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Transversal Vibrations Control and Load Bearing Capacity Enhancement of Beam System Using Smart Materials

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In the presented paper theoretical and numerical investigations of a coupled slender electro-mechanical system are demonstrated. The object of study comprises a core beam with both ends preventing natural longitudinal displacements and a "smart" piezoceramic material perfectly bonded to the beam's top and bottom surface. The problem is formulated upon Hamilton's principle, and the slenderness of the system classifies it into the Euler–Bernoulli beam theory. In order to study the system buckling behavior, the in-plane prescribed displacement of one of the supports is specified. Moreover, transversal vibrations related to the system's rectilinear shape are investigated. Depending on the actuator's poling direction, once the electric field is applied, based on its vector direction, it is possible to induce axial piezo force or force the system to bend. It is commonly used in precision engineering, especially in micropositioning, but also in shape and vibrations control and stability enhancement. In this work, the influence of the in-plane stress induction on system buckling behavior and transversal vibration control is studied in detail. Obtained numerical results show that the induced in-plane stress allows to control the system's vibrations in a significant manner. Moreover, by forcing the smart materials to induce the axial tensile force in the system, one may considerably increase buckling load.

topics: sandwich beam, buckling, dynamic response modification, piezoelectric actuation

1. Introduction

Since the fall of the 20th century, theoretical, numerical, and experimental investigations concerning the adjustment of static and dynamic response in slender beam and column structures using smart materials (mainly shape memory alloys and piezoelectric actuators) have been the subject of interest for many researchers. One of the fundamental works presenting the influence of piezoelectric (PZT) actuation on static and dynamic response in a slender pinned-pinned system are these presented by de Faria [1] as well as Zehetner and Irschik [2]. The presented results and discussion lead to a statement that the PZT actuation may be efficiently used in systems with both ends preventing longitudinal displacements, in order to enhance their critical load, modify transversal vibrations frequency, as well as revert the rectilinear shape under applied external load. Srinivasan et al. [3] presented experimental investigations concerning the application of shape memory alloy (SMA) actuators on a cantilever beam in order to control its vibration frequency. Stability and linear and nonlinear transversal vibration control via piezoelectric actuation in a two-member column with a spring subjected to Euler's load was presented in [4]. It was shown that the location of the spring connection and its stiffness have a crucial influence on the system stability and vibrations, however, induction of residual force makes it possible to change the magnitude of maximum load and control natural and nonlinear vibration frequency.

The main purpose of this work is to show a theoretical and numerical approach concerning the active/passive enhancement of critical load and modification of natural frequency in an inhomogeneous clamped-pinned Euler–Bernoulli beam, where both ends are restrained against longitudinal displacement. The inhomogeneity of the studied system results from the PZT material perfectly bonded to the top and bottom surface of an elastic, passive core aluminum alloy layer. As soon as an electric field is applied to the PZT material, depending on the polarization of patches and the electric field vector, the material is forced to expand or contract. Hence, in-plane stress in the system is generated, which in fact has a crucial influence on modifying the system's critical force and natural vibration frequency.

2. Physical model

The physical model of the studied system is presented in Fig. 1. Buckling load is achieved via one of the supports prescribed displacement δ. The magnitude of force P may then be easily determined from

Hooke's law. Before the load P is applied, the beam has an ideally rectilinear shape. Moreover, geometrical and physical imperfections are neglected. The slenderness of the system allows the application the Euler–Bernoulli beam theory, which was applied in many studies presented inter-alia in [5]. The PZT layers are bonded symmetrically in reference to the supporting bonds, and it is assumed that the electric field is evenly distributed along the piezosegment. The first and the third segment are identical in terms of mechanical and physical properties. The width B of piezo elements is exactly the same as the width of the core passive layer. The effect of piezo layers' delamination and the thickness of the adhesive layer between the core and actuators are not taken into consideration.

3. Mathematical model

In order to describe the problem of transversal vibrations in the three-segmented system, the following system of equations, which was formerly derived on the basis of Hamilton's principle for n-segmented beam in [6], is used

$$E_iJ_i\frac{\partial^4 W_i(x_i,t)}{\partial x_i^4}-E_iA_i\frac{\partial}{\partial x_i}\left\{\left[\frac{\partial U_i(x_i,t)}{\partial x_i}+\frac{1}{2}\left(\frac{\partial W_i(x_i,t)}{\partial x_i}\right)^2\right]\frac{\partial W_i(x_i,t)}{\partial x_i}\right\}\pm(P_u+F_r)\frac{\partial^2 W_i(x_i,t)}{\partial x_i^2}+\rho_iA_i\frac{\partial^2 W_i(x_i,t)}{\partial t^2}=0, \tag{1}$$

$$\frac{\partial}{\partial x_i}\left[\frac{\partial U_i(x_i,t)}{\partial x_i}+\frac{1}{2}\left(\frac{\partial W_i(x_i,t)}{\partial x_i}\right)^2\right]=0, \quad (i=1,2,3), \tag{2}$$

where E_i denotes the Young's moduli of i-th segment [N/m^2], J_i is the moment of inertia [m^4], A_i — cross-section area [m^2], P_u — axial force from the prescribed support displacement [N], F_r — axial residual force induced by PZT actuation [N], and ρ_i — mass density [N/m^3].

The axial residual force for the three-segmented system on the basis of [6] may be expressed as

$$F_r=\frac{2b\,e_{31}V}{1+\eta\left(\frac{L}{L_2}-1\right)}, \quad \eta=\frac{E_bA_b+E_pA_p}{E_bA_b}, \tag{3}$$

where b stands as piezoceramic width [m], e_{31} is the piezoelectric constant [C/m^2], V — controlled voltage [V], L — total beam length [m], L_2 — piezosegment length [m], "b" subscript denotes core beam element, and "p" subscript corresponds to the PZT element.

In the considered system, the total axial force can be expressed in the form of two components, therefore, the notation of (1) describing the transverse vibrations of i-th segment can be simplified to the following differential equation

$$E_iJ_i\frac{\partial^4 W_i(x_i,t)}{\partial x_i^4}\pm P\frac{\partial^2 W_i(x_i,t)}{\partial x_i^2}+\rho_iA_i\frac{\partial^2 W_i(x_i,t)}{\partial t^2}=0, \tag{4}$$

where P is the sum of prescribed support displacement P_u [N] and induced residual force F_r [N].

The sign for the load P in (4) depends on the direction of support displacement δ and the induced residual force F_r, which depends on the electric field vector direction. A system of geometrical and natural boundary conditions, including continuity conditions between segments, for $i = 1,\ 2$, is

$$W_1(x_1,t)\big|_{x_1=0}=W_3(x_3,t)\big|_{x_3=L_3}=0, \tag{5}$$

$$\left.\frac{\partial W_1(x_1,t)}{\partial x_1}\right|_{x_1=0}=\left.\frac{\partial^2 W_3(x_3,t)}{\partial x_3^2}\right|_{x_3=L_3}=0, \tag{6}$$

$$W_i(x_i,t)\big|_{x_i=L_i}=W_{i+1}(x_{i+1},t)\big|_{x_{i+1}=0}, \tag{7}$$

$$\left.\frac{\partial W_i(x_i,t)}{\partial x_i}\right|_{x_i=L_i}=\left.\frac{\partial W_{i+1}(x_{i+1},t)}{\partial x_{i+1}}\right|_{x_{i+1}=0}, \tag{8}$$

$$E_iJ_i\left.\frac{\partial^2 W_i(x_i,t)}{\partial x_i^2}\right|_{x_i=L_i}=E_{i+1}J_{i+1}\left.\frac{\partial^2 W_{i+1}(x_{i+1},t)}{\partial x_{i+1}^2}\right|_{x_{i+1}=0}, \tag{9}$$

$$E_iJ_i\left.\frac{\partial^3 W_i(x_i,t)}{\partial x_i^3}\right|_{x_i=L_i}=E_{i+1}J_{i+1}\left.\frac{\partial^3 W_{i+1}(x_{i+1},t)}{\partial x_{i+1}^3}\right|_{x_{i+1}=0}. \tag{10}$$

Due to the geometrical non-linearity occurring in the system, approximate solutions concerning critical load enhancement and modification of natural vibration frequency are obtained using the modified Lindstedt–Poincaré method (see [7]), which belongs to the perturbation techniques [8].

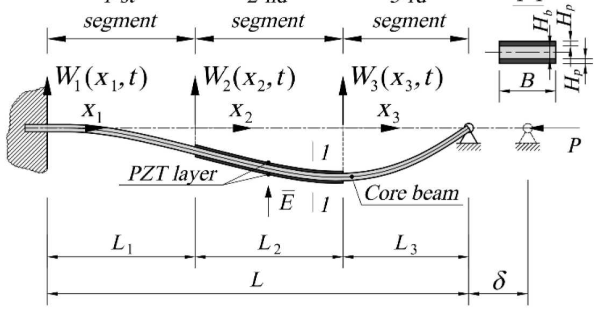

Fig. 1. Scheme of three-segmented beam with piezo patches located symmetrically between beam's ends.

4. Numerical simulations

Results in this section are presented using non-dimensional form, taking into account the following substitutions

$$l_i = \frac{L_i}{L}, \quad p_u = \sqrt{\frac{P_u L^2}{E_b I_b}}, \tag{11}$$

$$f_r = \pm\sqrt{\frac{F_r L^2}{E_b I_b}}, \quad f = \sqrt{\frac{2be_{31} V L^2}{E_b I_b}}, \tag{12}$$

$$\omega = \sqrt{\frac{\Omega^2 L^4 \rho_b A_b}{E_b I_b}}. \tag{13}$$

Geometry of a system is related to the total length of the beam, namely $b = B/L = 0.05$, $h_b = H_b/L = 0.0075$, $h_p = H_p/L = 0.00125$. For the core beam, an aluminum alloy is adopted, for which the Young's modulus is equal $E_b = 70.0$ GPa and $\rho_b = 2720$ kg/m^3. Two different PZT actuator materials are taken into account. According to the manufacturer data, the P-41 material [9] is described with following constants: $E_p = 83.33$ GPa, $\rho_p = 7450$ kg/m^3, piezoelectric constant necessary to determine the piezoelectric force ($F = 2be_{31}V$) is equal $d_{31} = 1.00 \times 10^{-10}$ C/N and maximum admissible electric field before material depolarization is $E_{\max} = 2.0$ kV/mm. The second studied material is NEC46 [10], for which $E_p = 76.923$ GPa, $\rho_p = 7700$ kg/m^3, $d_{31} = 1.30 \times 10^{-10}$ C/N, and $E_{\max} = 5.5$ kV/mm. The influence of piezosegment length on the non-dimensional first natural frequency ω_0 versus external load p_u is presented in Fig. 2. In Fig. 3 external load and first natural vibration frequency control via maximum admissible piezoelectric actuation is shown for two piezosegment lengths $l_2 = 0.25$ and 0.75.

Piezoelectric force f for both actuators is related to the critical load of a pinned-pinned beam ($p = \pi$). Hence, for the P-41 material, it obtains $f_{\max} = \pm 1.310\pi$, whereas for NEC-46, $f_{\max} = \pm 2.379\pi$. It should be noted that the residual force induced in the system f_r depends not only on the piezoelectric force f but also on the relation of axial and bending stiffness and the length of the piezosegment (see (3)).

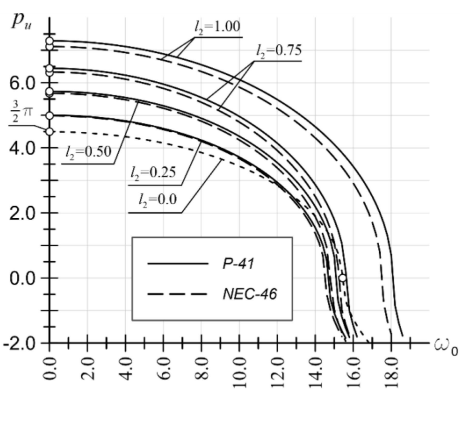

Fig. 2. The influence of piezosegment length on the non-dimensional first natural frequency ω_0 versus external load p_u in the nonactuated system $f = 0$.

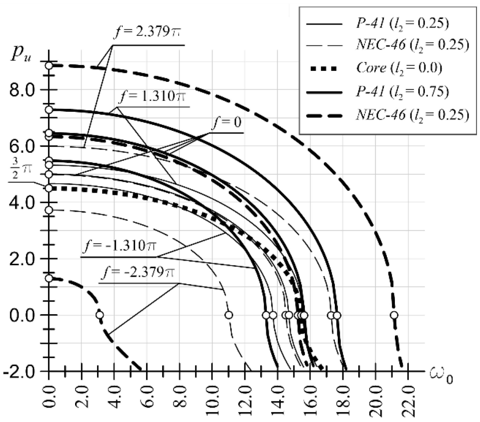

Fig. 3. Modification of non-dimensional first natural vibration frequency ω_0 versus external load p_u under piezoelectric actuation.

On the basis of the presented results, one can state that for the P-41 actuators, one obtains higher values in the p_u–ω_0 relationship in comparison to the NEC-46. The first natural vibration frequency ω_0 depends on the relation of piezosegment bending stiffness and the mass distribution per unit length of the beam. Hence, an irregular course of curves is obtained, when for the core beam $l_2 = 0$ at $p_u = 0$, the natural frequency is higher than in the studied systems (for P-41 — $l_2 < 0.75$ and for NEC-46 — $l_2 < 1.0$). When the maximum admissible voltage is applied to the actuators (see Fig. 3), it is clearly visible that the greater its length, the higher the range of possible critical load and vibrations frequency adjustment. By inducing in the system tensile residual force for the P-41 material at $l_2 = 0.75$, one can increase the critical load by 13.06%, whereas for NEC-46 by 39.93%, in regard to the non-actuated system. In the case of induction of tensile residual force in the system for $l_2 = 0.75$ with NEC-46 actuators, the ω_0 may be adjusted by 39.08%, whereas for a compressive residual force, ω_0 may be altered by 79.35%, in regard to the non-actuated system. For the identical length of P-41 actuators, one gets a 12.83% change for tensile residual force and 14.84% for the compressive one.

5. Conclusions

The numerical analysis performed in this paper shows that the geometrical and physical parameters of piezoceramic actuators have a crucial influence on the p_u–ω_0 course of curves and the resulting residual force induced in the system. The higher the piezosegment length, d_{31} constant, and admissible electric field, the greater the range of p_u–ω_0 relation adjustment. Hence, it has been shown that the piezoelectric actuation may be used as an efficient tool for the adjustment of critical load as well as natural vibrations frequency in slender beam systems.

References

[1] A.R. de Faria, *Comp. Struct.* **65**, 187 (2004).

[2] C. Zehetner, H. Irschik, *Smart Struct. and Sys.* **4**, 67 (2008).

[3] A.V. Srinivasan, D.M. Mcfarland, H.A. Canistraro, E.K. Begg, *J. Int. Mat. Sys. and Struct.* **8**, 202 (1997).

[4] K. Sokół, S. Uzny, *J. Appl. Math.* **10**, 2267 (2016).

[5] S. Uzny, K. Sokół, M. Osadnik, in: *22nd Int. Conf. Engineering Mechanics 2016*, 2016, p. 595.

[6] J. Przybylski, G. Gąsiorski, *J. Sound Vib.* **437** 150 (2018).

[7] K. Kuliński, J. Przybylski, *Mech. Res. Com.* **82**, 43 (2017).

[8] A.H. Nayfeh, *Perturbation methods*, Wiley & Sons, Wienheim 2008.

[9] Annon Piezo (date accessed: 4-th of April 2022).

[10] Noliac (date accessed: 4-th of April 2022).