i and thickness d_i, defined by

$$P_i = \begin{pmatrix} \mathrm{e}^{\mathrm{i}\, k_i d_i} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}\, k_i d_i} \end{pmatrix}. \tag{6}$$

The second component of the characteristic matrix M is the transmission matrix $\Phi_{i,i+1}$ at the interface i and $i+1$ defined as

$$\Phi_{i,i+1} = \frac{1}{t_{i,i+1}} \begin{pmatrix} 1 & r_{i,i+1} \\ r_{i,i+1} & 1 \end{pmatrix}, \tag{7}$$

which consists of the Fresnel transmission $t_{i,i+1}$ and reflectance $r_{i,i+1}$ coefficients, given by

$$t_{i,i+1} = \frac{2Z_{i+1}}{Z_{i+1} + Z_i}, \tag{8}$$

$$r_{i,i+1} = \frac{Z_{i+1} - Z_i}{Z_{i+1} + Z_i}. \tag{9}$$

The acoustic impedance Z_i for a given layer i is the product of the phase velocity c_i of the mechanical wave and the density of the material ϱ_i, from which the layer is made.

The transmission T of the mechanical wave through the multilayer structure is determined from the characteristic matrix M from the dependence

$$T = \frac{Z_{\mathrm{out}}}{Z_{\mathrm{in}}} \left| \frac{1}{M_{11}} \right|^2. \tag{10}$$

In lossless structures, the transmission T and the reflectance R are related to each other by the relationship from the equation $T + R = 1$.

3. Results and discussion

Mechanical wave transmission was analyzed for a source with a sound pressure level of 90 dB for a structure made of air as material A with a layer thickness of 1 cm and polylactic acid (PLA) as material B with a thickness of 1 mm. The structure was surrounded by air.

In this work, the air was assumed as material A, and material B was assumed to be PLA, for which the phase densities and velocities were respectively $\rho_A = 1.29$ kg/m^3 and $\rho_B = 1240$ kg/m^3, and $c_A = 331.45$ m/s and $c_B = 2220$ m/s [38–40].

Figure 1 shows the effect of increasing the number of layers on the transmission structure. Figure 1a shows the reference curve. Figures 1b–h show the sound pressure level as a function of frequency for 2 to 8 PLA layers in the structure, respectively. Above the frequency of 1 kHz for the given number of layers, the number of transmission peaks was 1 lower than the number of PLA layers. The vertical lines in Fig. 1 indicate the frequencies (4.1, 5.36, 7.6, and 9.3 kHz) for which there were common transmission peaks for more than one structure. Figure 2 shows the transmission curves from Fig. 1.

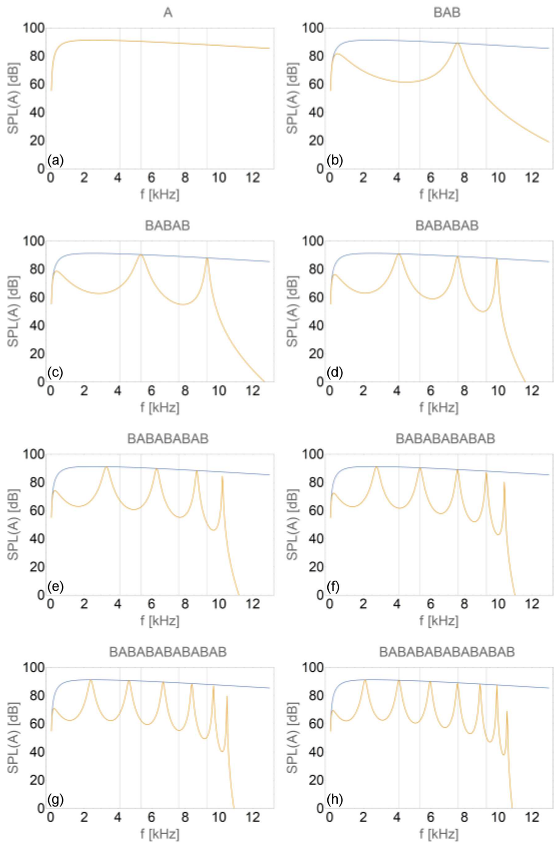

Fig. 1. The sound pressure level (SPL) as a function of frequency with increasing the number of PLA layers from 2 to 8.

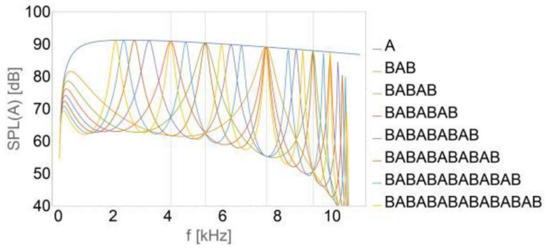

Fig. 2. Sound pressure level as a function of frequency for structures with a single PLA layer of different thicknesses and reference transmission (no PLA layer).

As shown in Figs. 1 and 2, increasing the number of layers increases the transmission peaks in the spectrum. For an even number of layers, there was a transmission peak common to all analyzed structures for a frequency value of 7.6 kHz.

As shown by the conducted research, the number of transmission peaks increases with an increase in the number of layers. In order to analyze the influence of the thickness of A and B materials on the transmission, a five-layer system was analyzed.

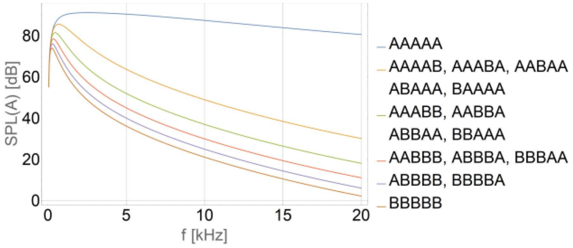

Fig. 3. Sound pressure level as a function of frequency for structures with a single PLA layer of different thicknesses and reference transmissions (no PLA layer).

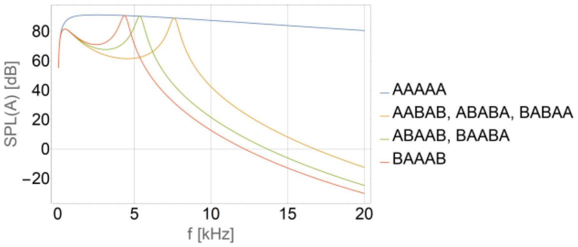

Fig. 4. Sound pressure level as a function of frequency for structures with two identical PLA layers and increasing distance between them and with reference transmission (no PLA layer).

TABLE I

Sound pressure level values [dB] for selected frequencies and given layer A thickness.

Thickness of the PLA layer [mm]	5 kHz	10 kHz	15 kHz	20 kHz
0	90.55	87.57	83.99	80.66
1	63.80	49.02	38.52	30.21
2	52.07	37.06	26.51	18.19
3	45.08	30.03	19.47	11.15
4	40.10	25.04	14.47	6.15
5	36.24	21.17	10.60	2.28

Figure 3 shows the transmission of the mechanical wave through structures with a single PLA layer of different thicknesses, where the number of repeats of the B layer determines the multiplier of the layer thickness. The transmission of the AAAAA structure has a reference character, where the transmission is equal to 100%, and the shape of the sound pressure curve results only from the shape of the correction curve A. As the thickness of the PLA layer increases, the sound pressure level decreases, which is shown in Table I for frequencies 5, 10, 15, and 20 kHz.

Figure 4 shows the sound pressure level as a function of frequency for structures in which there are two layers of the same thickness made of PLA with a changing distance between them. For a 1 cm distance between B layers, a single A layer had a peak for a frequency of 7582 Hz. Increasing the distance between the layers to 2 cm shifted the peak to 5341 Hz, and another increase in the distance between the layers lowered the peak frequency to 4356 Hz.

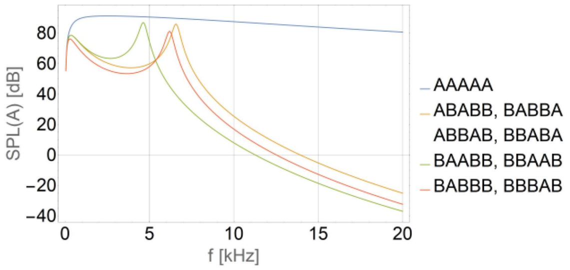

Fig. 5. The sound pressure level as a function of frequency for structures with two different thickness of PLA layers and increasing distance between them and with reference transmission (no PLA layer).

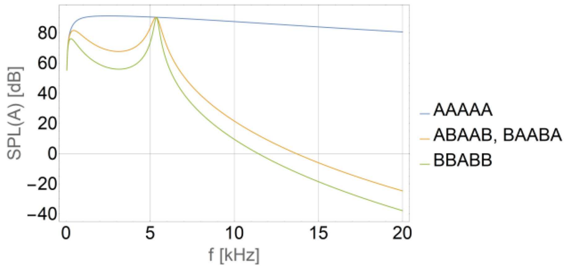

Fig. 6. The sound pressure level as a function of frequency with a simultaneous increase in the thickness of the layers and reduction of the distance between them.

The increase in the distance between layers of different thicknesses shown in Fig. 5 shifts the transmission peak towards lower frequencies, in a manner similar to that shown in Fig. 4 for layers with the same thicknesses. For the BABBB structure, as compared to the BABBA structure (Fig. 5), there was a slight shift of the frequency peak, and the increase in the thickness of the second PLA layer resulted in a decrease in the transmission value.

The simultaneous increase in the thickness of both layers in Fig. 6, while reducing the distance between them, decreased the transmission in the entire analyzed frequency range with a minimal shift of the transmission peak towards higher frequencies.

4. Conclusions

As part of the research, the properties of multilayer acoustic partitions modeled with the use of the TMM algorithm and made of PLA and air were analyzed.

It was shown that increasing the thickness of a single PLA layer decreased the transmission of the mechanical wave, but did not result in the occurrence of the audio band gap. Adding another layer created an additional peak in the transmission spectrum, and increasing the distance between the layers shifted the peak in the spectrum towards lower frequencies. The successive increase in the number of layers in the structure resulted in the appearance of successive peaks in the analyzed frequency range of the mechanical wave.

The observed 30 dB SPL reduction occurring in the selected frequency bands in the analyzed multilayer structures means that there is 99.9% of the reduction emitted by the energy source and indicates the presence of the band gap.

References

[1] R. Martínez-Sala, J. Sancho, J. V. Sánchez, V. Gómez, J. Llinares, F. Meseguer, *Nature* **378**, 6554 (1995).

[2] M. S. Kushwaha, P. Halevi, L. Dobrzynski, B. Djafari-Rouhani, *Phys. Rev. Lett.* **71**, 2022 (1993).

[3] Y. Tanaka, Y. Tomoyasu, S. Tamura, *Phys. Rev. B* **62**, 7387 (2000).

[4] Jin-Chen Hsu, Tsung-Tsong Wu, *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **53**, 1169 (2006).

[5] G. Wang, X. Wen, J. Wen, L. Shao, Y. Liu, *Phys. Rev. Lett.* **93**, 154302 (2004).

[6] S. Benchabane, A. Khelif, J.-Y. Rauch, L. Robert, V. Laude, *Phys. Rev. E* **73**, 065601 (2006).

[7] S. Garus, M. Szota, *Rev. Chim.* **69**, 735 (2018).

[8] S. Garus, W. Sochacki, *J. Appl. Math. Comput. Mech.* **16**, 17 (2017).

[9] J. Wang, G. Dai, J. Huang, *Science* **23**, 101637 (2020).

[10] C.J. Rupp, M.L. Dunn, K. Maute, *Appl. Phys. Lett.* **96**, 111902 (2010).

[11] Y. Pennec, B. Djafari-Rouhani, J.O. Vasseur, A. Khelif, P.A. Deymier, *Phys. Rev. E* **69**, 046608 (2004).

[12] M. Kafesaki, M.M. Sigalas, N. García, *Phys. Rev. Lett.* **85**, 4044 (2000).

[13] I. Kriegel, F. Scotognella, *Physica E Low Dimens. Syst. Nanostruct.* **85**, 34 (2017).

[14] S. Villa-Arango, R. Torres, P.A. Kyriacou, R. Lucklum, *Measurement* **102**, 20 (2017).

[15] X. Zhang, Y. Li, Y. Wang, Z. Jia, Y. Luo, *Int. J. Mech. Sci.* **212**, 106829 (2021).

[16] X. Zhang, Z. Liu, *Nat. Mater.* **7**, 435 (2008).

[17] R.H. Olsson, I.F. El-Kady, M.F. Su, M.R. Tuck, J.G. Fleming, *Sens. Actuators A Phys.* **145–146**, 87 (2008).

[18] L. Pomot, C. Payan, M. Remillieux, S. Guenneau, *Wave Motion* **92**, 102413 (2020).

[19] F. Cervera, L. Sanchis, J. V. Sśnchez-Pérez, R. Martínez-Sala, C. Rubio, F. Meseguer, C. López, D. Caballero, and J. Sánchez-Dehesa, *Phys. Rev. Lett.* **88**, 023902 (2001).

[20] C.G. Parazzoli, R.B. Greegor, J.A. Nielsen, M.A. Thompson, K. Li, A.M. Vetter, M.H. Tanielian, D.C. Vier, *Appl. Phys. Lett.* **84**, 3232 (2004).

[21] X. Hu, C.T. Chan, *Phys. Rev. Lett.* **95**, 154501 (2005).

[22] S. Huang, L. Peng, H. Sun, Q. Wang, W. Zhao, S. Wang, *Appl. Acoust.* **173**, 107692 (2021).

[23] X. Su, A.N. Norris, C.W. Cushing, M.R. Haberman, P.S. Wilson, *J. Acoust. Soc. Am.* **141**, 4408 (2017).

[24] J. Wen, D. Yu, L. Cai, X. Wen, *J. Phys. D Appl. Phys.* **42**, 115417 (2009).

[25] B. Morvan, A. Tinel, J.O. Vasseur, R. Sainidou, P. Rembert, A.-C. Hladky-Hennion, N. Swinteck, P.A. Deymier, *J. Appl. Phys.* **116**, 214901 (2014).

[26] A. Khelif, A. Choujaa, S. Benchabane, B. Djafari-Rouhani, V. Laude, *Appl. Phys. Lett.* **84**, 4400 (2004).

[27] S. Garus, *Rev. Chim.* **70**, 3671 (2019).

[28] X.-F. Li, X. Ni, L. Feng, M.-H. Lu, C. He, Y.-F. Chen, *Phys. Rev. Lett.* **106**, 084301 (2011).

[29] D. Richards, D.J. Pines, *J. Sound Vib.* **264**, 317 (2003).

[30] J.V. Sanchez-Perez, C. Rubio, R. Martinez-Sala, R. Sanchez-Grandia, V. Gomez, *Appl. Phys. Lett.* **81**, 5240 (2002).

[31] S.-H. Jo, H. Yoon, Y. C. Shin, B.D. Youn, *Int. J. Mech. Sci.* **183**, 105833 (2020).

[32] Z. Wen, Y. Jin, P. Gao, X. Zhuang, T. Rabczuk, B. Djafari-Rouhani, *Mech. Syst. Signal Process.* **162**, 108047 (2022).

[33] S.-H. Jo, H. Yoon, Y.C. Shin, B.D. Youn, *Int. J. Mech. Sci.* **193**, 106160 (2021).

[34] J. Madrigal-Melchor, A. Enciso-Muñoz, D.A. Contreras-Solorio, X. Saldaña-Saldaña, R.A. Reyes-Villagrana, *Open J. Acoust.* **07**, 39 (2017).

[35] P. Yeh, *Optical Waves in Layered Media*, Wiley, Hoboken (NJ) 2005.

[36] W.C. Elmore, M.A. Heald, *Physics of Waves*, Dover Publications, New York 1985.

[37] L.M. Brekhovskikh, *Waves in Layered Media*, Academic Press, 1980.

[38] D. Tarrazó-Serrano, S. Castiñeira-Ibáñez, E. Sánchez-Aparisi, A. Uris, C. Rubio, *Appl. Sci.* **8**, 2634 (2018).

[39] S. Yang, W.-D. Yu, N. Pan, *Physica B Condens. Matter* **406**, 963 (2011).

[40] Y. Wang, W. Song, E. Sun, R. Zhang, W. Cao, *Physica E Low Dimens. Syst. Nanostruct.* **60**, 37 (2014).